3 Graphing Linear Functions

3.1 Functions
3.2 Linear Functions
3.3 Function Notation
3.4 Graphing Linear Equations in Standard Form
3.5 Graphing Linear Equations in Slope-Intercept Form
3.6 Modeling Direct Variation
3.7 Transformations of Graphs of Linear Functions

Mathematical Thinking: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
Plotting Points (6.11)

Example 1  Plot the point \(A(-3, 4)\) in a coordinate plane. Describe the location of the point.

Start at the origin. Move 3 units left and 4 units up. Then plot the point.
The point is in Quadrant II.

Plot the point in a coordinate plane. Describe the location of the point.

1. \(A(3, 2)\)  
2. \(B(-5, 1)\)  
3. \(C(0, 3)\)  
4. \(D(-1, -4)\)  
5. \(E(-3, 0)\)  
6. \(F(2, -1)\)

Evaluating Expressions (6.3.D)

Example 2  Evaluate \(4x - 5\) when \(x = 3\).

\[
4x - 5 = 4(3) - 5 \quad \text{Substitute 3 for } x.
\]
\[
= 12 - 5 \quad \text{Multiply.}
\]
\[
= 7 \quad \text{Subtract.}
\]

Example 3  Evaluate \(-2x + 9\) when \(x = -8\).

\[
-2x + 9 = -2(-8) + 9 \quad \text{Substitute } -8 \text{ for } x.
\]
\[
= 16 + 9 \quad \text{Multiply.}
\]
\[
= 25 \quad \text{Add.}
\]

Evaluate the expression for the given value of \(x\).

7. \(3x - 4; x = 7\)  
8. \(-5x + 8; x = 3\)  
9. \(10x + 18; x = 5\)  
10. \(-9x - 2; x = -4\)  
11. \(24 - 8x; x = -2\)  
12. \(15x + 9; x = -1\)  
13. **ABSTRACT REASONING** Let \(a\) and \(b\) be positive real numbers. Describe how to plot \((a, b), (-a, b), (a, -b),\) and \((-a, -b)\).
Using a Graphing Calculator

**Core Concept**

**Standard and Square Viewing Windows**
A typical graphing calculator screen has a height to width ratio of 2 to 3. This means that when you use the standard viewing window of $-10$ to $10$ (on each axis), the graph will not be in its true perspective.

To see a graph in its true perspective, you need to use a square viewing window, in which the tick marks on the $x$-axis are spaced the same as the tick marks on the $y$-axis.

**Using a Graphing Calculator**

Use a graphing calculator to graph $y = 2x + 5$.

**SOLUTION**

Enter the equation $y = 2x + 5$ into your calculator. Then graph the equation. The standard viewing window does not show the graph in its true perspective. Notice that the tick marks on the $y$-axis are closer together than the tick marks on the $x$-axis. To see the graph in its true perspective, use a square viewing window.

**Monitoring Progress**

Determine whether the viewing window is square. Explain.

1. $-8 \leq x \leq 7$, $-3 \leq y \leq 7$
2. $-6 \leq x \leq 6$, $-9 \leq y \leq 9$
3. $-18 \leq x \leq 18$, $-12 \leq y \leq 12$

Use a graphing calculator to graph the equation. Use a square viewing window.

4. $y = x + 3$
5. $y = -x - 2$
6. $y = 2x - 1$
7. $y = -2x + 1$
8. $y = -\frac{1}{3}x - 4$
9. $y = \frac{1}{2}x + 2$

10. How does the appearance of the slope of a line change between a standard viewing window and a square viewing window?
Essential Question  What is a function?

A relation pairs inputs with outputs. When a relation is given as ordered pairs, the $x$-coordinates are inputs and the $y$-coordinates are outputs. A relation that pairs each input with exactly one output is a function.

### Describing a Function

Work with a partner. Functions can be described in many ways.

- by an equation
- by an input-output table
- using words
- by a graph
- as a set of ordered pairs

**a.** Explain why the graph shown represents a function.

**b.** Describe the function in two other ways.

### Identifying Functions

Work with a partner. Determine whether each relation represents a function. Explain your reasoning.

**a.**

<table>
<thead>
<tr>
<th>Input, $x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, $y$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

**b.**

<table>
<thead>
<tr>
<th>Input, $x$</th>
<th>8</th>
<th>8</th>
<th>8</th>
<th>8</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, $y$</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**c.**

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

**d.**

**e.** $(−2, 5), (−1, 8), (0, 6), (1, 6), (2, 7)$

**f.** $(−2, 0), (−1, 0), (−1, 1), (0, 1), (1, 2), (2, 2)$

**g.** Each radio frequency $x$ in a listening area has exactly one radio station $y$.

**h.** The same television station $x$ can be found on more than one channel $y$.

**i.** $x = 2$

**j.** $y = 2x + 3$

Communicate Your Answer

3. What is a function? Give examples of relations, other than those in Explorations 1 and 2, that (a) are functions and (b) are not functions.
What You Will Learn

- Determine whether relations are functions.
- Find the domain and range of a function.
- Identify the independent and dependent variables of functions.

Determining Whether Relations Are Functions

A relation pairs inputs with outputs. When a relation is given as ordered pairs, the \( x \)-coordinates are inputs and the \( y \)-coordinates are outputs. A relation that pairs each input with exactly one output is a function.

**EXAMPLE 1**  Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

a. \((-2, 2), (-1, 2), (0, 2), (1, 0), (2, 0)\)

b. \((4, 0), (8, 7), (6, 4), (4, 3), (5, 2)\)

c. 

<table>
<thead>
<tr>
<th>Input, ( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, ( y )</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

d. \(\begin{array}{c}
-1 \\
3 \\
11 \\
\end{array} \rightarrow \begin{array}{c}
4 \\
15 \\
\end{array}\)

**SOLUTION**

a. Every input has exactly one output. So, the relation is a function.

b. The input 4 has two outputs, 0 and 3. So, the relation is \textit{not} a function.

c. The input 0 has two outputs, 5 and 6. So, the relation is \textit{not} a function.

d. Every input has exactly one output. So, the relation is a function.

**EXAMPLE 2**  Determining Whether Relations Are Functions

Determine whether each relation is a function. Explain.

a. \(y = 5x\) with inputs \(x = 1, x = 2, x = 3\)

b. \(x = y^2\) with inputs \(x = 0\) and \(x = 1\)

**SOLUTION**

a. The input 1 has exactly one output, 5. The input 2 has exactly one output, 10. The input 3 has exactly one output, 15.

So, the relation is a function.

b. The input 0 has exactly one output, 0, but the input 1 has two outputs, 1 and \(-1\).

So, the relation is \textit{not} a function.
Core Concept

Vertical Line Test

Words  A graph represents a function when no vertical line passes through more than one point on the graph.

Examples  

<table>
<thead>
<tr>
<th>Function</th>
<th>Not a function</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="example1.png" alt="Graph" /></td>
<td><img src="example2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Using the Vertical Line Test

Determine whether each graph represents a function. Explain.

a. ![Graph](example3a.png)

SOLUTION

a. You can draw a vertical line through (2, 2) and (2, 5).

So, the graph does not represent a function.

b. ![Graph](example3b.png)

So, the graph represents a function.

Monitoring Progress

Determine whether the graph represents a function. Explain.

1. (−5, 0), (0, 0), (5, 0), (5, 10)
2. (−4, 8), (−1, 2), (2, −4), (5, −10)
3. | Input, x | 2 | 4 | 6 |
   | Output, y | 2.6 | 5.2 | 7.8 |
4. \(y = 2x + 4\) with inputs \(x = 2\) and \(x = 4\)

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Determine whether the relation is a function. Explain.

1. \((-5, 0), (0, 0), (5, 0), (5, 10)\)
2. \((-4, 8), (-1, 2), (2, -4), (5, -10)\)
3. | Input, x | 2 | 4 | 6 |
   | Output, y | 2.6 | 5.2 | 7.8 |
4. \(y = 2x + 4\) with inputs \(x = 2\) and \(x = 4\)
Finding the Domain and Range of a Function

Core Concept

The Domain and Range of a Function
The domain of a function is the set of all possible input values.
The range of a function is the set of all possible output values.

EXAMPLE 4 Finding the Domain and Range from a Graph

Find the domain and range of the function represented by the graph.

a. Find the domain and range of the function represented by the graph.

b. Identify the x- and y-values represented by the graph.

SOLUTION

a. Write the ordered pairs. Identify the inputs and outputs.

(-3, -2), (-1, 0), (1, 2), (3, 4)

The domain is -3, -1, 1, and 3.
The range is -2, 0, 2, and 4.

b. The domain is \(-2 \leq x \leq 3\).
The range is \(-1 \leq y \leq 2\).

STUDY TIP
A relation also has a domain and a range.

Monitoring Progress

Find the domain and range of the function represented by the graph.

7. 

8. 

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Identifying Independent and Dependent Variables

The variable that represents the input values of a function is the **independent variable** because it can be *any* value in the domain. The variable that represents the output values of a function is the **dependent variable** because it *depends* on the value of the independent variable. When an equation represents a function, the dependent variable is defined in terms of the independent variable. The statement “*y* is a function of *x*” means that *y* varies depending on the value of *x*.

\[ y = -x + 10 \]

**Example 5**  
**Identifying Independent and Dependent Variables**

The function \( y = -3x + 12 \) represents the amount \( y \) (in fluid ounces) of juice remaining in a bottle after you take \( x \) gulps.

**a.** Identify the independent and dependent variables.

**b.** The domain is 0, 1, 2, 3, and 4. What is the range?

**Solution**

**a.** The amount \( y \) of juice remaining depends on the number \( x \) of gulps.

So, \( y \) is the dependent variable, and \( x \) is the independent variable.

**b.** Make an input-output table to find the range.

<table>
<thead>
<tr>
<th>Input, ( x )</th>
<th>(-3x + 12)</th>
<th>Output, ( y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(-3(0) + 12)</td>
<td>12</td>
</tr>
<tr>
<td>1</td>
<td>(-3(1) + 12)</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>(-3(2) + 12)</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>(-3(3) + 12)</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>(-3(4) + 12)</td>
<td>0</td>
</tr>
</tbody>
</table>

The range is 12, 9, 6, 3, and 0.

**Monitoring Progress**

9. The function \( a = -4b + 14 \) represents the number \( a \) of avocados you have left after making \( b \) batches of guacamole.

**a.** Identify the independent and dependent variables.

**b.** The domain is 0, 1, 2, and 3. What is the range?

10. The function \( t = 19m + 65 \) represents the temperature \( t \) (in degrees Fahrenheit) of an oven after preheating for \( m \) minutes.

**a.** Identify the independent and dependent variables.

**b.** A recipe calls for an oven temperature of 350°F. Describe the domain and range of the function.
In Exercises 3–8, determine whether the relation is a function. Explain. (See Example 1.)

3. \((1, -2), (2, 1), (3, 6), (4, 13), (5, 22)\)
4. \((7, 4), (5, -1), (3, -8), (1, -5), (3, 6)\)
5. Input, \(x\) | Output, \(y\)
   | 0 | -3 |
   | 1 | 0  |
   | 2 | 3  |
   | 3 | 2  |

6. Input, \(x\) | Output, \(y\)
   | -10 | 2  |
   | -8  | -6 |
   | -6  | -4 |
   | -4  | -2 |

7. Input, \(x\) | 16 | 1 | 0 | 1 | 16
   Output, \(y\) | -2 | -1 | 0 | 1 | 2

8. Input, \(x\) | -3 | 0 | 3 | 6 | 9
   Output, \(y\) | 11 | 5 | -1 | -7 | -13

In Exercises 9–12, determine whether the relation is a function. Explain. (See Example 2.)

9. \(y = -3x\) with inputs \(x = 0, x = 1, \) and \(x = 2\)
10. \(y = 4x - 1\) with inputs \(x = -3, x = -2, \) and \(x = -1\)
11. \(x = y^2\) with inputs \(x = 4\) and \(x = 9\)
12. \(x = |y|\) with inputs \(x = 1, x = 2, \) and \(x = 3\)

In Exercises 13–16, determine whether the graph represents a function. Explain. (See Example 3.)

13. 

14. 

15. 

16. 

In Exercises 17–20, find the domain and range of the function represented by the graph. (See Example 4.)

17. 

18. 

19. 

20.
21. **MODELING WITH MATHEMATICS** The function 
\[ y = 25x + 500 \]
represents your monthly rent \( y \) (in dollars) when you pay \( x \) days late. 
*(See Example 5.)*

- a. Identify the independent and dependent variables.
- b. The domain is 0, 1, 2, 3, 4, and 5. What is the range?

22. **MODELING WITH MATHEMATICS** The function 
\[ y = 3.5x + 2.8 \]
represents the cost \( y \) (in dollars) of a taxi ride of \( x \) miles.

- a. Identify the independent and dependent variables.
- b. You have enough money to travel at most 20 miles in the taxi. Find the domain and range of the function.

**ERROR ANALYSIS** In Exercises 23 and 24, describe and correct the error in the statement about the relation shown in the table.

<table>
<thead>
<tr>
<th>Input, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output, ( y )</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

23. **✗** The relation is not a function. One output is paired with two inputs.

24. **✗** The relation is a function. The range is 1, 2, 3, 4, and 5.

**ANALYZING RELATIONSHIPS** In Exercises 25–28, identify the independent and dependent variables.

25. The length of your hair depends on the amount of time since your last haircut.

26. A baseball team’s rank depends on the number of games the team wins.

27. The number of quarters you put into a parking meter affects the amount of time you have on the meter.

28. The battery power remaining on your MP3 player is based on the amount of time you listen to it.

29. **MULTIPLE REPRESENTATIONS** The balance \( y \) (in dollars) of your savings account is a function of the month \( x \).

<table>
<thead>
<tr>
<th>Month, ( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance (dollars), ( y )</td>
<td>100</td>
<td>125</td>
<td>150</td>
<td>175</td>
<td>200</td>
</tr>
</tbody>
</table>

- a. Describe this situation in words.
- b. Write the function as a set of ordered pairs.
- c. Plot the ordered pairs in a coordinate plane.

30. **MULTIPLE REPRESENTATIONS** The function 
\[ 1.5x + 0.5y = 12 \]
represents the number of hardcover books \( x \) and softcover books \( y \) you can buy at a used book sale.

- a. Solve the equation for \( y \).
- b. Make an input-output table to find ordered pairs for the function.
- c. Plot the ordered pairs in a coordinate plane.

31. **OPEN-ENDED** Graph a relation that fails the Vertical Line Test at exactly one value of \( x \).

32. **OPEN-ENDED** List the inputs and outputs of a relation such that the relation is a function, but when the inputs and outputs are switched, the relation is not a function.

33. **ANALYZING RELATIONSHIPS** You select items in a vending machine by pressing one letter and then one number.

- a. Explain why the relation that pairs letter-number combinations with food or drink items is a function.
- b. Identify the independent and dependent variables.
- c. Find the domain and range of the function.
34. **HOW DO YOU SEE IT?** The graph represents the height \( h \) of a projectile after \( t \) seconds.

![Graph of a projectile's height](image)

- a. Explain why \( h \) is a function of \( t \).
- b. Approximate the height of the projectile after 0.5 second and after 1.25 seconds.
- c. Approximate the domain of the function.
- d. Is \( t \) a function of \( h \)? Explain.

35. **MAKING AN ARGUMENT** Your friend says that a line always represents a function. Is your friend correct? Explain.

36. **THOUGHT PROVOKING** Write a function in which the inputs and/or the outputs are not numbers. Identify the independent and dependent variables. Then find the domain and range of the function.

37. The selling price of an item is a function of the cost of making the item.

38. The sales tax on a purchased item in a given state is a function of the selling price.

39. A function pairs each student in your school with a homeroom teacher.

40. A function pairs each chaperone on a school trip with 10 students.

**REASONING** In Exercises 41–44, tell whether the statement is true or false. If it is false, explain why.

- 41. Every function is a relation.
- 42. Every relation is a function.
- 43. When you switch the inputs and outputs of any function, the resulting relation is a function.
- 44. When the domain of a function has an infinite number of values, the range always has an infinite number of values.

45. **MATHEMATICAL CONNECTIONS** Consider the triangle shown.

![Triangle](image)

- a. Write a function that represents the perimeter of the triangle.
- b. Identify the independent and dependent variables.
- c. Describe the domain and range of the function. (Hint: The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.)

**REASONING** In Exercises 46–49, find the domain and range of the function.

- 46. \( y = |x| \)
- 47. \( y = -|x| \)
- 48. \( y = |x| - 6 \)
- 49. \( y = 4 - |x| \)

**Maintaining Mathematical Proficiency**

Write the sentence as an inequality. (Section 2.1)

- 50. A number \( y \) is less than 16.
- 51. Three is no less than a number \( x \).
- 52. Seven is at most the quotient of a number \( d \) and \(-5\).
- 53. The sum of a number \( w \) and 4 is more than \(-12\).

Evaluate the expression. (Skills Review Handbook)

- 54. \( 11^2 \)
- 55. \( (-3)^4 \)
- 56. \( -5^2 \)
- 57. \( 2^5 \)
### Essential Question
How can you determine whether a function is linear or nonlinear?

#### Exploration 1
Finding Patterns for Similar Figures

**Work with a partner.** Copy and complete each table for the sequence of similar figures. (In parts (a) and (b), use the rectangle shown.) Graph the data in each table. Decide whether each pattern is linear or nonlinear. Justify your conclusion.

- **a.** perimeters of similar rectangles
- **b.** areas of similar rectangles
- **c.** circumferences of circles of radius $r$
- **d.** areas of circles of radius $r$

#### Communicate Your Answer

2. How do you know that the patterns you found in Exploration 1 represent functions?

3. How can you determine whether a function is linear or nonlinear?

4. Describe two real-life patterns: one that is linear and one that is nonlinear. Use patterns that are different from those described in Exploration 1.
What You Will Learn

- Identify linear functions using graphs, tables, and equations.
- Graph linear functions using discrete and continuous data.
- Write real-life problems to fit data.

Identifying Linear Functions

A linear equation in two variables, \( x \) and \( y \), is an equation that can be written in the form \( y = mx + b \), where \( m \) and \( b \) are constants. The graph of a linear equation is a line. Likewise, a linear function is a function whose graph is a nonvertical line. A linear function has a constant rate of change and can be represented by a linear equation in two variables. A nonlinear function does not have a constant rate of change. So, its graph is not a line.

**Example 1**

Identifying Linear Functions Using Graphs

Does the graph represent a linear or nonlinear function? Explain.

**Solution**

a. The graph is not a line. So, the function is nonlinear.

b. The graph is a line. So, the function is linear.

**Example 2**

Identifying Linear Functions Using Tables

Does the table represent a linear or nonlinear function? Explain.

**Solution**

a. As \( x \) increases by 3, \( y \) decreases by 6. The rate of change is constant. So, the function is linear.

b. As \( x \) increases by 2, \( y \) increases by different amounts. The rate of change is not constant. So, the function is nonlinear.
Monitoring Progress

Does the graph or table represent a linear or nonlinear function? Explain.

1. [Graph with a straight line through points (−2, 2) and (2, 2).]
2. [Graph with a downward curve through points (−2, 3) and (2, 3).]
3. | x | 0 | 1 | 2 | 3 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>
4. | x | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>16</td>
<td>8</td>
<td>4</td>
</tr>
</tbody>
</table>

Example 3

Identifying Linear Functions Using Equations

Which of the following equations represent linear functions? Explain.

\( y = 3.8, \ y = \sqrt{x}, \ y = 3^x, \ y = \frac{2}{x}, \ y = 6(x - 1), \) and \( x^2 - y = 0 \)

SOLUTION

You cannot rewrite the equations \( y = \sqrt{x}, \ y = 3^x, \ y = \frac{2}{x}, \) and \( x^2 - y = 0 \) in the form \( y = mx + b \). So, these equations cannot represent linear functions.

You can rewrite the equation \( y = 3.8 \) as \( y = 0x + 3.8 \) and the equation \( y = 6(x - 1) \) as \( y = 6x - 6 \). So, they represent linear functions.

Monitoring Progress

Does the equation represent a linear or nonlinear function? Explain.

5. \( y = x + 9 \)
6. \( y = \frac{3x}{5} \)
7. \( y = 5 - 2x^2 \)

Concept Summary

Representations of Functions

**Words**  An output is 3 more than the input.

**Equation**  \( y = x + 3 \)

**Input-Output Table**

<table>
<thead>
<tr>
<th>Input, ( x )</th>
<th>Output, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

**Mapping Diagram**

<table>
<thead>
<tr>
<th>Input, ( x )</th>
<th>Output, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

**Graph**
Graphing Linear Functions

A **solution of a linear equation in two variables** is an ordered pair \((x, y)\) that makes the equation true. The graph of a linear equation in two variables is the set of points \((x, y)\) in a coordinate plane that represents all solutions of the equation. Sometimes the points are distinct, and other times the points are connected.

**Core Concept**

Discrete and Continuous Domains

A **discrete domain** is a set of input values that consists of only certain numbers in an interval.

Example: Integers from 1 to 5

\[0, 1, 2, 3, 4, 5\]

A **continuous domain** is a set of input values that consists of all numbers in an interval.

Example: All numbers from 1 to 5

\[0, 1, 2, 3, 4, 5\]

**EXAMPLE 4** Graphing Discrete Data

The linear function \(y = 15.95x\) represents the cost \(y\) (in dollars) of \(x\) tickets for a museum. Each customer can buy a maximum of three tickets. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.

**SOLUTION**

a. You cannot buy part of a ticket, only a certain number of tickets. Because \(x\) represents the number of tickets, it must be a whole number. The maximum number of tickets a customer can buy is three.

So, the domain is 0, 1, 2, and 3, and it is discrete.

b. **Step 1** Make an input-output table to find the ordered pairs.

<table>
<thead>
<tr>
<th>Input, (x)</th>
<th>Output, (y = 15.95x)</th>
<th>((x, y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>15.95 (1)</td>
<td>(1, 15.95)</td>
</tr>
<tr>
<td>2</td>
<td>31.9 (2)</td>
<td>(2, 31.9)</td>
</tr>
<tr>
<td>3</td>
<td>47.85 (3)</td>
<td>(3, 47.85)</td>
</tr>
</tbody>
</table>

**Step 2** Plot the ordered pairs. The domain is discrete. So, the graph consists of individual points.

c. Use the input-output table to find the range.

The range is 0, 15.95, 31.9, and 47.85.

**Monitoring Progress**

8. The linear function \(m = 50 - 9d\) represents the amount \(m\) (in dollars) of money you have after buying \(d\) DVDs. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.
Graphing Continuous Data

A cereal bar contains 130 calories. The number $c$ of calories consumed is a function of the number $b$ of bars eaten.

a. Does this situation represent a linear function? Explain.

b. Find the domain of the function. Is the domain discrete or continuous? Explain.

c. Graph the function using its domain.

d. Find the range of the function.

**SOLUTION**

a. As $b$ increases by 1, $c$ increases by 130. The rate of change is constant.

   ▶ So, this situation represents a linear function.

b. You can eat part of a cereal bar. The number $b$ of bars eaten can be any value greater than or equal to 0.

   ▶ So, the domain is $b \geq 0$, and it is continuous.

c. **Step 1** Make an input-output table to find ordered pairs.

<table>
<thead>
<tr>
<th>Input, $b$</th>
<th>Output, $c$</th>
<th>$(b, c)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>130</td>
<td>(1, 130)</td>
</tr>
<tr>
<td>2</td>
<td>260</td>
<td>(2, 260)</td>
</tr>
<tr>
<td>3</td>
<td>390</td>
<td>(3, 390)</td>
</tr>
<tr>
<td>4</td>
<td>520</td>
<td>(4, 520)</td>
</tr>
</tbody>
</table>

**Step 2** Plot the ordered pairs.

**Step 3** Draw a line through the points. The line should start at $(0, 0)$ and continue to the right. Use an arrow to indicate that the line continues without end, as shown. The domain is continuous. So, the graph is a line with a domain of $b \geq 0$.

d. From the graph or table, you can see that the output values are greater than or equal to 0.

   ▶ So, the range is $c \geq 0$.

**Monitoring Progress**

9. Is the domain discrete or continuous? Explain.

<table>
<thead>
<tr>
<th>Input</th>
<th>Number of stories, $x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>Height of building (feet), $y$</td>
<td>12</td>
<td>24</td>
<td>36</td>
</tr>
</tbody>
</table>

10. A 20-gallon bathtub is draining at a rate of 2.5 gallons per minute. The number $g$ of gallons remaining is a function of the number $m$ of minutes. (a) Does this situation represent a linear function? Explain. (b) Find the domain of the function. Is the domain discrete or continuous? Explain. (c) Graph the function using its domain. (d) Find the range of the function.
Writing Real-Life Problems

Write a real-life problem to fit the data shown in each graph. Is the domain of each function discrete or continuous? Explain.

**SOLUTION**

a. You want to think of a real-life situation in which there are two variables, $x$ and $y$. Using the graph, notice that the sum of the variables is always 6, and the value of each variable must be a whole number from 0 to 6.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Discrete domain

One possibility is two people bidding against each other on six coins at an auction. Each coin will be purchased by one of the two people. Because it is not possible to purchase part of a coin, the domain is discrete.

b. You want to think of a real-life situation in which there are two variables, $x$ and $y$. Using the graph, notice that the sum of the variables is always 6, and the value of each variable can be any real number from 0 to 6.

\[ x + y = 6 \quad \text{or} \quad y = -x + 6 \]

Continuous domain

One possibility is two people bidding against each other on 6 ounces of gold dust at an auction. All the dust will be purchased by the two people. Because it is possible to purchase any portion of the dust, the domain is continuous.

Monitoring Progress

Write a real-life problem to fit the data shown in the graph. Is the domain of the function discrete or continuous? Explain.

11. [Graph with data points]

12. [Graph with line graph]
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A linear equation in two variables is an equation that can be written in the form __________, where \( m \) and \( b \) are constants.

2. **VOCABULARY** Compare linear functions and nonlinear functions.

3. **VOCABULARY** Compare discrete domains and continuous domains.

4. **WRITING** How can you tell whether a graph shows a discrete domain or a continuous domain?

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, determine whether the graph represents a **linear** or **nonlinear** function. Explain. (See Example 1.)

5. ![Graph](image1)

6. ![Graph](image2)

7. ![Graph](image3)

8. ![Graph](image4)

9. ![Graph](image5)

10. ![Graph](image6)

In Exercises 11–14, determine whether the table represents a **linear** or **nonlinear** function. Explain. (See Example 2.)

11. \[
\begin{array}{c|c|c|c|c}
\text{x} & 1 & 2 & 3 & 4 \\
\text{y} & 5 & 10 & 15 & 20 \\
\end{array}
\]

12. \[
\begin{array}{c|c|c|c|c}
\text{x} & 5 & 7 & 9 & 11 \\
\text{y} & -9 & -3 & -1 & 3 \\
\end{array}
\]

13. \[
\begin{array}{c|c|c|c|c}
\text{x} & 4 & 8 & 12 & 16 \\
\text{y} & 16 & 12 & 7 & 1 \\
\end{array}
\]

14. \[
\begin{array}{c|c|c|c|c}
\text{x} & -1 & 0 & 1 & 2 \\
\text{y} & 35 & 20 & 5 & -10 \\
\end{array}
\]

**ERROR ANALYSIS** In Exercises 15 and 16, describe and correct the error in determining whether the table or graph represents a linear function.

15. \[
\begin{array}{c|c|c|c|c}
\text{x} & 2 & 4 & 6 & 8 \\
\text{y} & 4 & 16 & 64 & 256 \\
\end{array}
\]

As \( x \) increases by 2, \( y \) increases by a constant factor of 4. So, the function is linear.

16. ![Graph](image7)

The graph is a line. So, the graph represents a linear function.
In Exercises 17–24, determine whether the equation represents a linear or nonlinear function. Explain. (See Example 3.)

17. \( y = x^2 + 13 \)  
18. \( y = 7 - 3x \)
19. \( y = \sqrt{8} - x \)  
20. \( y = 4x(8 - x) \)
21. \( 2 + \frac{1}{3}y = 3x + 4 \)  
22. \( y - x = 2x - \frac{2}{3}y \)
23. \( 18x - 2y = 26 \)  
24. \( 2x + 3y = 9xy \)

25. **CLASSIFYING FUNCTIONS** Which of the following equations do not represent linear functions? Explain.

- A. \( 12 = 2x^2 + 4y^2 \)
- B. \( y - x + 3 = x \)
- C. \( x = 8 \)
- D. \( x = 9 - \frac{3}{4}y \)
- E. \( y = \frac{5x}{11} \)
- F. \( y = \sqrt{x} + 3 \)

26. **USING STRUCTURE** Fill in the table so it represents a linear function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td>11</td>
</tr>
</tbody>
</table>

In Exercises 27 and 28, find the domain of the function represented by the graph. Determine whether the domain is discrete or continuous. Explain.

27. 

28. 

In Exercises 29–32, determine whether the domain is discrete or continuous. Explain.

29. **Input**

<table>
<thead>
<tr>
<th>Bags, ( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Marbles, ( y )</td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

30. **Input**

<table>
<thead>
<tr>
<th>Years, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Height of tree (feet), ( y )</td>
<td>6</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

31. **Input**

<table>
<thead>
<tr>
<th>Time (hours), ( x )</th>
<th>3</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Distance (miles), ( y )</td>
<td>150</td>
<td>300</td>
<td>450</td>
</tr>
</tbody>
</table>

32. **Input**

<table>
<thead>
<tr>
<th>Relay teams, ( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Athletes, ( y )</td>
<td>0</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

**ERROR ANALYSIS** In Exercises 33 and 34, describe and correct the error in the statement about the domain.

33. 

34. 

35. **MODELING WITH MATHEMATICS** The linear function \( m = 55 - 8.5b \) represents the amount \( m \) (in dollars) of money that you have after buying \( b \) books. (See Example 4.)

a. Find the domain of the function. Is the domain discrete or continuous? Explain.

b. Graph the function using its domain.

c. Find the range of the function.
36. **MODELING WITH MATHEMATICS** The number \( y \) of calories burned after \( x \) hours of rock climbing is represented by the linear function \( y = 650x \).

   a. Find the domain of the function. Is the domain discrete or continuous? Explain.
   
   b. Graph the function using its domain.
   
   c. Find the range of the function.

37. **MODELING WITH MATHEMATICS** You are researching the speed of sound waves in dry air at 86°F. The table shows the distances \( d \) (in miles) sound waves travel in \( t \) seconds. (See Example 5.)

<table>
<thead>
<tr>
<th>Time (seconds), ( t )</th>
<th>Distance (miles), ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.434</td>
</tr>
<tr>
<td>4</td>
<td>0.868</td>
</tr>
<tr>
<td>6</td>
<td>1.302</td>
</tr>
<tr>
<td>8</td>
<td>1.736</td>
</tr>
<tr>
<td>10</td>
<td>2.170</td>
</tr>
</tbody>
</table>

   a. Does this situation represent a linear function? Explain.
   
   b. Find the domain of the function. Is the domain discrete or continuous? Explain.
   
   c. Graph the function using its domain.
   
   d. Find the range of the function.

38. **MODELING WITH MATHEMATICS** The function \( y = 30 + 5x \) represents the cost \( y \) (in dollars) of having your dog groomed and buying \( x \) extra services.

   a. Does this situation represent a linear function? Explain.
   
   b. Find the domain of the function. Is the domain discrete or continuous? Explain.
   
   c. Graph the function using its domain.
   
   d. Find the range of the function.

39. **WRITING** In Exercises 39–42, write a real-life problem to fit the data shown in the graph. Determine whether the domain of the function is discrete or continuous. Explain. (See Example 6.)

40. 

41. 

42. 

43. **USING STRUCTURE** The table shows your earnings \( y \) (in dollars) for working \( x \) hours.

<table>
<thead>
<tr>
<th>Time (hours), ( x )</th>
<th>Earnings (dollars), ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>40.80</td>
</tr>
<tr>
<td>5</td>
<td>50.00</td>
</tr>
<tr>
<td>6</td>
<td>61.20</td>
</tr>
<tr>
<td>7</td>
<td>71.40</td>
</tr>
</tbody>
</table>

   a. What is the missing \( y \)-value that makes the table represent a linear function?
   
   b. What is your hourly pay rate?

44. **MAKING AN ARGUMENT** The linear function \( d = 50t \) represents the distance \( d \) (in miles) Car A is from a car rental store after \( t \) hours. The table shows the distances Car B is from the rental store.

<table>
<thead>
<tr>
<th>Time (hours), ( t )</th>
<th>Distance (miles), ( d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>5</td>
<td>310</td>
</tr>
</tbody>
</table>

   a. Does the table represent a linear or nonlinear function? Explain.
   
   b. Your friend claims Car B is moving at a faster rate. Is your friend correct? Explain.
**Mathematical Connections** In Exercises 45–48, tell whether the volume of the solid is a linear or nonlinear function of the missing dimension(s). Explain.

45. 

![Volume of a Rectangular Prism](image)

46. 

![Volume of a Triangular Prism](image)

47. 

![Volume of a Cylinder](image)

48. 

![Volume of a Cone](image)

**Reasoning** A water company fills two different-sized jugs. The first jug can hold $x$ gallons of water. The second jug can hold $y$ gallons of water. The company fills $A$ jugs of the first size and $B$ jugs of the second size. What does each expression represent? Does each expression represent a set of discrete or continuous values?

a. $x + y$

b. $A + B$

c. $Ax$

d. $Ax + By$

**Thought-Provoking** You go to a farmer’s market to buy tomatoes. Graph a function that represents the cost of buying tomatoes. Explain your reasoning.

**Classifying a Function** Is the function represented by the ordered pairs linear or nonlinear? Explain your reasoning.

$(0, 2), (3, 14), (5, 22), (9, 38), (11, 46)$

**How Do You See It?** You and your friend go running. The graph shows the distances you and your friend run.

**Writing** In Exercises 53 and 54, describe a real-life situation for the constraints.

53. The function has at least one negative number in the domain. The domain is continuous.

54. The function gives at least one negative number as an output. The domain is discrete.

---

**Maintaining Mathematical Proficiency** Reviewing what you learned in previous grades and lessons

Evaluate the expression when $x = 2$. *(Skills Review Handbook)*

55. $6x + 8$

56. $10 - 2x + 8$

57. $4(x + 2 - 5x)$

58. $\frac{x}{2} + 5x - 7$

Solve the equation. Check your solution. *(Section 1.2)*

59. $2x + 10 = 16$

60. $8 = 5 - 3p$

61. $\frac{5t + 2}{7} = 6$

62. $12 = y - 7 - 3y$

Solve the inequality. Graph the solution. *(Section 2.3)*

63. $8x \geq 4$

64. $-3t < 24$

65. $\frac{r}{16} \leq -1$

66. $-\frac{3}{5}a > 6$
Essential Question: How can you use function notation to represent a function?

The notation \( f(x) \), called function notation, is another name for \( y \). This notation is read as “the value of \( f \) at \( x \)” or “\( f \) of \( x \).” The parentheses do not imply multiplication. You can use letters other than \( f \) to name a function. The letters \( g, h, j, \) and \( k \) are often used to name functions.

### EXPLORATION 1 Matching Functions with Their Graphs

**Work with a partner.** Match each function with its graph.

- a. \( f(x) = 2x - 3 \)
- b. \( g(x) = -x + 2 \)
- c. \( h(x) = x^2 - 1 \)
- d. \( j(x) = 2x^2 - 3 \)

### EXPLORATION 2 Evaluating a Function

**Work with a partner.** Consider the function

\[ f(x) = -x + 3. \]

Locate the points \((x, f(x))\) on the graph. Explain how you found each point.

- a. \((-1, f(-1))\)
- b. \((0, f(0))\)
- c. \((1, f(1))\)
- d. \((2, f(2))\)

### Communicate Your Answer

3. How can you use function notation to represent a function? How are standard notation and function notation similar? How are they different?

<table>
<thead>
<tr>
<th>Standard Notation</th>
<th>Function Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x + 5 )</td>
<td>( f(x) = 2x + 5 )</td>
</tr>
</tbody>
</table>
What You Will Learn

- Use function notation to evaluate and interpret functions.
- Use function notation to solve and graph functions.
- Solve real-life problems using function notation.

Using Function Notation to Evaluate and Interpret

You know that a linear function can be written in the form \( y = mx + b \). By naming a linear function \( f \), you can also write the function using function notation:

\[
 f(x) = mx + b
 \]

**Function notation**

The notation \( f(x) \) is another name for \( y \). If \( f \) is a function, and \( x \) is in its domain, then \( f(x) \) represents the output of \( f \) corresponding to the input \( x \). You can use letters other than \( f \) to name a function, such as \( g \) or \( h \).

**EXAMPLE 1**  Evaluating a Function

Evaluate \( f(x) = -4x + 7 \) when \( x = 2 \) and \( x = -2 \).

**SOLUTION**

\[
 f(x) = -4x + 7 \quad \text{Write the function.} \quad f(x) = -4x + 7
\]

\[
 f(2) = -4(2) + 7 \quad \text{Substitute for } x. \quad f(-2) = -4(-2) + 7
\]

\[
 = -8 + 7 \quad \text{Multiply.} \quad = 8 + 7
\]

\[
 = -1 \quad \text{Add.} \quad = 15
\]

When \( x = 2 \), \( f(x) = -1 \), and when \( x = -2 \), \( f(x) = 15 \).

**EXAMPLE 2**  Interpreting Function Notation

Let \( f(t) \) be the outside temperature (°F) \( t \) hours after 6 A.M. Explain the meaning of each statement.

**a.** \( f(0) = 58 \)

**b.** \( f(6) = n \)

**c.** \( f(3) < f(9) \)

**SOLUTION**

**a.** The initial value of the function is 58. So, the temperature at 6 A.M. is 58°F.

**b.** The output of \( f \) when \( t = 6 \) is \( n \). So, the temperature at noon (6 hours after 6 A.M.) is \( n \)°F.

**c.** The output of \( f \) when \( t = 3 \) is less than the output of \( f \) when \( t = 9 \). So, the temperature at 9 A.M. (3 hours after 6 A.M.) is less than the temperature at 3 P.M. (9 hours after 6 A.M.).

**Monitoring Progress**

Evaluate the function when \( x = -4, 0, \) and 3.

1. \( f(x) = 2x - 5 \)

2. \( g(x) = -x - 1 \)

3. **WHAT IF?** In Example 2, let \( f(t) \) be the outside temperature (°F) \( t \) hours after 9 A.M. Explain the meaning of each statement.

   **a.** \( f(4) = 75 \)

   **b.** \( f(m) = 70 \)

   **c.** \( f(2) = f(9) \)

   **d.** \( f(6) > f(0) \)
Using Function Notation to Solve and Graph

EXAMPLE 3  Solving for the Independent Variable

For \( h(x) = \frac{2}{3}x - 5 \), find the value of \( x \) for which \( h(x) = -7 \).

SOLUTION

\[
\begin{align*}
  h(x) &= \frac{2}{3}x - 5 \\
  -7 &= \frac{2}{3}x - 5 \\
  +5 &= +5 \\
  -2 &= \frac{2}{3}x \\
  \frac{3}{2} \cdot (-2) &= \frac{3}{2} \cdot \frac{2}{3}x \\
  -3 &= x
\end{align*}
\]

When \( x = -3 \), \( h(x) = -7 \).

EXAMPLE 4  Graphing a Linear Function in Function Notation

Graph \( f(x) = 2x + 5 \).

SOLUTION

Step 1  Make an input-output table to find ordered pairs.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

Step 2  Plot the ordered pairs.

Step 3  Draw a line through the points.

STUDY TIP

The graph of \( y = f(x) \) consists of the points \( (x, f(x)) \).

Monitoring Progress

Find the value of \( x \) so that the function has the given value.

4. \( f(x) = 6x + 9; f(x) = 21 \)  
5. \( g(x) = -\frac{1}{2}x + 3; g(x) = -1 \)

Graph the linear function.

6. \( f(x) = 3x - 2 \)  
7. \( g(x) = -x + 4 \)  
8. \( h(x) = -\frac{3}{4}x - 1 \)
Solving Real-Life Problems

**EXAMPLE 5  Modeling with Mathematics**

The graph shows the number of miles a helicopter is from its destination after $x$ hours on its first flight. On its second flight, the helicopter travels 50 miles farther and increases its speed by 25 miles per hour. The function $f(x) = 350 - 125x$ represents the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after $x$ hours. Which flight takes less time? Explain.

**SOLUTION**

1. **Understand the Problem** You are given a graph of the first flight and an equation of the second flight. You are asked to compare the flight times to determine which flight takes less time.

2. **Make a Plan** Graph the function that represents the second flight. Compare the graph to the graph of the first flight. The $x$-value that corresponds to $f(x) = 0$ represents the flight time.

3. **Solve the Problem** Graph $f(x) = 350 - 125x$.

   **Step 1** Make an input-output table to find the ordered pairs.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>350</td>
</tr>
<tr>
<td>1</td>
<td>225</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>-25</td>
</tr>
</tbody>
</table>

   **Step 2** Plot the ordered pairs.

   **Step 3** Draw a line through the points. Note that the function only makes sense when $x$ and $f(x)$ are positive. So, only draw the line in the first quadrant.

   From the graph of the first flight, you can see that when $f(x) = 0$, $x = 3$. From the graph of the second flight, you can see that when $f(x) = 0$, $x$ is slightly less than 3. So, the second flight takes less time.

4. **Look Back** You can check that your answer is correct by finding the value of $x$ for which $f(x) = 0$.

   $f(x) = 350 - 125x$  
   $0 = 350 - 125x$  
   $-350 = -125x$  
   $2.8 = x$  

   So, the second flight takes 2.8 hours, which is less than 3.

**Monitoring Progress**

**WHAT IF?** Let $f(x) = 250 - 75x$ represent the second flight, where $f(x)$ is the number of miles the helicopter is from its destination after $x$ hours. Which flight takes less time? Explain.
In Exercises 3–10, evaluate the function when \( x = -2, 0, \) and \( 5. \) (See Example 1.)

3. \( f(x) = x + 6 \)
4. \( g(x) = 3x \)
5. \( h(x) = -2x + 9 \)
6. \( r(x) = -x - 7 \)
7. \( p(x) = -3 + 4x \)
8. \( b(x) = 18 - 0.5x \)
9. \( v(x) = 12 - 2x - 5 \)
10. \( n(x) = -1 - x + 4 \)

11. **INTERPRETING FUNCTION NOTATION** Let \( c(t) \) be the number of customers in a restaurant \( t \) hours after 8 A.M. Explain the meaning of each statement. (See Example 2.)
   a. \( c(0) = 0 \)
   b. \( c(3) = c(8) \)
   c. \( c(n) = 29 \)
   d. \( c(13) < c(12) \)

12. **INTERPRETING FUNCTION NOTATION** Let \( H(x) \) be the percent of U.S. households with Internet use \( x \) years after 1980. Explain the meaning of each statement.
   a. \( H(23) = 55 \)
   b. \( H(4) = k \)
   c. \( H(27) \geq 61 \)
   d. \( H(17) + H(21) = H(29) \)

In Exercises 13–18, find the value of \( x \) so that the function has the given value. (See Example 3.)

13. \( h(x) = -7x; h(x) = 63 \)
14. \( t(x) = 3x; t(x) = 24 \)
15. \( m(x) = 4x + 15; m(x) = 7 \)
16. \( k(x) = 6x - 12; k(x) = 18 \)
17. \( g(x) = \frac{1}{2}x - 3; g(x) = -4 \)
18. \( j(x) = -\frac{4}{5}x + 7; j(x) = -5 \)

In Exercises 19 and 20, find the value of \( x \) so that \( f(x) = 7. \)

19. \[ f(x) \]
20. \[ f(x) \]

21. **MODELING WITH MATHEMATICS** The function \( C(x) = 17.5x - 10 \) represents the cost (in dollars) of buying \( x \) tickets to the orchestra with a $10 coupon.
   a. How much does it cost to buy five tickets?
   b. How many tickets can you buy with $130?

22. **MODELING WITH MATHEMATICS** The function \( d(t) = 300,000t \) represents the distance (in kilometers) that light travels in \( t \) seconds.
   a. How far does light travel in 15 seconds?
   b. How long does it take light to travel 12 million kilometers?

In Exercises 23–28, graph the linear function. (See Example 4.)

23. \( p(x) = 4x \)
24. \( h(x) = -5 \)
25. \( d(x) = -\frac{1}{2}x - 3 \)
26. \( w(x) = \frac{3}{4}x + 2 \)
27. \( g(x) = -4 + 7x \)
28. \( f(x) = 3 - 6x \)
29. **PROBLEM SOLVING** The graph shows the percent \( p \) (in decimal form) of battery power remaining in a laptop computer after \( t \) hours of use. A tablet computer initially has 75% of its battery power remaining and loses 12.5% per hour. Which computer’s battery will last longer? Explain. (See Example 5.)

30. **PROBLEM SOLVING** The function \( C(x) = 25x + 50 \) represents the labor cost (in dollars) for Certified Remodeling to build a deck, where \( x \) is the number of hours of labor. The table shows sample labor costs from its main competitor, Master Remodeling. The deck is estimated to take 8 hours of labor. Which company would you hire? Explain.

<table>
<thead>
<tr>
<th>Hours</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$130</td>
</tr>
<tr>
<td>4</td>
<td>$160</td>
</tr>
<tr>
<td>6</td>
<td>$190</td>
</tr>
</tbody>
</table>

31. **MAKING AN ARGUMENT** Let \( P(x) \) be the number of people in the U.S. who own a cell phone \( x \) years after 1990. Your friend says that \( P(x + 1) > P(x) \) for any \( x \) because \( x + 1 \) is always greater than \( x \). Is your friend correct? Explain.

32. **THOUGHT PROVOKING** Let \( B(t) \) be your bank account balance after \( t \) days. Describe a situation in which \( B(0) < B(4) < B(2) \).

33. **MATHEMATICAL CONNECTIONS** Rewrite each geometry formula using function notation. Evaluate each function when \( r = 5 \) feet. Then explain the meaning of the result.
   a. Diameter, \( d = 2r \)
   b. Area, \( A = \pi r^2 \)
   c. Circumference, \( C = 2\pi r \)

34. **HOW DO YOU SEE IT?** The function \( y = A(x) \) represents the attendance at a high school \( x \) weeks after a flu outbreak. The graph of the function is shown.

   a. What happens to the school’s attendance after the flu outbreak?
   b. Estimate \( A(13) \) and explain its meaning.
   c. Use the graph to estimate the solution(s) of the equation \( A(x) = 400 \). Explain the meaning of the solution(s).
   d. What was the least attendance? When did that occur?
   e. How many students do you think are enrolled at this high school? Explain your reasoning.

35. **INTERPRETING FUNCTION NOTATION** Let \( f \) be a function. Use each statement to find the coordinates of a point on the graph of \( f \).
   a. \( f(5) \) is equal to 9.
   b. A solution of the equation \( f(n) = -3 \) is 5.

36. **REASONING** Given a function \( f \), tell whether the statement \( f(a + b) = f(a) + f(b) \) is true or false for all inputs \( a \) and \( b \). If it is false, explain why.

### Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. Graph the solution. (Section 2.5)

37. \(-2 \leq x - 11 \leq 6\)
38. \(5a < -35 \) or \( a - 14 > 1\)
39. \(-16 < 6k + 2 \leq 0\)
40. \(2d + 7 < -9 \) or \( 4d - 1 > -3\)
41. \(5 \leq 3y + 8 < 17\)
42. \(4v + 9 \leq 5 \) or \(-3v \geq -6\)
What Did You Learn?

Core Vocabulary

relation, p. 90
function, p. 90
domain, p. 92
range, p. 92
independent variable, p. 93
dependent variable, p. 93
linear equation in two variables, p. 98
linear function, p. 98
nonlinear function, p. 98
solution of a linear equation in two variables, p. 100
discrete domain, p. 100
continuous domain, p. 100
function notation, p. 108

Core Concepts

Section 3.1
Determining Whether Relations Are Functions, p. 90
Vertical Line Test, p. 91
The Domain and Range of a Function, p. 92
Independent and Dependent Variables, p. 93

Section 3.2
Linear and Nonlinear Functions, p. 98
Representations of Functions, p. 99
Discrete and Continuous Domains, p. 100

Section 3.3
Using Function Notation, p. 108

Mathematical Thinking

1. How can you use technology to confirm your answers in Exercises 46–49 on page 96?
2. How can you use patterns to solve Exercise 43 on page 105?
3. How can you make sense of the quantities in the function in Exercise 21 on page 111?

Staying Focused during Class

As soon as class starts, quickly review your notes from the previous class and start thinking about math.
Repeat what you are writing in your head.
When a particular topic is difficult, ask for another example.
Determine whether the relation is a function. Explain. (Section 3.1)

1. \[
\begin{array}{c|c|c|c|c}
\text{Input, } x & -1 & 0 & 1 & 2 & 3 \\
\hline
\text{Output, } y & 0 & 1 & 4 & 4 & 8 \\
\end{array}
\]

2. \((-10, 2), (-8, 3), (-6, 5), (-8, 8), (-10, 6)\)

Find the domain and range of the function represented by the graph. (Section 3.1)

3. 

4. 

5. 

Determine whether the graph, table, or equation represents a linear or nonlinear function. Explain. (Section 3.2)

6. 

7. 

8. \(y = x(2 - x)\)

Determine whether the domain is discrete or continuous. Explain. (Section 3.2)

9. 

10. 

11. For \(w(x) = -2x + 7\), find the value of \(x\) for which \(w(x) = -3\). (Section 3.3)

Graph the linear function. (Section 3.3)

12. \(g(x) = x + 3\)

13. \(p(x) = -3x - 1\)

14. \(m(x) = \frac{2}{3}x\)

15. The function \(m = 30 - 3r\) represents the amount \(m\) (in dollars) of money you have after renting \(r\) video games. (Section 3.1 and Section 3.2)
   a. Identify the independent and dependent variables.
   b. Find the domain of the function. Is the domain discrete or continuous? Explain.
   c. Graph the function using its domain.
   d. Find the range of the function.

16. The function \(d(x) = 1375 - 110x\) represents the distance (in miles) a high-speed train is from its destination after \(x\) hours. (Section 3.3)
   a. How far is the train from its destination after 8 hours?
   b. How long does the train travel before reaching its destination?
Essential Question: How can you describe the graph of the equation $Ax + By = C$?

**Exploration 1** Using a Table to Plot Points

**Work with a partner.** You sold a total of $16 worth of tickets to a fundraiser. You lost track of how many of each type of ticket you sold. Adult tickets are $4 each. Child tickets are $2 each.

<table>
<thead>
<tr>
<th>Number of adult tickets</th>
<th>Number of child tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

a. Let $x$ represent the number of adult tickets. Let $y$ represent the number of child tickets. Use the verbal model to write an equation that relates $x$ and $y$.

b. Copy and complete the table to show the different combinations of tickets you might have sold.

c. Plot the points from the table. Describe the pattern formed by the points.

d. If you remember how many adult tickets you sold, can you determine how many child tickets you sold? Explain your reasoning.

**Exploration 2** Rewriting and Graphing an Equation

**Work with a partner.** You sold a total of $48 worth of cheese. You forgot how many pounds of each type of cheese you sold. Swiss cheese costs $8 per pound. Cheddar cheese costs $6 per pound.

<table>
<thead>
<tr>
<th>Pounds of Swiss</th>
<th>Pounds of cheddar</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

a. Let $x$ represent the number of pounds of Swiss cheese. Let $y$ represent the number of pounds of cheddar cheese. Use the verbal model to write an equation that relates $x$ and $y$.

b. Solve the equation for $y$. Then use a graphing calculator to graph the equation. Given the real-life context of the problem, find the domain and range of the function.

c. The **x-intercept** of a graph is the $x$-coordinate of a point where the graph crosses the $x$-axis. The **y-intercept** of a graph is the $y$-coordinate of a point where the graph crosses the $y$-axis. Use the graph to determine the $x$- and $y$-intercepts.

d. How could you use the equation you found in part (a) to determine the $x$- and $y$-intercepts? Explain your reasoning.

e. Explain the meaning of the intercepts in the context of the problem.

**Communicate Your Answer**

3. How can you describe the graph of the equation $Ax + By = C$?
4. Write a real-life problem that is similar to those shown in Explorations 1 and 2.
What You Will Learn

- Graph equations of horizontal and vertical lines.
- Graph linear equations in standard form using intercepts.
- Find zeros of functions.
- Use linear equations to solve real-life problems.

Horizontal and Vertical Lines

The standard form of a linear equation is $Ax + By = C$, where $A$, $B$, and $C$ are real numbers and $A$ and $B$ are not both zero.

Consider what happens when $A = 0$ or when $B = 0$. When $A = 0$, the equation becomes $By = C$, or $y = \frac{C}{B}$. Because $\frac{C}{B}$ is a constant, you can write $y = b$. Similarly, when $B = 0$, the equation becomes $Ax = C$, or $x = \frac{C}{A}$, and you can write $x = a$.

Core Concept

Horizontal and Vertical Lines

The graph of $y = b$ is a horizontal line. The line passes through the point $(0, b)$.

The graph of $x = a$ is a vertical line. The line passes through the point $(a, 0)$.

Example 1

Graph (a) $y = 4$ and (b) $x = -2$.

Solution

a. For every value of $x$, the value of $y$ is 4. The graph of the equation $y = 4$ is a horizontal line 4 units above the $x$-axis.

b. For every value of $y$, the value of $x$ is $-2$. The graph of the equation $x = -2$ is a vertical line 2 units to the left of the $y$-axis.

STUDY TIP

For every value of $x$, the ordered pair $(x, 4)$ is a solution of $y = 4$.

Monitoring Progress

Graph the linear equation.

1. $y = -2.5$
2. $x = 5$
Using Intercepts to Graph Linear Equations

You can use the fact that two points determine a line to graph a linear equation. Two convenient points are the points where the graph crosses the axes.

**Core Concept**

**Using Intercepts to Graph Equations**

The **x-intercept** of a graph is the x-coordinate of a point where the graph crosses the x-axis. It occurs when \( y = 0 \).

The **y-intercept** of a graph is the y-coordinate of a point where the graph crosses the y-axis. It occurs when \( x = 0 \).

To graph the linear equation \( Ax + By = C \), find the intercepts and draw the line that passes through the two intercepts.

- To find the x-intercept, let \( y = 0 \) and solve for \( x \).
- To find the y-intercept, let \( x = 0 \) and solve for \( y \).

**EXAMPLE 2 Using Intercepts to Graph a Linear Equation**

Use intercepts to graph the equation \( 3x + 4y = 12 \).

**SOLUTION**

**Step 1** Find the intercepts.

To find the x-intercept, substitute 0 for \( y \) and solve for \( x \).

\[
3x + 4y = 12 \quad \text{Write the original equation.}
\]

\[
3x + 4(0) = 12 \quad \text{Substitute 0 for } y.
\]

\[
x = 4 \quad \text{Solve for } x.
\]

To find the y-intercept, substitute 0 for \( x \) and solve for \( y \).

\[
3x + 4y = 12 \quad \text{Write the original equation.}
\]

\[
3(0) + 4y = 12 \quad \text{Substitute 0 for } x.
\]

\[
y = 3 \quad \text{Solve for } y.
\]

**Step 2** Plot the points and draw the line.

The x-intercept is 4, so plot the point \((4, 0)\). The y-intercept is 3, so plot the point \((0, 3)\). Draw a line through the points.

**Monitoring Progress**

Use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

3. \( 2x - y = 4 \)

4. \( x + 3y = -9 \)
Finding the Zeros of Functions

A zero of a function \( f \) is an \( x \)-value for which \( f(x) = 0 \) (or \( y = 0 \)). A zero of a function is an \( x \)-intercept of the graph of the function.

**EXAMPLE 3** Finding the Zero of a Function

Find the zero of the function \( f(x) = 2x + 8 \).

**SOLUTION**

Substitute 0 for \( f(x) \) in the function and solve for \( x \).

\[
\begin{align*}
\text{Write the function.} \\
\text{Substitute 0 for } f(x). \\
\text{Solve for } x.
\end{align*}
\]

\( x = -4 \)

\( \boxed{\text{The zero of the function is } -4} \)

**Monitoring Progress**

Find the zero of the function.

5. \( h(x) = 4x - 16 \)
6. \( r(x) = -3x - 12 \)
7. \( r(x) = -\frac{1}{2}x + 5 \)
8. \( g(x) = 2x + \frac{2}{3} \)

**Solving Real-Life Problems**

**EXAMPLE 4** Modeling with Mathematics

An artist rents a booth at an art show for $300. The function \( f(x) = 50x - 300 \) represents the artist’s profit, where \( x \) is the number of paintings the artist sells. Find the zero of the function. Explain what the zero means in this situation.

**SOLUTION**

Substitute 0 for \( f(x) \) in the function and solve for \( x \).

\[
\begin{align*}
\text{Write the function.} \\
\text{Substitute 0 for } f(x). \\
\text{Solve for } x.
\end{align*}
\]

\( x = 6 \)

\( \boxed{\text{The zero of the function is } 6} \). Because the function represents the artist’s profit, the zero of the function represents the number of paintings the artist must sell to recover the cost of renting the booth. So, the artist must sell 6 paintings to recover the cost of renting the booth.

**Monitoring Progress**

9. The function \( f(t) = -2t + 25 \) represents the amount (in gallons) of water remaining in a tub after \( t \) seconds. Find the zero of the function. Explain what the zero means in this situation.
You are planning an awards banquet for your school. You need to rent tables to seat 180 people. Tables come in two sizes. Small tables seat 6 people, and large tables seat 10 people. The equation $6x + 10y = 180$ models this situation, where $x$ is the number of small tables and $y$ is the number of large tables.

a. Graph the equation. Interpret the intercepts.

b. Find four possible solutions in the context of the problem.

**SOLUTION**

1. **Understand the Problem** You know the equation that models the situation. You are asked to graph the equation, interpret the intercepts, and find four solutions.

2. **Make a Plan** Use intercepts to graph the equation. Then use the graph to interpret the intercepts and find other solutions.

3. **Solve the Problem**

   a. Use intercepts to graph the equation. Neither $x$ nor $y$ can be negative, so only graph the equation in the first quadrant.

   b. Only whole-number values of $x$ and $y$ make sense in the context of the problem. Besides the intercepts, it appears that the line passes through the points $(10, 12)$ and $(20, 6)$. To verify that these points are solutions, check them in the equation, as shown.

   So, four possible combinations of tables that will seat 180 people are 0 small and 18 large, 10 small and 12 large, 20 small and 6 large, and 30 small and 0 large.

4. **Look Back** The graph shows that as the number $x$ of small tables increases, the number $y$ of large tables decreases. This makes sense in the context of the problem. So, the graph is reasonable.

**Monitoring Progress**

10. **WHAT IF?** You decide to rent tables from a different company. The situation can be modeled by the equation $4x + 6y = 180$, where $x$ is the number of small tables and $y$ is the number of large tables. Graph the equation and interpret the intercepts.
1. **WRITING** How are x-intercepts and y-intercepts alike? How are they different?

2. **WHICH ONE DOESN’T BELONG?** Which point does not belong with the other three? Explain your reasoning.

   - (0, −3)
   - (0, 0)
   - (4, −3)
   - (4, 0)

### Monitoring Progress and Modeling with Mathematics

#### In Exercises 3–8, graph the linear equation.

(See Example 1.)

3. \(x = 4\)
4. \(y = 2\)
5. \(y = −3\)
6. \(x = −1\)
7. \(x = −\frac{1}{3}\)
8. \(y = \frac{7}{2}\)

In Exercises 9–12, find the x- and y-intercepts of the graph of the linear equation.

9. \(y = -x + 4\)
10. \(y = x + 5\)
11. \(y = \frac{1}{2}x - 2\)
12. \(y = -\frac{3}{2}x - 4\)

In Exercises 19–28, use intercepts to graph the linear equation. Label the points corresponding to the intercepts. (See Example 2.)

19. \(5x + 3y = 30\)
20. \(4x + 6y = 12\)
21. \(-12x + 3y = 24\)
22. \(-2x + 6y = 18\)
23. \(-4x + 3y = -30\)
24. \(-2x + 7y = -21\)
25. \(-x + 2y = 7\)
26. \(3x - y = -5\)
27. \(-\frac{5}{2}x + y = 10\)
28. \(-\frac{1}{2}x + y = -4\)

**ERROR ANALYSIS** In Exercises 29 and 30, describe and correct the error in finding the intercepts of the graph of the equation.

29. \(3x + 12y = 24\)
   \(3x + 12(0) = 24\)
   \(3x = 24\)
   \(x = 8\)
   \(y = 2\)
   The intercept is at (8, 2).

30. \(4x + 10y = 20\)
   \(4x + 10(0) = 20\)
   \(4x = 20\)
   \(x = 5\)
   \(y = 2\)
   The x-intercept is at (0, 5), and the y-intercept is at (2, 0).
In Exercises 31–34, use the graph to find the zero of the function.

31. \( f(x) = \frac{1}{2}x - 3 \)  
32. \( f(x) = x + 4 \)

33. \( f(x) = -2x - 2 \)  
34. \( f(x) = -\frac{1}{3}x - 1 \)

In Exercises 35–44, find the zero of the function. 
(See Example 3.)

35. \( f(x) = x + 2 \)  
36. \( r(x) = x - 5 \)

37. \( m(x) = 3x - 18 \)  
38. \( g(x) = 2x + 4 \)

39. \( q(x) = -5x - 20 \)  
40. \( n(x) = -6x + 18 \)

41. \( h(x) = 4x + 2 \)  
42. \( f(x) = -3x + 1 \)

43. \( g(x) = -\frac{1}{3}x + 7 \)  
44. \( p(x) = \frac{1}{3}x - 3 \)

45. **MODELING WITH MATHEMATICS** The function \( f(x) = -200x + 1000 \) represents the balance (in dollars) in a checking account after \( x \) months. Find the zero of the function. Explain what the zero means in this situation. (See Example 4.)

46. **MODELING WITH MATHEMATICS** The function \( f(t) = -10t + 3000 \) represents the height (in feet) of a skydiver \( t \) seconds after opening the parachute. Find the zero of the function. Explain what the zero means in this situation.

47. **REASONING** The function \( c(x) = 9 + 1.50x \) represents the total cost (in dollars) of a large pizza, where \( x \) is the number of additional toppings. Find the zero of the function. Does the zero make sense in this situation? Explain.

48. **OPEN-ENDED** Consider the equation \( 8 = 4x + 16 \).
Write a function so that the solution of the equation is the zero of the function. Explain your reasoning.

49. **MODELING WITH MATHEMATICS** A football team has an away game, and the bus breaks down. The coaches decide to drive the players to the game in cars and vans. Four players can ride in each car. Six players can ride in each van. There are 48 players on the team. The equation \( 4x + 6y = 48 \) models this situation, where \( x \) is the number of cars and \( y \) is the number of vans. (See Example 5.)

a. Graph the equation. Interpret the intercepts.
b. Find four possible solutions in the context of the problem.

50. **MODELING WITH MATHEMATICS** You are ordering shirts for the math club at your school. Short-sleeved shirts cost $10 each. Long-sleeved shirts cost $12 each. You have a budget of $300 for the shirts. The equation \( 10x + 12y = 300 \) models the total cost, where \( x \) is the number of short-sleeved shirts and \( y \) is the number of long-sleeved shirts.

a. Graph the equation. Interpret the intercepts.
b. Twelve students decide they want short-sleeved shirts. How many long-sleeved shirts can you order?

51. **MAKING AN ARGUMENT** You overhear your friend explaining how to find intercepts to a classmate. Your friend says, “When you want to find the \( x \)-intercept, just substitute 0 for \( x \) and continue to solve the equation.” Is your friend’s explanation correct? Explain.
52. **ANALYZING RELATIONSHIPS** You lose track of how many 2-point baskets and 3-point baskets a team makes in a basketball game. The team misses all the 1-point baskets and still scores 54 points. The equation $2x + 3y = 54$ models the total points scored, where $x$ is the number of 2-point baskets made and $y$ is the number of 3-point baskets made.

a. Find and interpret the intercepts.

b. Can the number of 3-point baskets made be odd? Explain your reasoning.

c. Graph the equation. Find two more possible solutions in the context of the problem.

53. **MULTIPLE REPRESENTATIONS** In Exercises 53–56, match the equation with its graph.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5x + 3y = 30$</td>
<td>A</td>
</tr>
<tr>
<td>$5x + 3y = -30$</td>
<td>B</td>
</tr>
<tr>
<td>$5x - 3y = 30$</td>
<td>C</td>
</tr>
<tr>
<td>$5x - 3y = -30$</td>
<td>D</td>
</tr>
</tbody>
</table>

54. Simplify the expression. (Skills Review Handbook)

63. $\frac{2 - (-2)}{4 - (-4)}$

64. $\frac{14 - 18}{0 - 2}$

65. $\frac{-3 - 9}{8 - (-7)}$

66. $\frac{12 - 17}{-5 - (-2)}$
3.5 Graphing Linear Equations in Slope-Intercept Form

**Essential Question** How can you describe the graph of the equation \( y = mx + b \)?

**Slope** is the rate of change between any two points on a line. It is the measure of the steepness of the line.

To find the slope of a line, find the ratio of the change in \( y \) (vertical change) to the change in \( x \) (horizontal change).

\[
\text{slope} = \frac{\text{change in } y}{\text{change in } x}
\]

**EXPLORATION 1** Finding Slopes and \( y \)-Intercepts

Work with a partner. Find the slope and \( y \)-intercept of each line.

a. \( y = \frac{2}{3}x + 2 \)

b. \( y = -2x - 1 \)

**EXPLORATION 2** Writing a Conjecture

Work with a partner. Graph each equation. Then copy and complete the table. Use the completed table to write a conjecture about the relationship between the graph of \( y = mx + b \) and the values of \( m \) and \( b \).

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description of graph</th>
<th>Slope of graph</th>
<th>( y )-Intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. ( y = -\frac{2}{3}x + 3 )</td>
<td>Line</td>
<td>(-\frac{2}{3})</td>
<td>3</td>
</tr>
<tr>
<td>b. ( y = 2x - 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. ( y = -x + 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. ( y = x - 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Communicate Your Answer**

3. How can you describe the graph of the equation \( y = mx + b \)?

a. How does the value of \( m \) affect the graph of the equation?

b. How does the value of \( b \) affect the graph of the equation?

c. Check your answers to parts (a) and (b) by choosing one equation from Exploration 2 and (1) varying only \( m \) and (2) varying only \( b \).
What You Will Learn

- Find the slope of a line.
- Use the slope-intercept form of a linear equation.
- Use slopes and \( y \)-intercepts to solve real-life problems.

The Slope of a Line

**Core Concept**

**Slope**

The slope \( m \) of a nonvertical line passing through two points \((x_1, y_1)\) and \((x_2, y_2)\) is the ratio of the rise (change in \( y \)) to the run (change in \( x \)).

\[
\text{slope } m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

When the line rises from left to right, the slope is positive. When the line falls from left to right, the slope is negative.

**EXAMPLE 1** Finding the Slope of a Line

Describe the slope of each line. Then find the slope.

**a.**

\[
\begin{align*}
&\text{The line rises from left to right.} \\
&\text{So, the slope is positive.} \\
&m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-2)}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}
\end{align*}
\]

**b.**

\[
\begin{align*}
&m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-1)}{2 - 0} = \frac{-2}{2} = -1
\end{align*}
\]

**STUDY TIP**

When finding slope, you can label either point as \((x_1, y_1)\) and the other point as \((x_2, y_2)\). The result is the same.

**READING**

In the slope formula, \( x_1 \) is read as “\( x \) sub one” and \( y_2 \) is read as “\( y \) sub two.” The numbers 1 and 2 in \( x_1 \) and \( y_2 \) are called subscripts.

**Monitoring Progress**

Describe the slope of the line. Then find the slope.

1. \((−4, 3), (1, 1)\)
2. \((3, 3), (3, 1)\)
3. \((5, 4), (2, −3)\)
### Example 2: Finding Slope from a Table

The points represented by each table lie on a line. How can you find the slope of each line from the table? What is the slope of each line?

<table>
<thead>
<tr>
<th></th>
<th>a.</th>
<th>b.</th>
<th>c.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>14</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

**SOLUTION**

**a.** Choose any two points from the table and use the slope formula. Use the points \((x_1, y_1) = (4, 20)\) and \((x_2, y_2) = (7, 14)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 20}{7 - 4} = \frac{-6}{3} = -2
\]

The slope is \(-2\).

**b.** Note that there is no change in \(y\). Choose any two points from the table and use the slope formula. Use the points \((x_1, y_1) = (-1, 2)\) and \((x_2, y_2) = (5, 2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{5 - (-1)} = \frac{0}{6} = 0
\]

The change in \(y\) is 0.

The slope is 0.

**c.** Note that there is no change in \(x\). Choose any two points from the table and use the slope formula. Use the points \((x_1, y_1) = (-3, 0)\) and \((x_2, y_2) = (-3, 6)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{-3 - (-3)} = \frac{6}{0}
\]

The change in \(x\) is 0.

Because division by zero is undefined, the slope of the line is undefined.

### Monitoring Progress

The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

<table>
<thead>
<tr>
<th></th>
<th>4.</th>
<th>5.</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>y</td>
<td>10</td>
<td>-12</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>-9</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>-6</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>-3</td>
</tr>
</tbody>
</table>

### Concept Summary

**Slope**

<table>
<thead>
<tr>
<th>Positive slope</th>
<th>Negative slope</th>
<th>Slope of 0</th>
<th>Undefined slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>The line rises from left to right.</td>
<td>The line falls from left to right.</td>
<td>The line is horizontal.</td>
<td>The line is vertical.</td>
</tr>
</tbody>
</table>
A linear equation written in the form \( y = 0x + b \), or \( y = b \), is a **constant function**. The graph of a constant function is a horizontal line.

**EXAMPLE 3**  Identifying Slopes and \( y \)-Intercepts

Find the slope and the \( y \)-intercept of the graph of each linear equation.

a. \( y = 3x - 4 \)  
b. \( y = 6.5 \)

c. \( -5x - y = -2 \)

**SOLUTION**

a. \( y = mx + b \)  
   Write the slope-intercept form.

\[
 y = 3x + (-4) \quad \text{Rewrite the original equation in slope-intercept form.}
\]

\( y \)-intercept is \(-4\).

b. The equation represents a constant function. The equation can also be written as \( y = 0x + 6.5 \).

\( y \)-intercept is 6.5.

c. Rewrite the equation in slope-intercept form by solving for \( y \).

\[
-5x - y = -2 \quad \text{Write the original equation.}
\]

\[
+5x \quad +5x \quad \text{Add 5x to each side.}
\]

\[
-y = 5x - 2 \quad \text{Simplify.}
\]

\[
-y = \frac{5x - 2}{-1} \quad \text{Divide each side by -1.}
\]

\[
y = -5x + 2 \quad \text{Simplify.}
\]

\( y \)-intercept is 2.

**STUDY TIP**

For a constant function, every input has the same output. For instance, in Example 3b, every input has an output of 6.5.

**STUDY TIP**

When you rewrite a linear equation in slope-intercept form, you are expressing \( y \) as a function of \( x \).
**EXAMPLE 4** Using Slope-Intercept Form to Graph

Graph \(2x + y = 2\). Identify the \(x\)-intercept.

**SOLUTION**

Step 1 Rewrite the equation in slope-intercept form.

\[ y = -2x + 2 \]

Step 2 Find the slope and the \(y\)-intercept.

\[ m = -2 \quad \text{and} \quad b = 2 \]

Step 3 The \(y\)-intercept is 2. So, plot \((0, 2)\).

Step 4 Use the slope to find another point on the line.

\[ \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} \]

Plot the point that is 1 unit right and 2 units down from \((0, 2)\). Draw a line through the two points. The line crosses the \(x\)-axis at \((1, 0)\). So, the \(x\)-intercept is 1.

**EXAMPLE 5** Graphing from a Verbal Description

A linear function \(g\) models a relationship in which the dependent variable increases 3 units for every 1 unit the independent variable increases. Graph \(g\) when \(g(0) = 3\). Identify the slope, \(y\)-intercept, and \(x\)-intercept of the graph.

**SOLUTION**

Because the function \(g\) is linear, it has a constant rate of change. Let \(x\) represent the independent variable and \(y\) represent the dependent variable.

Step 1 Find the slope. When the dependent variable increases by 3, the change in \(y\) is +3. When the independent variable increases by 1, the change in \(x\) is +1.

So, the slope is \(\frac{3}{1}\), or 3.

Step 2 Find the \(y\)-intercept. The statement \(g(0) = 3\) indicates that when \(x = 0, y = 3\). So, the \(y\)-intercept is 3. Plot \((0, 3)\).

Step 3 Use the slope to find another point on the line. A slope of 3 can be written as \(\frac{-3}{-1}\). Plot the point that is 1 unit left and 3 units down from \((0, 3)\). Draw a line through the two points. The line crosses the \(x\)-axis at \((-1, 0)\). So, the \(x\)-intercept is \(-1\).

The slope is 3, the \(y\)-intercept is 3, and the \(x\)-intercept is \(-1\).

**Monitoring Progress**

Graph the linear equation. Identify the \(x\)-intercept.

9. \(y = 4x - 4\) \hspace{1cm} 10. \(3x + y = -3\) \hspace{1cm} 11. \(x + 2y = 6\)

12. A linear function \(h\) models a relationship in which the dependent variable decreases 2 units for every 5 units the independent variable increases. Graph \(h\) when \(h(0) = 4\). Identify the slope, \(y\)-intercept, and \(x\)-intercept of the graph.
Solving Real-Life Problems
In most real-life problems, slope is interpreted as a rate, such as miles per hour, dollars per hour, or people per year.

EXAMPLE 6  Modeling with Mathematics
A submersible that is exploring the ocean floor begins to ascend to the surface. The elevation \( h \) (in feet) of the submersible is modeled by the function 
\[ h(t) = 650t - 13,000, \]
where \( t \) is the time (in minutes) since the submersible began to ascend.

a. Graph the function and identify its domain and range.
b. Interpret the slope and the intercepts of the graph.

SOLUTION
1. Understand the Problem  You know the function that models the elevation. You are asked to graph the function and identify its domain and range. Then you are asked to interpret the slope and intercepts of the graph.

2. Make a Plan  Use the slope-intercept form of a linear equation to graph the function. Only graph values that make sense in the context of the problem. Examine the graph to interpret the slope and the intercepts.

3. Solve the Problem
   a. The time \( t \) must be greater than or equal to 0. The elevation \( h \) is below sea level and must be less than or equal to 0. Use the slope of 650 and the \( h \)-intercept of \(-13,000\) to graph the function in Quadrant IV.

   The domain is \( 0 \leq t \leq 20 \), and the range is \(-13,000 \leq h \leq 0\).

   b. The slope is 650. So, the submersible ascends at a rate of 650 feet per minute. The \( h \)-intercept is \(-13,000\). So, the elevation of the submersible after 0 minutes, or when the ascent begins, is \(-13,000\) feet. The \( t \)-intercept is 20. So, the submersible takes 20 minutes to reach an elevation of 0 feet, or sea level.

4. Look Back  You can check that your graph is correct by substituting the \( t \)-intercept for \( t \) in the function. If \( h = 0 \) when \( t = 20 \), the graph is correct.

   \[
   h = 650(20) - 13,000 \quad \text{Substitute 20 for} \ t \text{ in the original equation.}
   \]

   \[
   h = 0 \quad \checkmark \text{Simplify.}
   \]

Monitoring Progress
Help in English and Spanish at BigIdeasMath.com

13. WHAT IF?  The elevation of the submersible is modeled by \( h(t) = 500t - 10,000 \).
   (a) Graph the function and identify its domain and range. (b) Interpret the slope and the intercepts of the graph.
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The ________ of a nonvertical line passing through two points is the ratio of the rise to the run.

2. **VOCABULARY** What is a constant function? What is the slope of a constant function?

3. **WRITING** What is the slope-intercept form of a linear equation? Explain why this form is called the slope-intercept form.

4. **WHICH ONE DOESN'T BELONG?** Which equation does not belong with the other three? Explain your reasoning.

   \[ y = -5x - 1 \quad 2x - y = 8 \quad y = x + 4 \quad y = -3x + 13 \]

Monitoring Progress and Modeling with Mathematics

In Exercises 5–8, describe the slope of the line. Then find the slope. (See Example 1.)

5. \[
\begin{array}{c}
\text{points: } (-2, 1) \quad (2, -2) \\
\text{slope: } \frac{1}{3}
\end{array}
\]

6. \[
\begin{array}{c}
\text{points: } (4, 3) \quad (1, -1) \\
\text{slope: } 4
\end{array}
\]

7. \[
\begin{array}{c}
\text{points: } (-2, -3) \quad (2, -3) \\
\text{slope: } 0
\end{array}
\]

8. \[
\begin{array}{c}
\text{points: } (0, 3) \quad (5, -1) \\
\text{slope: } -
\end{array}
\]

In Exercises 9–12, the points represented by the table lie on a line. Find the slope of the line. (See Example 2.)

9. \[
\begin{array}{c|c|c|c|c}
\text{x} & -9 & -5 & -1 & 3 \\
\hline
\text{y} & -2 & 0 & 2 & 4 \\
\end{array}
\]

10. \[
\begin{array}{c|c|c|c|c}
\text{x} & -1 & 2 & 5 & 8 \\
\hline
\text{y} & -6 & -6 & -6 & -6 \\
\end{array}
\]

11. \[
\begin{array}{c|c|c|c|c}
\text{x} & 0 & 0 & 0 & 0 \\
\hline
\text{y} & -4 & 0 & 4 & 8 \\
\end{array}
\]

12. \[
\begin{array}{c|c|c|c|c}
\text{x} & -4 & -3 & -2 & -1 \\
\hline
\text{y} & 2 & -5 & -12 & -19 \\
\end{array}
\]

13. **ANALYZING A GRAPH** The graph shows the distance \( y \) (in miles) that a bus travels in \( x \) hours. Find and interpret the slope of the line.

   \[
   \begin{array}{|c|c|c|c|}
   \hline
   \text{Time (hours)} & 0 & 1 & 2 \\
   \text{Distance (miles)} & 0 & 60 & 120 \\
   \hline
   \end{array}
   \]

14. **ANALYZING A TABLE** The table shows the amount \( x \) (in hours) of time you spend at a theme park and the admission fee \( y \) (in dollars) to the park. The points represented by the table lie on a line. Find and interpret the slope of the line.

   \[
   \begin{array}{|c|c|c|}
   \hline
   \text{Time (hours), } x & 6 & 7 & 8 \\
   \text{Admission (dollars), } y & 54.99 & 54.99 & 54.99 \\
   \hline
   \end{array}
   \]
In Exercises 15–22, find the slope and the y-intercept of the graph of the linear equation. (See Example 3.)

15. \( y = -3x + 2 \)  
16. \( y = 4x - 7 \)  
17. \( y = 6x \)  
18. \( y = -1 \)  
19. \( -2x + y = 4 \)  
20. \( x + y = -6 \)  
21. \( -5x = 8 - y \)  
22. \( 0 = 1 - 2y + 14x \)

**ERROR ANALYSIS** In Exercises 23 and 24, describe and correct the error in finding the slope and the y-intercept of the graph of the equation.

23. \( x = -4y \)  
The slope is \(-4\), and the y-intercept is 0.

24. \( y = 3x - 6 \)  
The slope is 3, and the y-intercept is 6.

In Exercises 25–32, graph the linear equation. Identify the x-intercept. (See Example 4.)

25. \( y = -x + 7 \)  
26. \( y = \frac{1}{2}x + 3 \)  
27. \( y = 2x \)  
28. \( y = -x \)  
29. \( 3x + y = -1 \)  
30. \( x + 4y = 8 \)  
31. \( -y + 5x = 0 \)  
32. \( 2x - y + 6 = 0 \)

In Exercises 33 and 34, graph the function with the given description. Identify the slope, y-intercept, and x-intercept of the graph. (See Example 5.)

33. A linear function \( f \) models a relationship in which the dependent variable decreases 4 units for every 2 units the independent variable increases. The value of the function at 0 is \(-2\).

34. A linear function \( h \) models a relationship in which the dependent variable increases 1 unit for every 5 units the independent variable decreases. The value of the function at 0 is 3.

35. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function \( r \) models the growth of your right index fingernail. The length of the fingernail increases 0.7 millimeter every week. Graph \( r \) when \( r(0) = 12 \). Identify the slope and interpret the y-intercept of the graph.

36. **GRAPHING FROM A VERBAL DESCRIPTION** A linear function \( m \) models the amount of milk sold by a farm per month. The amount decreases 500 gallons for every $1 increase in price. Graph \( m \) when \( m(0) = 3000 \). Identify the slope and interpret the x- and y-intercepts of the graph.

37. **MODELING WITH MATHEMATICS** The function shown models the depth \( d \) (in inches) of snow on the ground during the first 9 hours of a snowstorm, where \( t \) is the time (in hours) after the snowstorm begins. (See Example 6.)

\[ d(t) = \frac{1}{2}t + 6 \]

a. Graph the function and identify its domain and range.

b. Interpret the slope and the \( d \)-intercept of the graph.

38. **MODELING WITH MATHEMATICS** The function \( c(x) = 0.5x + 70 \) represents the cost \( c \) (in dollars) of renting a truck from a moving company, where \( x \) is the number of miles you drive the truck.

a. Graph the function and identify its domain and range.

b. Interpret the slope and the \( c \)-intercept of the graph.

39. **COMPARING FUNCTIONS** A linear function models the cost of renting a truck from a moving company. The table shows the cost \( y \) (in dollars) when you drive the truck \( x \) miles. Graph the function and compare the slope and the y-intercept of the graph with the slope and the \( c \)-intercept of the graph in Exercise 38.

<table>
<thead>
<tr>
<th>Miles, ( x )</th>
<th>Cost, ( y ) (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>100</td>
<td>120</td>
</tr>
</tbody>
</table>
ERROR ANALYSIS  In Exercises 40 and 41, describe and correct the error in graphing the function.

40. \[ y + 1 = 3x \]

41. \[ -4x + y = -2 \]

42. **MATHEMATICAL CONNECTIONS**  Graph the four equations in the same coordinate plane.
   
   \[
   \begin{align*}
   3y &= -x - 3 \\
   2y - 14 &= 4x \\
   4x - 3 &= y \\
   x - 12 &= -3y
   \end{align*}
   \]

   a. What enclosed shape do you think the lines form? Explain.
   
   b. Write a conjecture about the equations of parallel lines.

43. **MATHEMATICAL CONNECTIONS**  The graph shows the relationship between the width \( y \) and the length \( x \) of a rectangle in inches. The perimeter of a second rectangle is 10 inches less than the perimeter of the first rectangle.

   a. Graph the relationship between the width and length of the second rectangle.
   
   b. How does the graph in part (a) compare to the graph shown?

44. **MATHEMATICAL CONNECTIONS**  The graph shows the relationship between the base length \( x \) and the side length (of the two equal sides) \( y \) of an isosceles triangle in meters. The perimeter of a second isosceles triangle is 8 meters more than the perimeter of the first triangle.

   a. Graph the relationship between the base length and the side length of the second triangle.
   
   b. How does the graph in part (a) compare to the graph shown?

45. **ANALYZING EQUATIONS**  Determine which of the equations could be represented by each graph.

   \[
   \begin{align*}
   y &= -3x + 8 \\
   y &= -x - \frac{4}{3} \\
   y &= -7x \\
   y &= 2x - 4 \\
   y &= \frac{7}{4}x - \frac{1}{4} \\
   y &= \frac{1}{3}x + 5 \\
   y &= -4x - 9 \\
   y &= 6
   \end{align*}
   \]

   a. 
   
   b. 
   
   c. 
   
   d. 

46. **MAKING AN ARGUMENT**  Your friend says that you can write the equation of any line in slope-intercept form. Is your friend correct? Explain your reasoning.
47. **Writing** Write the definition of the slope of a line in two different ways.

48. **Thought Provoking** Your family goes on vacation to a beach 300 miles from your house. You reach your destination 6 hours after departing. Draw a graph that describes your trip. Explain what each part of your graph represents.

49. **Analyzing a Graph** The graphs of the functions $g(x) = 6x + a$ and $h(x) = 2x + b$, where $a$ and $b$ are constants, are shown. They intersect at the point $(p, q)$.

![Graph of linear functions](image)

a. Label the graphs of $g$ and $h$.

b. What do $a$ and $b$ represent?

c. Starting at the point $(p, q)$, trace the graph of $g$ until you get to the point with the $x$-coordinate $p + 2$. Mark this point $C$. Do the same with the graph of $h$. Mark this point $D$. How much greater is the $y$-coordinate of point $C$ than the $y$-coordinate of point $D$?

50. **How Do You See It?** You commute to school by walking and by riding a bus. The graph represents your commute.

![Commute to School Graph](image)

a. Describe your commute in words.

b. Calculate and interpret the slopes of the different parts of the graph.

51. $y = 4kx - 5; m = \frac{1}{2}$

52. $y = -\frac{1}{3}x + \frac{5}{2}k; b = -10$

53. **Abstract Reasoning** To show that the slope of a line is constant, let $(x_1, y_1)$ and $(x_2, y_2)$ be any two points on the line $y = mx + b$. Use the equation of the line to express $y_1$ in terms of $x_1$ and $y_2$ in terms of $x_2$. Then use the slope formula to show that the slope between the points is $m$.

---

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Solve the inequality. *(Section 2.4)*

54. $8a - 7 \leq 2(3a - 1)$

55. $-3(2p + 4) > -6p - 5$

56. $4(3h + 1.5) \geq 6(2h - 2)$

57. $-4(x + 6) < 2(x - 9)$

Determine whether the graph or table represents a linear or nonlinear function. Explain. *(Section 3.2)*

58. ![Graph of linear function](image)

59. ![Graph of linear function](image)

60. | $x$ | 0 | 1 | 2 | 3 | 4 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

61. | $x$ | 2 | 4 | 6 | 8 | 10 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
</tr>
</tbody>
</table>
**Essential Question**  How can you describe the relationship between two quantities that vary directly?

Two quantities $x$ and $y$ show **direct variation** when $y = ax$ and $a \neq 0$.

**EXPLORATION 1**  Identifying Direct Variation

Work with a partner. Determine whether $x$ and $y$ show direct variation. Explain your reasoning.

a. Money

<table>
<thead>
<tr>
<th>Hours worked</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings (dollars)</td>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
</tr>
</tbody>
</table>

b. Helicopter

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (meters)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

c. Tickets

<table>
<thead>
<tr>
<th>Number of tickets</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (dollars)</td>
<td>0</td>
<td>40</td>
<td>80</td>
<td>120</td>
</tr>
</tbody>
</table>

d. Pizzas

<table>
<thead>
<tr>
<th>Number of pizzas</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (dollars)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
</tbody>
</table>

e. Laps, $x$

| Time (seconds), $y$ | 90 | 200 | 325 | 480 |

f. Cups of sugar, $x$

| Cups of flour, $y$ | 1 | 2 | 3 | 4 |

**EXPLORATION 2**  Analyzing Relationships

Work with a partner. For the relationships that show direct variation in Exploration 1, do the following.

- Find the slope of the line.
- Find the value of $y$ for the ordered pair $(1, y)$.

What do you notice? What does the value of $y$ represent?

**Communicate Your Answer**

3. How can you describe the relationship between two quantities that vary directly?

4. Give a real-life example of two quantities that show direct variation. Write an equation that represents the relationship and sketch its graph.
What You Will Learn

- Determine whether two quantities show direct variation.
- Write direct variation equations.
- Use direct variation equations to solve real-life problems.

Direct Variation

Two quantities \( x \) and \( y \) show direct variation when \( y = ax \) and \( a \neq 0 \). The number \( a \) is called the constant of variation, and \( y \) is said to vary directly with \( x \). The equation \( y = 2x \) is an example of direct variation, and the constant of variation is 2.

Notice that a direct variation equation \( y = ax \) is a linear equation in slope-intercept form, \( y = mx + b \), with \( m = a \) and \( b = 0 \). The graph of a direct variation equation is a line with a slope of \( a \) that passes through the origin.

EXAMPLE 1  Identifying Direct Variation

Determine whether \( x \) and \( y \) show direct variation. If so, identify the constant of variation.

a. \( 2x - 3y = 0 \)

SOLUTION

a. Solve the equation for \( y \).

\[
2x - 3y = 0
\]

\[
-3y = -2x
\]

\[
y = \frac{2}{3}x
\]

The equation can be rewritten in the form \( y = ax \). So, \( x \) and \( y \) show direct variation. The constant of variation is \( \frac{2}{3} \).

b. \( -x + y = 4 \)

SOLUTION

b. Solve the equation for \( y \).

\[
-x + y = 4
\]

\[
y = x + 4
\]

The equation cannot be rewritten in the form \( y = ax \). So, \( x \) and \( y \) do not show direct variation.

Monitoring Progress

Determine whether \( x \) and \( y \) show direct variation. If so, identify the constant of variation.

1. \( -x + y = 1 \)

2. \( 2x + y = 0 \)

3. \( -4x = -5y \)
**EXAMPLE 2** Identifying Direct Variation

Determine whether $x$ and $y$ show direct variation. Explain.

**a.**

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−1</td>
<td>1</td>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

**b.**

<table>
<thead>
<tr>
<th>$x$</th>
<th>−2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

**SOLUTION**

Plot the ordered pairs. Then draw a line through the points.

- **a.**
  - The line does not pass through the origin. So, $x$ and $y$ do not show direct variation.

- **b.**
  - The line passes through the origin. So, $x$ and $y$ show direct variation.

**Monitoring Progress**

Determine whether $x$ and $y$ show direct variation. Explain.

4. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−4</td>
<td>−6</td>
<td>−8</td>
<td>−10</td>
</tr>
</tbody>
</table>

5. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−2</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

**Writing Direct Variation Equations**

The direct variation equation $y = ax$ can be rewritten as $\frac{y}{x} = a$ for $x \neq 0$. So, in a direct variation, the ratio of $y$ to $x$ is constant for all nonzero data pairs $(x, y)$.

**EXAMPLE 3** Writing a Direct Variation Equation

The table shows the costs $C$ (in dollars) of downloading $s$ songs from a music website.

<table>
<thead>
<tr>
<th>Number of songs, $s$</th>
<th>Cost (dollars), $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.97</td>
</tr>
<tr>
<td>5</td>
<td>4.95</td>
</tr>
<tr>
<td>7</td>
<td>6.93</td>
</tr>
</tbody>
</table>

**a.** Explain why $C$ varies directly with $s$.

**b.** Write a direct variation equation that relates $s$ and $C$.

**SOLUTION**

- **a.** Find the ratio $\frac{C}{s}$ for each data pair $(s, C)$.
  
  \[
  \frac{2.97}{3} = 0.99, \quad \frac{4.95}{5} = 0.99, \quad \frac{6.93}{7} = 0.99 \]

  All of the ratios are equal to 0.99. So, $C$ varies directly with $s$.

- **b.** Because $\frac{C}{s} = 0.99$, the constant of variation is 0.99. So, a direct variation equation is $C = 0.99s$. 

**APPLYING MATHEMATICS**

For real-world data, the ratios may not be exactly equal. You may still be able to use direct variation when the ratios are approximately equal.
6. **WHAT IF?** The website in Example 3 charges a total of $1.99 for the first 5 songs you download and $0.99 for each song after the first 5. Is it reasonable to use a direct variation model for this situation? Explain.

**Solving Real-Life Problems**

**EXAMPLE 4** **Modeling with Mathematics**

The number $s$ of tablespoons of sea salt needed in a saltwater fish tank varies directly with the number $w$ of gallons of water in the tank. A pet shop owner recommends that you add 100 tablespoons of salt to a 20-gallon tank. How many tablespoons of salt should you add to a 30-gallon tank?

**SOLUTION**

1. **Understand the Problem** You know that a 20-gallon tank requires 100 tablespoons of salt and that the amount of salt varies directly with the capacity of the tank. You are asked to find the amount of salt you should add to a 30-gallon tank.

2. **Make a Plan** Use the direct variation equation $y = ax$ and the given values to write a direct variation equation for this situation. Then solve the equation when $w = 30$.

3. **Solve the Problem**

   **Step 1** Write a direct variation equation. Because $s$ varies directly with $w$, you can use the equation $s = aw$. Also use the fact that $s = 100$ when $w = 20$.

   
   $\begin{align*}
   s &= aw \\
   (100) &= a(20) \\
   5 &= a
   \end{align*}$

   A direct variation equation is $s = 5w$.

   **Step 2** Find the number of tablespoons of salt that you should add to a 30-gallon tank. Use the direct variation equation from Step 1.

   
   $\begin{align*}
   s &= 5w \\
   s &= 5(30) \\
   s &= 150
   \end{align*}$

   You should add 150 tablespoons of salt to a 30-gallon tank.

4. **Look Back** Find the ratio $\frac{w}{s}$ for the given data pair and the data calculated in the solution.

   
   $\frac{100}{20} = 5, \quad \frac{150}{30} = 5$

   Both ratios are equal to 5, so the solution makes sense.

7. An object that weighs 100 pounds on Earth would weigh just 6 pounds on Pluto. Assume that weight $P$ on Pluto varies directly with weight $E$ on Earth. What would a boulder that weighs 45 pounds on Pluto weigh on Earth?
1. **COMPLETE THE SENTENCE** Two quantities \( x \) and \( y \) show ________ when \( y = ax \) and \( a \neq 0 \).

2. **WRITING** A line has a slope of \(-3\) and a \(y\)-intercept of 4. Does the equation of the line represent direct variation? Explain.

---

**In Exercises 3–10, determine whether \( x \) and \( y \) show direct variation. If so, identify the constant of variation.** *(See Example 1.)*

3. \( y = x \)
4. \( y = 2x \)
5. \( y = \frac{1}{2}x - 1 \)
6. \( y = 3x + 2 \)
7. \( 4x + y = 1 \)
8. \( -\frac{1}{2}x + y = 0 \)
9. \( -x - 3y = 0 \)
10. \( 4 - 6x = 2y \)

**11. ERROR ANALYSIS** Describe and correct the error in determining whether \( x \) and \( y \) show direct variation.

\[ 6x - y = 0 \]

*Because the equation is not in the form \( y = ax \), it does not represent direct variation.*

**12. ERROR ANALYSIS** Describe and correct the error in identifying the constant of variation for the direct variation equation.

\[ -5x + 3y = 0 \]

\[ 3y = 5x \]

*The constant of variation is 5.*

---

**In Exercises 13–16, determine whether \( x \) and \( y \) show direct variation. Explain.** *(See Example 2.)*

13. \[
\begin{array}{cccccc}
\hline
x & 1 & 2 & 3 & 4 & 6 \\
\hline
y & 5 & 10 & 15 & 20 & 30 \\
\hline
\end{array}
\]
14. \[
\begin{array}{cccccc}
\hline
x & -3 & -1 & 1 & 3 & 5 \\
\hline
y & -2 & 0 & 2 & 4 & 6 \\
\hline
\end{array}
\]

**15.**

\[
\begin{array}{cccccc}
\hline
x & 1 & 3 & 5 & 7 & 9 \\
\hline
y & 6 & 12 & 18 & 24 & 30 \\
\hline
\end{array}
\]

**16.**

\[
\begin{array}{cccccc}
\hline
x & 2 & 4 & 6 & 8 & 10 \\
\hline
y & 1 & 2 & 4 & 8 & 16 \\
\hline
\end{array}
\]

---

**In Exercises 17–22, the ordered pair is a solution of a direct variation equation. Write the equation and identify the constant of variation.**

17. \((5, 6)\)
18. \((2, 1)\)
19. \((-1, 3)\)
20. \((-3, -6)\)
21. \((-2, -7)\)
22. \((-5, 2)\)

---

**In Exercises 23 and 24, determine whether the situation shows direct variation. Explain your reasoning.**

23. A canoe rental costs $20 plus $5 for each hour of the rental.
24. New carpet costs $4 per square foot.

---

25. **MODELING WITH MATHEMATICS** At a recycling center, computers and computer accessories can be recycled for a fee \( f \) based on weight \( w \), as shown in the table. *(See Example 3.)*

<table>
<thead>
<tr>
<th>Weight (pounds), ( w )</th>
<th>10</th>
<th>15</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fee (dollars), ( f )</td>
<td>2.50</td>
<td>3.75</td>
<td>7.50</td>
</tr>
</tbody>
</table>

a. Explain why \( f \) varies directly with \( w \).
b. Write a direct variation equation that relates \( w \) and \( f \).
c. Find the total recycling fee for a computer that weighs 18 pounds and a printer that weighs 10 pounds.
26. **MODELING WITH MATHEMATICS** A jewelry store sells gold chain by the inch. The table shows the prices of various lengths of gold chain.

<table>
<thead>
<tr>
<th>Length (inches), ℓ</th>
<th>7</th>
<th>9</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (dollars), p</td>
<td>8.75</td>
<td>11.25</td>
<td>20.00</td>
<td>22.50</td>
</tr>
</tbody>
</table>

a. Explain why p varies directly with ℓ.
b. Write a direct variation equation that relates ℓ and p.
c. You have $30. What is the longest chain that you can buy?

27. **MODELING WITH MATHEMATICS** At a company, the number h of vacation hours an employee earns varies directly with the number w of weeks the employee works. An employee who works 2 weeks earns 3 vacation hours. Find the number of vacation hours an employee earns for working 8 weeks. *(See Example 4.)*

28. **MODELING WITH MATHEMATICS** Landscapers plan to spread a layer of stone on a path. The number s of bags of stone needed varies directly with the depth d (in inches) of the layer. They need 20 bags to spread a layer of stone that is 2 inches deep. How deep will the layer of stone be when they use 15 bags of stone?

29. **REASONING** The slope of a line is $-\frac{1}{3}$ and the point $(-6, 2)$ lies on the line. Determine whether the equation of the line is a direct variation equation. Explain.

30. **THOUGHT PROVOKING** Two quantities x and y show inverse variation when $y = \frac{a}{x}$ and $a \neq 0$. Give an example of a real-life situation that shows inverse variation.

31. **REASONING** Consider the distance equation $d = rt$, where d is the distance (in feet), r is the rate (in feet per second), and t is the time (in seconds).
a. You run 6 feet per second. Do distance and time vary directly? Explain.
b. You run for 50 seconds. Do distance and rate vary directly? Explain.
c. You run 300 feet. Do rate and time vary directly? Explain.

32. **HOW DO YOU SEE IT?** Consider the graph shown.

33. **CRITICAL THINKING** Consider an equation where y varies directly with x. Does x vary directly with y? If so, what is the relationship between the constants of variation?

---

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Find the coordinates of the figure after the transformation. *(Skills Review Handbook)*

34. Translate the rectangle 4 units left.

35. Dilate the triangle with respect to the origin using a scale factor of 2.

36. Reflect the trapezoid in the y-axis.
Essential Question  How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?

Comparing Graphs of Functions  Work with a partner. The graph of $f(x) = x$ is shown. Sketch the graph of each function, along with $f$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

a. $g(x) = x + 4$

b. $g(x) = x + 2$

c. $g(x) = x - 2$

d. $g(x) = x - 4$

Comparing Graphs of Functions  Work with a partner. Sketch the graph of each function, along with $f$, on the same set of coordinate axes. Use a graphing calculator to check your results. What can you conclude?

a. $h(x) = \frac{1}{2}x$

b. $h(x) = 2x$

c. $h(x) = -\frac{1}{2}x$

d. $h(x) = -2x$

Matching Functions with Their Graphs  Work with a partner. Match each function with its graph. Use a graphing calculator to check your results. Then use the results of Explorations 1 and 2 to compare the graph of $k$ to the graph of $f(x) = x$.

a. $k(x) = 2x - 4$

c. $k(x) = \frac{1}{2}x + 4$

b. $k(x) = -2x + 2$

d. $k(x) = -\frac{1}{2}x - 2$

Communicate Your Answer

4. How does the graph of the linear function $f(x) = x$ compare to the graphs of $g(x) = f(x) + c$ and $h(x) = f(cx)$?
What You Will Learn

- Translate and reflect graphs of linear functions.
- Stretch and shrink graphs of linear functions.
- Combine transformations of graphs of linear functions.

Translating and Reflections

A family of functions is a group of functions with similar characteristics. The most basic function in a family of functions is the parent function. For nonconstant linear functions, the parent function is $f(x) = x$. The graphs of all other nonconstant linear functions are transformations of the graph of the parent function. A transformation changes the size, shape, position, or orientation of a graph.

Core Vocabulary

- family of functions, p. 140
- parent function, p. 140
- transformation, p. 140
- reflection, p. 141
- horizontal shrink, p. 142
- horizontal stretch, p. 142
- vertical stretch, p. 142
- vertical shrink, p. 142

Core Concept

A translation is a transformation that shifts a graph horizontally or vertically but does not change the size, shape, or orientation of the graph.

Horizontal Translations

The graph of $y = f(x - h)$ is a horizontal translation of the graph of $y = f(x)$, where $h \neq 0$.

Vertical Translations

The graph of $y = f(x) + k$ is a vertical translation of the graph of $y = f(x)$, where $k \neq 0$.

EXAMPLE 1

Horizontal and Vertical Translations

Let $f(x) = 2x - 1$. Graph (a) $g(x) = f(x) + 3$ and (b) $t(x) = f(x + 3)$. Describe the transformations from the graph of $f$ to the graphs of $g$ and $t$.

SOLUTION

a. The function $g$ is of the form $y = f(x) + k$, where $k = 3$. So, the graph of $g$ is a vertical translation 3 units up of the graph of $f$.

b. The function $t$ is of the form $y = f(x - h)$, where $h = -3$. So, the graph of $t$ is a horizontal translation 3 units left of the graph of $f$. 
### Reflections in the x-axis

The graph of \( y = -f(x) \) is a reflection in the x-axis of the graph of \( y = f(x) \).

- Multiplying the outputs by \(-1\) changes their signs.

### Reflections in the y-axis

The graph of \( y = f(-x) \) is a reflection in the y-axis of the graph of \( y = f(x) \).

- Multiplying the inputs by \(-1\) changes their signs.

### Reflections in the x-axis and the y-axis

Let \( f(x) = \frac{1}{2}x + 1 \). Graph (a) \( g(x) = -f(x) \) and (b) \( t(x) = f(-x) \). Describe the transformations from the graph of \( f \) to the graphs of \( g \) and \( t \).

#### SOLUTION

a. To find the outputs of \( g \), multiply the outputs of \( f \) by \(-1\). The graph of \( g \) consists of the points \((x, -f(x))\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-4)</th>
<th>(-2)</th>
<th>(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>(-1)</td>
<td>(0)</td>
<td>(1)</td>
</tr>
<tr>
<td>(-f(x))</td>
<td>(1)</td>
<td>(0)</td>
<td>(-1)</td>
</tr>
</tbody>
</table>

The graph of \( g \) is a reflection in the x-axis of the graph of \( f \).

b. To find the outputs of \( t \), multiply the inputs by \(-1\) and then evaluate \( f \). The graph of \( t \) consists of the points \((x, f(-x))\).

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(0)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-x)</td>
<td>(2)</td>
<td>(0)</td>
<td>(-2)</td>
</tr>
<tr>
<td>( f(-x) )</td>
<td>(2)</td>
<td>(1)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

The graph of \( t \) is a reflection in the y-axis of the graph of \( f \).

### Monitoring Progress

Using \( f \), graph (a) \( g \) and (b) \( h \). Describe the transformations from the graph of \( f \) to the graphs of \( g \) and \( h \).

1. \( f(x) = 3x + 1; \ g(x) = f(x) - 2; \ h(x) = f(x - 2) \)
2. \( f(x) = -4x - 2; \ g(x) = -f(x); \ h(x) = f(-x) \)
Stretches and Shrinks

You can transform a function by multiplying all the x-coordinates (inputs) by the same factor $a$. When $a > 1$, the transformation is a **horizontal shrink** because the graph shrinks toward the y-axis. When $0 < a < 1$, the transformation is a **horizontal stretch** because the graph stretches away from the y-axis. In each case, the y-intercept stays the same.

You can also transform a function by multiplying all the y-coordinates (outputs) by the same factor $a$. When $a > 1$, the transformation is a **vertical stretch** because the graph stretches away from the x-axis. When $0 < a < 1$, the transformation is a **vertical shrink** because the graph shrinks toward the x-axis. In each case, the x-intercept stays the same.

**Core Concept**

**Horizontal Stretches and Shrinks**

The graph of $y = f(ax)$ is a **horizontal stretch** or **shrink** by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

**Vertical Stretches and Shrinks**

The graph of $y = a \cdot f(x)$ is a vertical **stretch** or **shrink** by a factor of $a$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

**EXAMPLE 3** Horizontal and Vertical Stretches

Let $f(x) = x - 1$. Graph (a) $g(x) = f\left(\frac{1}{3}x\right)$ and (b) $h(x) = 3f(x)$. Describe the transformations from the graph of $f$ to the graphs of $g$ and $h$.

**SOLUTION**

a. To find the outputs of $g$, multiply the inputs by $\frac{1}{3}$.

\[
\begin{array}{ccc}
    x & f(x) = x - 1 & g(x) = f\left(\frac{1}{3}x\right) \\
    -3 & -4 & 0 \\
    0 & -1 & 1 \\
    3 & 2 & 3 \\
\end{array}
\]

The graph of $g$ is a horizontal stretch of the graph of $f$ by a factor of $1 \div \frac{1}{3} = 3$.

b. To find the outputs of $h$, multiply the outputs of $f$ by 3.

\[
\begin{array}{ccc}
    x & f(x) = x - 1 & 3f(x) = 3f(x) \\
    0 & -1 & 0 \\
    1 & 0 & 3 \\
    2 & 1 & 3 \\
\end{array}
\]

The graph of $h$ is a vertical stretch of the graph of $f$ by a factor of 3.

**STUDY TIP**

The graphs of $y = f(-ax)$ and $y = -a \cdot f(x)$ represent a stretch or shrink and a reflection in the x- or y-axis of the graph of $y = f(x)$. 
EXAMPLE 4  Horizontal and Vertical Shrinks

Let $f(x) = x + 2$. Graph (a) $g(x) = f(4x)$ and (b) $h(x) = \frac{1}{4}f(x)$. Describe the transformations from the graph of $f$ to the graphs of $g$ and $h$.

SOLUTION

a. To find the outputs of $g$, multiply the inputs by 4. Then evaluate $f$. The graph of $g$ consists of the points $(x, f(4x))$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-1</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4x$</td>
<td>-4</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$f(4x)$</td>
<td>-2</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

The graph of $g$ is a horizontal shrink of the graph of $f$ by a factor of $\frac{1}{4}$.

b. To find the outputs of $h$, multiply the outputs of $f$ by $\frac{1}{4}$. The graph of $h$ consists of the points $(x, \frac{1}{4}f(x))$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-2</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\frac{1}{4}f(x)$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
</tr>
</tbody>
</table>

The graph of $h$ is a vertical shrink of the graph of $f$ by a factor of $\frac{1}{4}$.

Monitoring Progress  Help in English and Spanish at BigIdeasMath.com

Using $f$, graph (a) $g$ and (b) $h$. Describe the transformations from the graph of $f$ to the graphs of $g$ and $h$.

3. $f(x) = 4x - 2; g(x) = f\left(\frac{1}{2}x\right); h(x) = 2f(x)$
4. $f(x) = -3x + 4; g(x) = f(2x); h(x) = \frac{1}{2}f(x)$

STUDY TIP
You can perform transformations on the graph of any function $f$ using these steps.

Core Concept

Transformations of Graphs

The graph of $y = a \cdot f(x - h) + k$ or the graph of $y = f(ax - h) + k$ can be obtained from the graph of $y = f(x)$ by performing these steps.

Step 1 Translate the graph of $y = f(x)$ horizontally $h$ units.

Step 2 Use $a$ to stretch or shrink the resulting graph from Step 1.

Step 3 Reflect the resulting graph from Step 2 when $a < 0$.

Step 4 Translate the resulting graph from Step 3 vertically $k$ units.
Combining Transformations

Graph \( f(x) = x \) and \( g(x) = -2x + 3 \). Describe the transformations from the graph of \( f \) to the graph of \( g \).

**SOLUTION**

Note that you can rewrite \( g \) as \( g(x) = -2f(x) + 3 \).

**Step 1** There is no horizontal translation from the graph of \( f \) to the graph of \( g \).

**Step 2** Stretch the graph of \( f \) vertically by a factor of 2 to get the graph of \( h(x) = 2x \).

**Step 3** Reflect the graph of \( h \) in the \( x \)-axis to get the graph of \( r(x) = -2x \).

**Step 4** Translate the graph of \( r \) vertically 3 units up to get the graph of \( g(x) = -2x + 3 \).

**SAE Example 6**

**Solving a Real-Life Problem**

A cable company charges customers $60 per month for its service, with no installation fee. The cost to a customer is represented by \( c(m) = 60m \), where \( m \) is the number of months of service. To attract new customers, the cable company reduces the monthly fee to $30 but adds an installation fee of $45. The cost to a new customer is represented by \( r(m) = 30m + 45 \), where \( m \) is the number of months of service. Describe the transformations from the graph of \( c \) to the graph of \( r \).

**SOLUTION**

Note that you can rewrite \( r \) as \( r(m) = \frac{1}{2}c(m) + 45 \). In this form, you can use the order of operations to get the outputs of \( r \) from the outputs of \( c \). First, multiply the outputs of \( c \) by \( \frac{1}{2} \) to get \( h(m) = 30m \). Then add 45 to the outputs of \( h \) to get \( r(m) = 30m + 45 \).

The transformations are a vertical shrink by a factor of \( \frac{1}{2} \) and then a vertical translation 45 units up.

**Monitoring Progress**

5. Graph \( f(x) = x \) and \( h(x) = \frac{1}{4}x - 2 \). Describe the transformations from the graph of \( f \) to the graph of \( h \).
3.7 Exercises

Vocabulary and Core Concept Check

1. WRITING Describe the relationship between \( f(x) = x \) and all other nonconstant linear functions.

2. VOCABULARY Name four types of transformations. Give an example of each and describe how it affects the graph of a function.

3. WRITING How does the value of \( a \) in the equation \( y = af(x) \) affect the graph of \( y = f(x) \)? How does the value of \( a \) in the equation \( y = f(ax) \) affect the graph of \( y = f(x) \)?

4. REASONING The functions \( f \) and \( g \) are linear functions. The graph of \( g \) is a vertical shrink of the graph of \( f \). What can you say about the \( x \)-intercepts of the graphs of \( f \) and \( g \)? Is this always true? Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the graphs of \( f \) and \( g \) to describe the transformation from the graph of \( f \) to the graph of \( g \). (See Example 1.)

5. \( g(x) = f(x) + 2 \)

6. \( g(x) = f(x + 4) \)

7. \( f(x) = \frac{1}{2}x + 3; g(x) = f(x) - 3 \)

8. \( f(x) = -3x + 4; g(x) = f(x) + 1 \)

9. \( f(x) = -x - 2; g(x) = f(x + 5) \)

10. \( f(x) = \frac{1}{2}x - 5; g(x) = f(x - 3) \)

11. MODELING WITH MATHEMATICS You and a friend start biking from the same location. Your distance \( d \) (in miles) after \( t \) minutes is given by the function \( d(t) = \frac{1}{2}t \). Your friend starts biking 5 minutes after you. Your friend’s distance \( f \) is given by the function \( f(t) = d(t - 5) \). Describe the transformation from the graph of \( d \) to the graph of \( f \).

12. MODELING WITH MATHEMATICS The total cost \( C \) (in dollars) to cater an event with \( p \) people is given by the function \( C(p) = 18p + 50 \). The set-up fee increases by $25. The new total cost \( T \) is given by the function \( T(p) = C(p) + 25 \). Describe the transformation from the graph of \( C \) to the graph of \( T \).

In Exercises 13–16, use the graphs of \( f \) and \( h \) to describe the transformation from the graph of \( f \) to the graph of \( h \). (See Example 2.)

13. \( h(x) = -f(x) \)

14. \( h(x) = f(-x) \)

15. \( f(x) = -5 - x; h(x) = f(-x) \)

16. \( f(x) = \frac{1}{4}x - 2; h(x) = -f(x) \)
In Exercises 17–22, use the graphs of \( f \) and \( r \) to describe the transformation from the graph of \( f \) to the graph of \( r \). (See Example 3.)

17. \[ f(x) = \frac{3}{2}x - 1 \]

18. \[ f(x) = -x \]

19. \( f(x) = -2x - 4; \quad r(x) = f\left(\frac{1}{3}x\right) \)

20. \( f(x) = 3x + 5; \quad r(x) = f\left(\frac{1}{3}x\right) \)

21. \( f(x) = \frac{1}{2}x + 1; \quad r(x) = 3f(x) \)

22. \( f(x) = -\frac{1}{2}x - 2; \quad r(x) = 4f(x) \)

In Exercises 23–28, use the graphs of \( f \) and \( h \) to describe the transformation from the graph of \( f \) to the graph of \( h \). (See Example 4.)

23. \[ h(x) = f(3x) \]

24. \[ h(x) = \frac{1}{3}f(x) \]

25. \( f(x) = 3x - 12; \quad h(x) = \frac{1}{6}f(x) \)

26. \( f(x) = -x + 1; \quad h(x) = f(2x) \)

27. \( f(x) = -2x - 2; \quad h(x) = f(5x) \)

28. \( f(x) = 4x + 8; \quad h(x) = \frac{3}{4}f(x) \)

In Exercises 29–34, use the graphs of \( f \) and \( g \) to describe the transformation from the graph of \( f \) to the graph of \( g \).

29. \( f(x) = x - 2; \quad g(x) = \frac{1}{3}f(x) \)

30. \( f(x) = -4x + 8; \quad g(x) = -f(x) \)

31. \( f(x) = -2x - 7; \quad g(x) = f(x - 2) \)

32. \( f(x) = 3x + 8; \quad g(x) = f\left(\frac{2}{3}x\right) \)

33. \( f(x) = x - 6; \quad g(x) = 6f(x) \)

34. \( f(x) = -x; \quad g(x) = f(x) - 3 \)

In Exercises 35–38, write a function \( g \) in terms of \( f \) so that the statement is true.

35. The graph of \( g \) is a horizontal translation 2 units right of the graph of \( f \).

36. The graph of \( g \) is a reflection in the \( y \)-axis of the graph of \( f \).

37. The graph of \( g \) is a vertical stretch by a factor of 4 of the graph of \( f \).

38. The graph of \( g \) is a horizontal shrink by a factor of \( \frac{1}{2} \) of the graph of \( f \).

ERROR ANALYSIS In Exercises 39 and 40, describe and correct the error in graphing \( g \).

39. \[ g(x) = f(x - 2) \]

40. \[ g(x) = f(-x) \]

In Exercises 41–46, graph \( f \) and \( h \). Describe the transformations from the graph of \( f \) to the graph of \( h \). (See Example 5.)

41. \( f(x) = x; \quad h(x) = \frac{1}{3}x + 1 \)

42. \( f(x) = x; \quad h(x) = 4x - 2 \)

43. \( f(x) = x; \quad h(x) = -3x - 4 \)

44. \( f(x) = x; \quad h(x) = -\frac{1}{2}x + 3 \)

45. \( f(x) = 2x; \quad h(x) = 6x - 5 \)

46. \( f(x) = 3x; \quad h(x) = -3x - 7 \)
47. **MODELING WITH MATHEMATICS** The function \( t(x) = -4x + 72 \) represents the temperature from 5 P.M. to 11 P.M., where \( x \) is the number of hours after 5 P.M. The function \( d(x) = 4x + 72 \) represents the temperature from 10 A.M. to 4 P.M., where \( x \) is the number of hours after 10 A.M. Describe the transformation from the graph of \( t \) to the graph of \( d \).

48. **MODELING WITH MATHEMATICS** A school sells T-shirts to promote school spirit. The school’s profit is given by the function \( P(x) = 8x - 150 \), where \( x \) is the number of T-shirts sold. During the play-offs, the school increases the price of the T-shirts. The school’s profit during the play-offs is given by the function \( Q(x) = 16x - 200 \), where \( x \) is the number of T-shirts sold. Describe the transformations from the graph of \( P \) to the graph of \( Q \). (See Example 6.)

49. **USING STRUCTURE** The graph of \( g(x) = a \cdot f(x - b) + c \) is a transformation of the graph of the linear function \( f \). Select the word or value that makes each statement true.

<table>
<thead>
<tr>
<th>reflection</th>
<th>translation</th>
<th>stretch</th>
<th>shrink</th>
<th>left</th>
<th>right</th>
<th>y-axis</th>
<th>x-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1</td>
<td>0</td>
<td>0.1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. The graph of \( g \) is a vertical ______ of the graph of \( f \) when \( a = 1, b = 2, \) and \( c = 0 \).
b. The graph of \( g \) is a horizontal translation ______ of the graph of \( f \) when \( a = 1, b = 2, \) and \( c = 0 \).
c. The graph of \( g \) is a vertical translation 1 unit up of the graph of \( f \) when \( a = 1, b = 0, \) and \( c = 72 \).

50. **USING STRUCTURE** The graph of \( h(x) = a \cdot f(bx - c) + d \) is a transformation of the graph of the linear function \( f \). Select the word or value that makes each statement true.

<table>
<thead>
<tr>
<th>vertical</th>
<th>horizontal</th>
<th>stretch</th>
<th>shrink</th>
<th>y-axis</th>
<th>x-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>( \frac{1}{5} )</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. The graph of \( h \) is a ______ shrink of the graph of \( f \) when \( a = \frac{1}{3}, b = 1, \) and \( c = 0 \).
b. The graph of \( h \) is a reflection in the ______ of the graph of \( f \) when \( a = 1, b = -1, \) and \( c = 0 \).
c. The graph of \( h \) is a horizontal stretch of the graph of \( f \) by a factor of 5 when \( a = 1, b = _____, \) and \( c = 0 \).

51. **ANALYZING GRAPHS** Which of the graphs are related by only a translation? Explain.

a. A larger hose is found. Then the pool is filled at a rate of 1360 gallons per hour.

b. Before filling up the pool with a hose, a water truck adds 2000 gallons of water to the pool.
53. **ANALYZING RELATIONSHIPS** You have $50 to spend on fabric for a blanket. The amount $m$ (in dollars) of money you have after buying $y$ yards of fabric is given by the function $m(y) = -9.98y + 50$. How does the graph of $m$ change in each situation?

![Fabric: $9.98/yard](image)

a. You receive an additional $10 to spend on the fabric.

b. The fabric goes on sale, and each yard now costs $4.99.

54. **THOUGHT PROVOKING** Write a function $g$ whose graph passes through the point (4, 2) and is a transformation of the graph of $f(x) = x$.

In Exercises 55–60, graph $f$ and $g$. Write $g$ in terms of $f$. Describe the transformation from the graph of $f$ to the graph of $g$.

55. $f(x) = 2x - 5$; $g(x) = 2x - 8$
56. $f(x) = 4x + 1$; $g(x) = -4x - 1$
57. $f(x) = 3x + 9$; $g(x) = 3x + 15$
58. $f(x) = -x - 4$; $g(x) = x - 4$
59. $f(x) = x + 2$; $g(x) = \frac{1}{3}x + 2$
60. $f(x) = x - 1$; $g(x) = 3x - 3$

56. **REASONING** The graph of $f(x) = x + 5$ is a vertical translation 5 units up of the graph of $f(x) = x$. How can you obtain the graph of $f(x) = x + 5$ from the graph of $f(x) = x$ using a horizontal translation?

62. **HOW DO YOU SEE IT?** Match each function with its graph. Explain your reasoning.

- a. $a(x) = f(-x)$
- b. $g(x) = f(x) - 4$
- c. $h(x) = f(x) + 2$
- d. $k(x) = f(3x)$

**REASONING** In Exercises 63–66, find the value of $r$.

![Graphs of functions](image)

63. $f(x) = \frac{2}{3}x + 2$
64. $g(x) = f(x - r)$
65. $f(x) = 3x - 6$
66. $f(x) = \frac{1}{2}x + 8$

67. **CRITICAL THINKING** When is the graph of $y = f(x) + 9$ the same as the graph of $y = f(x) + w$ for linear functions? Explain your reasoning.

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

Solve the formula for the indicated variable. (Section 1.4)

68. Solve for $h$.
   \[ V = \pi r^2 h \]

69. Solve for $w$.
   \[ P = 2 \ell + 2w \]

Solve the inequality. Graph the solution, if possible. (Section 2.2)

70. $x - 3 \leq 14$
71. $16 < x + 4$
72. $-25 \geq x + 7$
73. $x - 5 > -15$
3.4–3.7 What Did You Learn?

Core Vocabulary

standard form, p. 116
x-intercept, p. 117
y-intercept, p. 117
zero of a function, p. 118
slope, p. 124
rise, p. 124
run, p. 124
slope-intercept form, p. 126
constant function, p. 126
direct variation, p. 134
constant of variation, p. 134
family of functions, p. 140
parent function, p. 140
transformation, p. 140
translation, p. 140
reflection, p. 141
horizontal shrink, p. 142
horizontal stretch, p. 142
vertical stretch, p. 142
vertical shrink, p. 142

Core Concepts

Section 3.4
Horizontal and Vertical Lines, p. 116
Using Intercepts to Graph Equations, p. 117
Finding Zeros of Functions, p. 118

Section 3.5
Slope, p. 124
Slope-Intercept Form, p. 126

Section 3.6
Identifying Direct Variation Equations, p. 134
Writing Direct Variation Equations, p. 135

Section 3.7
Horizontal Translations, p. 140
Vertical Translations, p. 140
Reflections in the x-axis, p. 141
Reflections in the y-axis, p. 141
Horizontal Stretches and Shrinks, p. 142
Vertical Stretches and Shrinks, p. 142
Transformations of Graphs, p. 143

Mathematical Thinking

1. Explain how you determined what units of measure to use for the horizontal and vertical axes in Exercise 37 on page 130.

2. Explain your plan for solving Exercise 48 on page 147.

Performance Task

The Cost of a T-Shirt

You receive bids for making T-shirts for your class fundraiser from four companies. To present the pricing information, one company uses a table, one company uses a written description, one company uses an equation, and one company uses a graph. How will you compare the different representations and make the final choice?

To explore the answer to this question and more, go to BigIdeasMath.com.
3.1 Functions (pp. 89–96)

Determine whether the relation is a function. Explain.

Every input has exactly one output.

So, the relation is a function.

Determine whether the relation is a function. Explain.

1. Every input has exactly one output.

So, the relation is a function.

2. Input, x  
   Output, y
   2  5
   5  11
   7  19
   9  12
   14 3

Determine whether the relation is a function. Explain.

3. y = −x + 6 with inputs
   x = 0 and x = 3

4. The function y = 10x + 100 represents the amount y (in dollars) of money in your bank account after you babysit for x hours.
   a. Identify the independent and dependent variables.
   b. You babysit for 4 hours. Find the domain and range of the function.

3.2 Linear Functions (pp. 97–106)

Does the table or equation represent a linear or nonlinear function? Explain.

a. y = 3x − 4
   The equation is in the form y = mx + b.
   So, the equation represents a linear function.

b. So, the function is nonlinear.

Does the table or graph represent a linear or nonlinear function? Explain.

5. Every input has exactly one output.

So, the relation is a function.

6. y = 60 − 8x represents the amount y (in dollars) of money you have after buying x movie tickets. (a) Find the domain of the function. Is the domain discrete or continuous? Explain. (b) Graph the function using its domain. (c) Find the range of the function.
### 3.3 Function Notation (pp. 107–112)

**a.** Evaluate \( f(x) = 3x - 9 \) when \( x = 2 \).

\[
\begin{align*}
  f(x) &= 3x - 9 \\
  f(2) &= 3(2) - 9 \\
  &= 6 - 9 \\
  &= -3
\end{align*}
\]

When \( x = 2 \), \( f(x) = -3 \).

**b.** For \( f(x) = 4x \), find the value of \( x \) for which \( f(x) = 12 \).

\[
\begin{align*}
  f(x) &= 4x \\
  12 &= 4x \\
  3 &= x
\end{align*}
\]

When \( x = 3 \), \( f(x) = 12 \).

### 3.4 Graphing Linear Equations in Standard Form (pp. 115–122)

**Step 1** Find the intercepts. To find the \( x \)-intercept, substitute 0 for \( y \) and solve for \( x \). To find the \( y \)-intercept, substitute 0 for \( x \) and solve for \( y \).

\[
\begin{align*}
  2x + 3y &= 6 \\
  2x + 3(0) &= 6 \\
  x &= 3
\end{align*}
\]

Step 2 Plot the points and draw the line. The \( x \)-intercept is 3, so plot the point \((3, 0)\). The \( y \)-intercept is 2, so plot the point \((0, 2)\). Draw a line through the points.

**Graph the linear equation.**

12. \( 8x - 4y = 16 \)  
13. \( -12x - 3y = 36 \)  
14. \( y = -5 \)  
15. \( x = 6 \)

**Find the zero of the function.**

16. \( p(x) = 2x - 10 \)  
17. \( v(x) = 0.25x + 3 \)  
18. \( b(x) = -8x + 4 \)
3.5 Graphing Linear Equations in Slope-Intercept Form (pp. 123–132)

a. The points represented by the table lie on a line. How can you find the slope of the line from the table? What is the slope of the line?

Choose any two points from the table and use the slope formula.

Use the points \((x_1, y_1) = (1, -7)\) and \((x_2, y_2) = (4, 2)\).

\[
\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-7)}{4 - 1} = \frac{9}{3} = 3
\]

The slope is 3.

b. Graph \(-\frac{1}{2}x + y = 1\). Identify the \(x\)-intercept.

Step 1 Rewrite the equation in slope-intercept form.

\[y = \frac{1}{2}x + 1\]

Step 2 Find the slope and the \(y\)-intercept.

\[m = \frac{1}{2} \quad \text{and} \quad b = 1\]

Step 3 The \(y\)-intercept is 1. So, plot \((0, 1)\).

Step 4 Use the slope to find another point on the line.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{1}{2}
\]

Plot the point that is 2 units right and 1 unit up from \((0, 1)\). Draw a line through the two points.

The line crosses the \(x\)-axis at \((-2, 0)\). So, the \(x\)-intercept is \(-2\).

The points represented by the table lie on a line. Find the slope of the line.

19. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>16</td>
<td>21</td>
</tr>
<tr>
<td>21</td>
<td>27</td>
</tr>
</tbody>
</table>

20. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-5</td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

21. | \(x\) | \(y\) |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-1</td>
</tr>
<tr>
<td>-3</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>9</td>
<td>-1</td>
</tr>
</tbody>
</table>

Graph the linear equation. Identify the \(x\)-intercept.

22. \(y = 2x + 4\)  
23. \(-5x + y = -10\)  
24. \(x + 3y = 9\)

25. A linear function \(h\) models a relationship in which the dependent variable decreases 2 units for every 3 units the independent variable increases. Graph \(h\) when \(h(0) = 2\). Identify the slope, \(y\)-intercept, and \(x\)-intercept of the graph.
Modeling Direct Variation (pp. 133–138)

a. Determine whether $x$ and $y$ show direct variation. Explain.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
</tr>
</tbody>
</table>

Plot the ordered pairs. Then draw a line through the points.

The line does not pass through the origin. So, $x$ and $y$ do not show direct variation.

b. The table shows the costs $C$ (in dollars) for $h$ hours of repair work on your car. Explain why $C$ varies directly with $h$. Then write a direct variation equation that relates $h$ and $C$.

<table>
<thead>
<tr>
<th>Number of hours, $h$</th>
<th>Cost (dollars), $C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>360</td>
</tr>
<tr>
<td>5</td>
<td>450</td>
</tr>
<tr>
<td>6</td>
<td>540</td>
</tr>
</tbody>
</table>

Find the ratio $\frac{C}{h}$ for each data pair $(h, C)$.

\[
\frac{360}{4} = 90, \quad \frac{450}{5} = 90, \quad \frac{540}{6} = 90
\]

All of the ratios are equal to 90. So, $C$ varies directly with $h$.

Because $\frac{C}{h} = 90$, the constant of variation is 90. So, a direct variation equation is $C = 90h$.

Determine whether $x$ and $y$ show direct variation. Explain.

26. $-6x - y = 1$

27. $4x + y = 0$

28. | $x$ | 0 | 1 | 2 | 3 |
    |----|---|---|---|---|
    | $y$ | 4 | 1 | -2 | -5 |

29. | $x$ | -4 | -2 | 0 | 2 |
    |----|----|---|---|
    | $y$ | -6 | -3 | 0 | 3 |

30. The table shows the number $p$ of gallons of paint needed to cover $w$ square feet of wall space.

a. Explain why $p$ varies directly with $w$. Then write a direct variation equation that relates $w$ and $p$.

b. How much paint do you need to cover 1400 square feet of wall space?

<table>
<thead>
<tr>
<th>Wall space (ft$^2$), $w$</th>
<th>Paint (gallons), $p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>2</td>
</tr>
<tr>
<td>2100</td>
<td>6</td>
</tr>
<tr>
<td>3500</td>
<td>10</td>
</tr>
</tbody>
</table>
3.7 Transformations of Graphs of Linear Functions  (pp. 139–148)

a. Let \( f(x) = -\frac{1}{2}x + 2 \). Graph \( t(x) = f(x - 3) \). Describe the transformation from the graph of \( f \) to the graph of \( t \).

The function \( t \) is of the form \( y = f(x - h) \), where \( h = 3 \). So, the graph of \( t \) is a horizontal translation 3 units right of the graph of \( f \).

![Graph showing transformation from \( f(x) \) to \( t(x) \)](image)

b. Graph \( f(x) = x \) and \( g(x) = -3x - 2 \). Describe the transformations from the graph of \( f \) to the graph of \( g \).

Note that you can rewrite \( g \) as \( g(x) = -3f(x) - 2 \).

**Step 1** There is no horizontal translation from the graph of \( f \) to the graph of \( g \).
**Step 2** Stretch the graph of \( f \) vertically by a factor of 3 to get the graph of \( h(x) = 3x \).
**Step 3** Reflect the graph of \( h \) in the \( x \)-axis to get the graph of \( r(x) = -3x \).
**Step 4** Translate the graph of \( r \) vertically 2 units down to get the graph of \( g(x) = -3x - 2 \).

![Graph showing transformations from \( f(x) \) to \( g(x) \)](image)

Let \( f(x) = 3x + 4 \). Graph \( f \) and \( h \). Describe the transformation from the graph of \( f \) to the graph of \( h \).

31. \( h(x) = f(x + 3) \)  
32. \( h(x) = f(x) + 1 \)
33. \( h(x) = f(-x) \)  
34. \( h(x) = -f(x) \)
35. \( h(x) = 3f(x) \)  
36. \( h(x) = f(6x) \)
37. Graph \( f(x) = x \) and \( g(x) = 5x + 1 \). Describe the transformations from the graph of \( f \) to the graph of \( g \).
Determine whether the relation is a function. If the relation is a function, determine whether the function is linear or nonlinear. Explain.

1. \[
\begin{array}{c|cccc}
   x & -1 & 0 & 1 & 2 \\
   \hline
   y & 6 & 5 & 9 & 14 \\
\end{array}
\]

2. \[y = -2x + 3\]

3. \[x = -2\]

Graph the equation and identify the intercept(s). Find the slope of the line.

4. \[2x - 3y = 6\]

5. \[y = 4.5\]

6. \[y = -7x\]

Find the domain and range of the function represented by the graph. Determine whether the domain is discrete or continuous. Explain.

7. [Graph of a linear function]

8. [Graph of another linear function]

Graph \(f\) and \(g\). Describe the transformations from the graph of \(f\) to the graph of \(g\).

9. \[f(x) = 2x + 4; \ g(x) = \frac{1}{2}f(x)\]

10. \[f(x) = x; \ g(x) = -x + 3\]

Find the zero of the function.

11. \[h(x) = \frac{1}{3}x - 1\]

12. \[d(x) = -8x - 2\]

13. Function A represents the amount of money in a jar based on the number of quarters in the jar. Function B represents your distance from home over time. Compare the domains.

14. A mountain climber is scaling a 500-foot cliff. The graph shows the elevation of the climber over time.
   a. Find and interpret the slope and the \(y\)-intercept of the graph.
   b. Explain two ways to find \(f(3)\). Then find \(f(3)\) and interpret its meaning.
   c. How long does it take the climber to reach the top of the cliff? Justify your answer.

15. Without graphing, compare the slopes and the intercepts of the graphs of the functions \(f(x) = x + 1\) and \(g(x) = f(2x)\).

16. You are making cranberry sauce. The number \(c\) of cups of fresh cranberries varies directly with the number \(s\) of servings of cranberry sauce. A recipe that serves 16 people calls for 4 cups of fresh cranberries. How many cups of fresh cranberries should you use to make 28 servings of cranberry sauce?
1. Which statement is true for the function whose graph is shown? (TEKS A.2.A)
   
   - A) The domain is \( x \leq 0 \).
   - B) The domain is \( x \leq -2 \).
   - C) The range is \( y \leq -2 \).
   - D) The range is \( y \geq -2 \).

2. You want to buy a jacket at a clothing store, and you can spend at most $30. You have a coupon for 10% off any item in the store. Which inequality describes the original prices \( p \) (in dollars) of jackets you can buy? (TEKS A.5.B)
   
   - F) \( p \geq 27.27 \)
   - G) \( p \leq 27.27 \)
   - H) \( p \leq 33.33 \)
   - J) \( p \geq 33.33 \)

3. GRIDDED ANSWER The graph shows the numbers of T-shirts and tank tops you can buy with the amount of money you have. Suppose you buy only T-shirts. What is the greatest number you can buy? (TEKS A.3.C)

4. What is the solution of \( 3x + 24 = 18x - 5 \)? (TEKS A.5.A)
   
   - A) \( x = \frac{15}{29} \)
   - B) \( x = \frac{29}{15} \)
   - C) \( x = -\frac{15}{29} \)
   - D) \( x = \frac{19}{21} \)

5. The value of \( y \) varies directly with \( x \). Which function represents the relationship between \( x \) and \( y \) if \( y = \frac{10}{3} \) when \( x = 60 \)? (TEKS A.2.D)
   
   - F) \( y = 200x \)
   - G) \( y = \frac{1}{18}x \)
   - H) \( y = \frac{190}{3}x \)
   - J) \( y = 18x \)

6. On the five science tests you have taken this semester, you received the following scores: 75, 82, 90, 84, and 71. You want a mean score of at least 80 after you take the sixth test. Which inequality describes the scores \( s \) you can earn on your sixth test to meet your goal? (TEKS A.5.B)
   
   - A) \( s > 76 \)
   - B) \( s > 78 \)
   - C) \( s \geq 78 \)
   - D) \( s \geq 80 \)
7. The number of hours of daylight in Austin, Texas, during the month of March can be modeled by the function \( \ell(x) = 0.03x + 11.5 \), where \( x \) is the day of the month. The number of hours of darkness can be modeled by \( d \). The graphs of \( \ell \) and \( d \) are shown. Which equation describes the relationship between \( \ell \) and \( d \)? (TEKS A.3.E)

- [F] \( d(x) = 24 - \ell(x) \)
- [G] \( \ell(x) = 24 + d(x) \)
- [H] \( \ell(x) = 12 - d(x) \)
- [J] none of the above

8. Solve the literal equation \( xy = 2z - 3y \) for \( y \). (TEKS A.12.E)

- [A] \( y = \frac{1}{3x}(2z) \)
- [B] \( y = \frac{2z}{x + 3} \)
- [C] \( y = \frac{1}{3}(2z - xy) \)
- [D] \( y = 2z - x - 3 \)

9. What is the slope of the line shown? (TEKS A.3.A)

- [F] 5
- [G] \( \frac{1}{5} \)
- [H] 0
- [J] -5

10. For which value of \( a \) does the solution of the compound inequality \( a < 3x + 8 \) or \( a < -4x - 1 \) consist of numbers greater than 5 or less than -6? (TEKS A.5.B)

- [A] 16
- [B] 19
- [C] 23
- [D] 26

11. For the function \( f, f(-5) = 3 \), and \( f(3) = -2 \). If \( y = f(x) \), what is the value of \( y \) when \( x = 3 \)? (TEKS A.12.B)

- [F] -5
- [G] -2
- [H] 3
- [J] 5