1 Linear Functions

1.1 Interval Notation and Set Notation
1.2 Parent Functions and Transformations
1.3 Transformations of Linear and Absolute Value Functions
1.4 Solving Absolute Value Equations
1.5 Solving Absolute Value Inequalities
1.6 Modeling with Linear Functions

Mathematical Thinking: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
Maintaining Mathematical Proficiency

Evaluating Expressions \((6.7.A)\)

**Example 1** Evaluate the expression \(36 \div (3^2 \times 2) - 3\).

\[
\begin{align*}
36 \div (3^2 \times 2) - 3 &= 36 \div (9 \times 2) - 3 \\
&= 36 \div 18 - 3 \\
&= 2 - 3 \\
&= -1
\end{align*}
\]

Evaluate the power within parentheses. Multiply within parentheses. Divide. Subtract.

Evaluate.

1. \(5 \cdot 2^3 + 7\)  
2. \(4 - 2(3 + 2)^2\)  
3. \(48 \div 4^2 + \frac{3}{5}\)  
4. \(50 \div 5^2 \cdot 2\)  
5. \(\frac{1}{2}(2^5 + 22)\)  
6. \(\frac{1}{6}(6 + 18) - 2^2\)

Transformations of Figures \((8.10.C)\)

**Example 2** Reflect the black rectangle in the \(x\)-axis. Then translate the new rectangle 5 units to the left and 1 unit down.

Graph the transformation of the figure.

7. Translate the rectangle 1 unit right and 4 units up.
8. Reflect the triangle in the \(y\)-axis. Then translate 2 units left.
9. Translate the trapezoid 3 units down. Then reflect in the \(x\)-axis.

10. **ABSTRACT REASONING** Give an example to show why the order of operations is important when evaluating a numerical expression. Is the order of transformations of figures important? Justify your answer.
Techniques for Using a Graphing Calculator

Core Concept

Standard and Square Viewing Windows
A typical screen on a graphing calculator has a height-to-width ratio of 2 to 3. This means that when you view a graph using the standard viewing window of \( -10 \) to 10 (on each axis), the graph will not be shown in its true perspective.

To view a graph in its true perspective, you need to change to a square viewing window, where the tick marks on the x-axis are spaced the same as the tick marks on the y-axis.

EXAMPLE 1 Using a Graphing Calculator

Use a graphing calculator to graph \( y = |x| - 3 \).

SOLUTION

In the standard viewing window, notice that the tick marks on the y-axis are closer together than those on the x-axis. This implies that the graph is not shown in its true perspective.

In a square viewing window, notice that the tick marks on both axes have the same spacing. This implies that the graph is shown in its true perspective.

Monitoring Progress

Use a graphing calculator to graph the equation using the standard viewing window and a square viewing window. Describe any differences in the graphs.

1. \( y = 2x - 3 \) 2. \( y = |x + 2| \) 3. \( y = -x^2 + 1 \)
4. \( y = \sqrt{x - 1} \) 5. \( y = x^3 - 2 \) 6. \( y = 0.25x^3 \)

Determine whether the viewing window is square. Explain.

7. \( -8 \leq x \leq 8, \ -2 \leq y \leq 8 \) 8. \( -7 \leq x \leq 8, \ -2 \leq y \leq 8 \)
9. \( -6 \leq x \leq 9, \ -2 \leq y \leq 8 \) 10. \( -2 \leq x \leq 2, \ -3 \leq y \leq 3 \)
11. \( -4 \leq x \leq 5, \ -3 \leq y \leq 3 \) 12. \( -4 \leq x \leq 4, \ -3 \leq y \leq 3 \)
1.1 Interval Notation and Set Notation

**Essential Question** When is it convenient to use set-builder notation to represent a set of numbers?

A collection of objects is called a **set**. You can use braces { } to represent a set by listing its members or by using **set-builder notation** to define the set in terms of the properties of its members. For instance, the set of the numbers 1, 2, and 3 can be denoted as

\[
\{1, 2, 3\}
\]

and the set of all odd whole numbers can be denoted as

\[
\{x \mid x \text{ is a whole number and } x \text{ is odd}\}
\]

which is read “The set of all real numbers \(x\) such that \(x\) is a whole number and \(x\) is odd.”

If all of the members of a set \(A\) are also members of a set \(B\), then set \(A\) is a **subset** of set \(B\).

For instance, if set \(A = \{a, b\}\) and set \(B = \{a, b, c, d\}\), then set \(A\) is a subset of set \(B\).

**EXPLORATION 1** Writing Subsets in Set Notation

**Work with a partner.** Write all the nonempty subsets of each set.

<table>
<thead>
<tr>
<th>a. {4, 5}</th>
<th>b. {c, d}</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. {2, 4, 6}</td>
<td>d. {e, f, g, h}</td>
</tr>
</tbody>
</table>

**EXPLORATION 2** Writing Subsets in Set Notation

**Work with a partner.** Write each given subset of the real numbers in set-builder notation. Describe each set-subset relationship among these sets.

<table>
<thead>
<tr>
<th>a. the integers</th>
<th>b. the whole numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>c. the natural numbers</td>
<td>d. the rational numbers</td>
</tr>
<tr>
<td>e. the irrational numbers</td>
<td>f. the positive integers</td>
</tr>
</tbody>
</table>

**EXPLORATION 3** Writing Subsets in Set Notation

**Work with a partner.** Write each indicated set of numbers using either braces to list its members or set-builder notation. Explain your choice of notation.

| a. the whole numbers 50 through 54 |
| b. the real numbers 0 through 4 |
| c. the prime whole numbers |
| d. the integers −100 through 100 |

**Communicate Your Answer**

4. When is it convenient to use set-builder notation to represent a set of numbers?

5. What are some relationships between subsets of the real numbers?
What You Will Learn

- Represent intervals using interval notation.
- Represent intervals using set-builder notation.

Using Interval Notation

In mathematics, a collection of objects is called a set. You can use braces { } to represent a set by listing its members or elements. For instance, the set

\{1, 2, 3\}  \hspace{1cm} \text{A set with three members}

contains the three numbers 1, 2, and 3. Many sets are also described in words, such as the set of real numbers.

If all the members of a set \(A\) are also members of a set \(B\), then set \(A\) is a subset of set \(B\). The set of natural numbers \(\{1, 2, 3, 4, \ldots\}\) is a subset of the set of real numbers. The diagram shows several important subsets of the real numbers.

Many subsets of the real numbers can be represented as intervals on the real number line.

Core Concept

Bounded Intervals on the Real Number Line

Let \(a\) and \(b\) be two real numbers such that \(a < b\). Then \(a\) and \(b\) are the endpoints of four different bounded intervals on the real number line, as shown below. A bracket or closed circle indicates that the endpoint is included in the interval and a parenthesis or open circle indicates that the endpoint is not included in the interval.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Interval Notation</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a \leq x \leq b)</td>
<td>([a, b])</td>
<td>![Graph for (a \leq x \leq b)]</td>
</tr>
<tr>
<td>(a &lt; x &lt; b)</td>
<td>((a, b))</td>
<td>![Graph for (a &lt; x &lt; b)]</td>
</tr>
<tr>
<td>(a \leq x &lt; b)</td>
<td>([a, b))</td>
<td>![Graph for (a \leq x &lt; b)]</td>
</tr>
<tr>
<td>(a &lt; x \leq b)</td>
<td>((a, b])</td>
<td>![Graph for (a &lt; x \leq b)]</td>
</tr>
</tbody>
</table>

The length of any bounded interval, \([a, b]\), \((a, b)\), \([a, b)\), or \((a, b]\), is the distance between its endpoints: \(b - a\). Any bounded interval has a finite length. An interval that does not have a finite length is called unbounded or infinite.
Writing Interval Notation

Write each interval in interval notation.

a. \(-2 \leq x \leq 3\)

b. \(x > -1\)

c. [Graph of interval from -5 to 5]

d. [Graph of interval from -5 to 5]

SOLUTION

a. The graph of \(-2 \leq x \leq 3\) is the bounded interval \([-2, 3]\).

b. The graph of \(x > -1\) is the unbounded interval \((-1, \infty)\).

c. The graph represents all the real numbers between \(-3\) and 4, including the endpoint \(-3\). This is the bounded interval \([-3, 4]\).

d. The graph represents all the real numbers less than or equal to 3. This is the unbounded interval \((\neg \infty, 3]\).

Monitoring Progress

Write the interval in interval notation.

1. \(-7 < x < -4\)

2. \(x \leq 5\)

3. [Graph of interval from -5 to 5]
Using Set-Builder Notation

Another way to represent intervals is to write them in set-builder notation.

**Core Concept**

**Set-Builder Notation**

Set-builder notation uses symbols to define a set in terms of the properties of the members of the set.

**Words**

Set-builder notation

\{x | x ≤ a or x > b\}

\{x | x ≠ a\}

**Graph**

The set of all real numbers \(x\) such that \(x\) is less than \(b\)

**EXAMPLE 2**

Using Set-Builder Notation

Sketch the graph of each set of numbers.

\[a. \{x | 2 < x ≤ 5\} \quad b. \{x | x ≤ 0 or x > 4\}\]

**SOLUTION**

\[a. \text{ The real numbers in the set satisfy both } x > 2 \text{ and } x ≤ 5.\]

\[b. \text{ The real numbers in the set satisfy either } x ≤ 0 \text{ or } x > 4.\]

**EXAMPLE 3**

Writing Set-Builder Notation

Write the set of numbers in set-builder notation.

\[a. \text{ the set of all integers greater than 5} \quad b. \text{ (−∞, −1) or (−1, ∞)}\]

**SOLUTION**

\[a. x \text{ is greater than 5 and } x \text{ is an integer.} \quad b. x \text{ can be any real number except } −1.\]

\[\{x | x > 5 \text{ and } x ∈ ℤ\} \quad \{x | x ≠ −1\}\]

**Monitoring Progress**

Sketch the graph of the set of numbers.

\[4. \{x | −6 < x ≤ −2\} \quad 5. \{x | x ≤ 0 or x ≥ 10\}\]

Write the set of numbers in set-builder notation.

\[6. \text{ (−∞, −1) or (1, ∞)} \quad 7. \text{ the set of all integers except } −4\]

**Understanding Mathematical Terms**

The symbol \(∈\) denotes membership in a set. The expression \(x ∈ ℤ\) means that \(x\) is a member (or element) of the set of integers.
Section 1.1

1. **COMPLETE THE SENTENCE** Two real numbers \( a \) and \( b \) are the ________ of four different ________ intervals on the real number line.

2. **WHICH ONE DOESN'T BELONG?** The graph of which set of numbers does not belong with the other three? Explain.

\[ (-3, 5] \quad \{x \mid -3 < x \leq 5 \]  
the set of all integers greater than -3 and less than or equal to 5

In Exercises 3–6, use braces to list the elements of the set.

3. the set of whole numbers less than 10
4. the set of odd whole numbers less than 24
5. the set of integers greater than 50
6. the set of integers less than -8

In Exercises 7–16, write the interval in interval notation. (See Example 1.)

7. \( 3 < x < 9 \)  
8. \( -5 < x < 20 \)
9. \( x \geq -13 \)  
10. \( x \leq 58 \)
11. \[ -6 \quad -4 \quad -2 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \]  
12. \[ -20 \quad -10 \quad 0 \quad 10 \quad 20 \quad 30 \quad 40 \]
13. \[ -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \]
14. \[ -10 \quad -5 \quad 0 \quad 5 \quad 10 \quad 15 \quad 20 \quad 25 \]
15. the real numbers from -10 through 10
16. the real numbers between 110 and 220

In Exercises 17–20, sketch the graph of the set of numbers. (See Example 2.)

17. \( \{x \mid 3 < x < 12\} \)
18. \( \{x \mid -10 \leq x \leq 15\} \)
19. \( \{x \mid x < 5 \text{ or } x > 10\} \)
20. \( \{x \mid x \neq 4\} \)

In Exercises 21–28, write the set of numbers in set-builder notation. (See Example 3.)

21. \[ [-5, 16] \]
22. \[ (22, 98] \]
23. \( (-\infty, -4] \text{ or } [4, \infty) \)
24. \( (-\infty, 5] \text{ or } [14, \infty) \)
25. the set of all integers less than -20
26. the set of all real numbers greater than 19 and less than 32
27. the set of all real numbers except 100
28. the set of all whole numbers except 50
29. **ERROR ANALYSIS** Describe and correct the error in rewriting the interval \( (-\infty, -8] \) in set-builder notation.

\[ \{x \mid x < -8\} \]

30. **ERROR ANALYSIS** Describe and correct the error in rewriting the interval \( [-7, 24) \) in set-builder notation.

\[ \{x \mid x \geq -7 \text{ or } x < 24\} \]
31. **MODELING WITH MATHEMATICS** The elevation relative to sea level in the United States ranges from −282 feet in Death Valley, California to 20,320 feet on Mount McKinley, Alaska. Write the range of elevations in interval notation and in set-builder notation.

32. **MODELING WITH MATHEMATICS** The main floor of an auditorium ranges from 6 feet below the stage to 8 feet above the stage. The floor of the balcony ranges from 26 to 37 feet above the stage. Write the range of the floor levels relative to the stage in interval notation and in set-builder notation.

33. **MAKING AN ARGUMENT** Your friend says that it is impossible to write the set
\[ \{ x \mid x \geq 30 \text{ and } x \leq 60, \text{ and } x \in \mathbb{W} \} \]
in interval notation. Is your friend correct? Explain.

34. **HOW DO YOU SEE IT?** The graphs show the legal driving speeds (in miles per hour) on two different roadways.

- **A**
  - Legal speeds (miles per hour)
  - 0 10 20 30 40 50 60

- **B**
  - Legal speeds (miles per hour)
  - 20 30 40 50 60 70 80

a. Write the legal driving speeds shown in Graph A in interval notation, in set-builder notation, and in words.

b. Write the legal driving speeds shown in Graph B in interval notation, in set-builder notation, and in words.

c. One of the roadways is a state highway and the other is a residential street. Which graph represents each roadway?

35. **NUMBER SENSE** Write each set using braces to list the elements, in interval notation, and in set-builder notation. If not possible, explain why.

   a. the set of even whole numbers
   b. the set of real numbers less than −4
   c. the set of real numbers 10 or more units from 50

36. **THOUGHT PROVOKING** Explain how you can add the same number to each member of the set of whole numbers to produce another important subset of the real numbers.

37. **MATHEMATICAL CONNECTIONS** You are marking a rectangular paintball zone that must be 34 meters wide and have a perimeter of at least 140 meters but not more than 260 meters. Find the interval for the length \( x \) of the rectangular paintball zone.

38. **MATHEMATICAL CONNECTIONS** You have 20 gallons of roof coating to apply to the roof of a mobile home that is 16 feet wide. Twenty gallons covers 760 to 1000 square feet. Find the interval for the length \( x \) that you will cover before you need to buy more roof coating.

### Maintaining Mathematical Proficiency

**Reviewing what you learned in previous grades and lessons**

Complete the table of values for the function \( f \). Then graph the function. *(Skills Review Handbook)*

39. \( f(x) = 4x \)  
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

40. \( f(x) = 2x + 2 \)  
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

41. \( f(x) = 3x^2 \)  
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

42. \( f(x) = 2x^2 − 3 \)  
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>−2</th>
<th>−1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Essential Question What are the characteristics of some of the basic parent functions?

An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is

\[ f(x) = |x|. \]

### EXPLORATION 1 Graphing the Parent Absolute Value Function

**Work with a partner.** Complete the table. Then use the values in the table to sketch the graph of the parent absolute value function \( f(x) = |x|. \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### EXPLORATION 2 Identifying Basic Parent Functions

**Work with a partner.** Graphs of four basic parent functions are shown below. Classify each function as **constant**, **linear**, **quadratic**, or **exponential**. Justify your reasoning.

a. 

b. 

c. 

d. 

### Communicate Your Answer

3. What are the characteristics of some of the basic parent functions?

4. Write an equation for each function whose graph is shown in Exploration 2. Then use a graphing calculator to verify that your equations are correct.
What You Will Learn

- Identify families of functions.
- Describe transformations of parent functions.
- Describe combinations of transformations.

Identifying Function Families

Functions that belong to the same *family* share key characteristics. The *parent function* is the most basic function in a family. Functions in the same family are transformations of their parent function.

Core Vocabulary

- absolute value function, p. 9
- parent function, p. 10
- transformation, p. 11
- reflection, p. 11
- vertical stretch, p. 12
- vertical shrink, p. 12

Core Concept

Parent Functions

<table>
<thead>
<tr>
<th>Family</th>
<th>Constant</th>
<th>Linear</th>
<th>Absolute Value</th>
<th>Quadratic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rule</td>
<td>$f(x) = 1$</td>
<td>$f(x) = x$</td>
<td>$f(x) =</td>
<td>x</td>
</tr>
<tr>
<td>Graph</td>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
<tr>
<td>Domain</td>
<td>All real numbers</td>
<td>All real numbers</td>
<td>All real numbers</td>
<td>All real numbers</td>
</tr>
<tr>
<td>Range</td>
<td>$y = 1$</td>
<td>All real numbers</td>
<td>$y \geq 0$</td>
<td>$y \geq 0$</td>
</tr>
</tbody>
</table>

EXAMPLE 1  Identifying a Function Family

Identify the function family to which $f$ belongs. Compare the graph of $f$ to the graph of its parent function.

**SOLUTION**

The graph of $f$ is V-shaped, so $f$ is an absolute value function.

The graph is shifted up and is narrower than the graph of the parent absolute value function. The domain of each function is all real numbers, but the range of $f$ is $\{y \mid y \geq 1\}$ and the range of the parent absolute value function is $\{y \mid y \geq 0\}$.

Monitoring Progress  Help in English and Spanish at BigIdeasMath.com

1. Identify the function family to which $g$ belongs. Compare the graph of $g$ to the graph of its parent function.
Describing Transformations

A **transformation** changes the size, shape, position, or orientation of a graph.

A **translation** is a transformation that shifts a graph horizontally and/or vertically but does not change its size, shape, or orientation.

### Example 2

**Graphing and Describing Translations**

Graph \( g(x) = x - 4 \) and its parent function. Then describe the transformation.

**SOLUTION**

The function \( g \) is a linear function with a slope of 1 and a \( y \)-intercept of \(-4\). So, draw a line through the point \((0, -4)\) with a slope of 1.

The graph of \( g \) is 4 units below the graph of the parent linear function \( f \).

- So, the graph of \( g(x) = x - 4 \) is a vertical translation 4 units down of the graph of the parent linear function.

A **reflection** is a transformation that flips a graph over a line called the **line of reflection**. A reflected point is the same distance from the line of reflection as the original point but on the opposite side of the line.

### Example 3

**Graphing and Describing Reflections**

Graph \( p(x) = -x^2 \) and its parent function. Then describe the transformation.

**SOLUTION**

The function \( p \) is a quadratic function. Use a table of values to graph each function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 )</th>
<th>( y = -x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>-4</td>
</tr>
</tbody>
</table>

The graph of \( p \) is the graph of the parent function flipped over the \( x \)-axis.

- So, \( p(x) = -x^2 \) is a reflection in the \( x \)-axis of the parent quadratic function.

**Monitoring Progress**

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Graph the function and its parent function. Then describe the transformation.

2. \( g(x) = x + 3 \) 
3. \( h(x) = (x - 2)^2 \) 
4. \( n(x) = -|x| \)
Another way to transform the graph of a function is to multiply all of the y-coordinates by the same positive factor (other than 1). When the factor is greater than 1, the transformation is a vertical stretch. When the factor is greater than 0 and less than 1, it is a vertical shrink.

**Graphing and Describing Stretches and Shrinks**

Graph each function and its parent function. Then describe the transformation.

a. \( g(x) = 2|x| \)

**SOLUTION**

a. The function \( g \) is an absolute value function. Use a table of values to graph the functions.

| \( x \) | \( y = |x| \) | \( y = 2|x| \) |
|---|---|---|
| \(-2\) | 2 | 4 |
| \(-1\) | 1 | 2 |
| 0 | 0 | 0 |
| 1 | 1 | 2 |
| 2 | 2 | 4 |

The \( y \)-coordinate of each point on \( g \) is two times the \( y \)-coordinate of the corresponding point on the parent function.

So, the graph of \( g(x) = 2|x| \) is a vertical stretch of the graph of the parent absolute value function.

b. The function \( h \) is a quadratic function. Use a table of values to graph the functions.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x^2 )</th>
<th>( y = \frac{1}{2}x^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2)</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>(-1)</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

The \( y \)-coordinate of each point on \( h \) is one-half of the \( y \)-coordinate of the corresponding point on the parent function.

So, the graph of \( h(x) = \frac{1}{2}x^2 \) is a vertical shrink of the graph of the parent quadratic function.

**Monitoring Progress**

Graph the function and its parent function. Then describe the transformation.

5. \( g(x) = 3x \)

6. \( h(x) = \frac{3}{2}x^2 \)

7. \( c(x) = 0.2|x| \)
Combinations of Transformations

You can use more than one transformation to change the graph of a function.

EXAMPLE 5 Describing Combinations of Transformations

Use a graphing calculator to graph \( g(x) = -|x + 5| - 3 \) and its parent function. Then describe the transformations.

SOLUTION

The function \( g \) is an absolute value function.

\[ g(x) = -|x + 5| - 3 \]

is a reflection in the \( x \)-axis followed by a translation 5 units left and 3 units down of the graph of the parent absolute value function.

EXAMPLE 6 Modeling with Mathematics

The table shows the height \( y \) of a dirt bike \( x \) seconds after jumping off a ramp. What type of function can you use to model the data? Estimate the height after 1.75 seconds.

SOLUTION

1. Understand the Problem
   You are asked to identify the type of function that can model the table of values and then to find the height at a specific time.

2. Make a Plan
   Create a scatter plot of the data. Then use the relationship shown in the scatter plot to estimate the height after 1.75 seconds.

3. Solve the Problem
   Create a scatter plot.

   The data appear to lie on a curve that resembles a quadratic function. Sketch the curve.

   So, you can model the data with a quadratic function. The graph shows that the height is about 15 feet after 1.75 seconds.

4. Look Back
   To check that your solution is reasonable, analyze the values in the table. Notice that the heights decrease after 1 second. Because 1.75 is between 1.5 and 2, the height must be between 20 feet and 8 feet.

   \[ 8 < 15 < 20 \]

Monitoring Progress

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Use a graphing calculator to graph the function and its parent function. Then describe the transformations.

8. \( h(x) = -\frac{1}{2}x + 5 \)

9. \( d(x) = 3(x - 5)^2 - 1 \)

10. The table shows the amount of fuel in a chainsaw over time. What type of function can you use to model the data? When will the tank be empty?

<table>
<thead>
<tr>
<th>Time (minutes), ( x )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel remaining (fluid ounces), ( y )</td>
<td>15</td>
<td>12</td>
<td>9</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
1. **COMPLETE THE SENTENCE** The function \( f(x) = x^2 \) is the ____ of \( f(x) = 2x^2 - 3 \).

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What are the coordinates of the vertices after a reflection in the \( x \)-axis, followed by a translation 2 units right?

What are the coordinates of the vertices after a translation 6 units up and 2 units right?

What are the coordinates of the vertices after a translation 2 units right, followed by a reflection in the \( x \)-axis?

What are the coordinates of the vertices after a translation 6 units up, followed by a reflection in the \( x \)-axis?

3. In Exercises 3–6, identify the function family to which \( f \) belongs. Compare the graph of \( f \) to the graph of its parent function. (See Example 1.)

4. \[ f(x) = 2|x + 2| - 8 \]

5. \[ f(x) = -2x^2 + 3 \]

6. \[ f(x) = 5x - 2 \]

7. **MODELING WITH MATHEMATICS** At 8:00 a.m., the temperature is 43°F. The temperature increases 2°F each hour for the next 7 hours. Graph the temperatures over time \( t \) (\( t = 0 \) represents 8:00 a.m.). What type of function can you use to model the data? Explain.

8. **MODELING WITH MATHEMATICS** You purchase a car from a dealership for $10,000. The trade-in value of the car each year after the purchase is given by the function \( f(x) = 10,000 - 250x^2 \). What type of function can you use to model the data?

In Exercises 9–18, graph the function and its parent function. Then describe the transformation. (See Examples 2 and 3.)

9. \( g(x) = x + 4 \)
10. \( f(x) = x - 6 \)
11. \( f(x) = x^2 - 1 \)
12. \( h(x) = (x + 4)^2 \)
13. \( g(x) = |x - 5| \)
14. \( f(x) = 4 + |x| \)
15. \( h(x) = -x^2 \)
16. \( g(x) = -x \)
17. \( f(x) = 3 \)
18. \( f(x) = -2 \)
In Exercises 19–26, graph the function and its parent function. Then describe the transformation. (See Example 4.)

19. \(f(x) = \frac{1}{3}x\)  
20. \(g(x) = 4x\)
21. \(f(x) = 2x^2\)  
22. \(h(x) = \frac{1}{3}x^2\)
23. \(h(x) = \frac{3}{4}x\)  
24. \(g(x) = \frac{4}{3}x\)
25. \(h(x) = 3|x|\)  
26. \(f(x) = \frac{1}{2}|x|\)

In Exercises 27–34, use a graphing calculator to graph the function and its parent function. Then describe the transformations. (See Example 5.)

27. \(f(x) = 3x + 2\)  
28. \(h(x) = -x + 5\)
29. \(h(x) = -3|x| - 1\)  
30. \(f(x) = \frac{3}{2}|x| + 1\)
31. \(g(x) = \frac{1}{2}x^2 - 6\)  
32. \(f(x) = 4x^2 - 3\)
33. \(f(x) = -(x + 3)^2 + \frac{1}{4}\)  
34. \(g(x) = -|x - 1| - \frac{1}{2}\)

**ERROR ANALYSIS** In Exercises 35 and 36, identify and correct the error in describing the transformation of the parent function.

35. 

The graph is a reflection in the x-axis and a vertical shrink of the parent quadratic function.

36. 

The graph is a translation 3 units right of the parent absolute value function, so the function is \(f(x) = |x + 3|\).

**MATHEMATICAL CONNECTIONS** In Exercises 37 and 38, find the coordinates of the figure after the transformation.

37. Translate 2 units down.  
38. Reflect in the x-axis.

**USING TOOLS** In Exercises 39–44, identify the function family and describe the domain and range. Use a graphing calculator to verify your answer.

39. \(g(x) = |x + 2| - 1\)  
40. \(h(x) = |x - 3| + 2\)
41. \(g(x) = 3x + 4\)  
42. \(f(x) = -4x + 11\)
43. \(f(x) = 5x^2 - 2\)  
44. \(f(x) = -2x^2 + 6\)

**MODELING WITH MATHEMATICS** The table shows the speeds of a car as it travels through an intersection with a stop sign. What type of function can you use to model the data? Estimate the speed of the car when it is 20 yards past the intersection. (See Example 5.)

<table>
<thead>
<tr>
<th>Displacement from sign (yards), (x)</th>
<th>Speed (miles per hour), (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>40</td>
</tr>
<tr>
<td>-50</td>
<td>20</td>
</tr>
<tr>
<td>-10</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>40</td>
</tr>
</tbody>
</table>

45. **THOUGHT PROVOKING** In the same coordinate plane, sketch the graph of the parent quadratic function and the graph of a quadratic function that has no \(x\)-intercepts. Describe the transformation(s) of the parent function.

46. **USING STRUCTURE** Graph the functions \(f(x) = |x - 4|\) and \(g(x) = |x| - 4\). Are they equivalent? Explain.
48. **HOW DO YOU SEE IT?** Consider the graphs of \(f\), \(g\), and \(h\).

![Graph of functions \(f\), \(g\), and \(h\).]

a. Does the graph of \(g\) represent a vertical stretch or a vertical shrink of the graph of \(f\)? Explain your reasoning.

b. Describe how to transform the graph of \(f\) to obtain the graph of \(h\).

49. **MAKING AN ARGUMENT** Your friend says two different translations of the graph of the parent linear function can result in the graph of \(f(x) = x - 2\). Is your friend correct? Explain.

50. **DRAWING CONCLUSIONS** A person swims at a constant speed of 1 meter per second. What type of function can be used to model the distance the swimmer travels? If the person has a 10-meter head start, what type of transformation does this represent? Explain.

51. **PROBLEM SOLVING** You are playing basketball with your friends. The height (in feet) of the ball above the ground \(t\) seconds after a shot is made is modeled by the function \(f(t) = -16t^2 + 32t + 5.2\).

   a. Without graphing, identify the type of function that models the height of the basketball.
   
   b. What is the value of \(t\) when the ball is released from your hand? Explain your reasoning.
   
   c. How many feet above the ground is the ball when it is released from your hand? Explain.

52. **MODELING WITH MATHEMATICS** The table shows the battery lives of a computer over time. What type of function can you use to model the data? Interpret the meaning of the \(x\)-intercept in this situation.

<table>
<thead>
<tr>
<th>Time (hours), (x)</th>
<th>Battery life remaining, (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>80%</td>
</tr>
<tr>
<td>3</td>
<td>40%</td>
</tr>
<tr>
<td>5</td>
<td>0%</td>
</tr>
<tr>
<td>6</td>
<td>20%</td>
</tr>
<tr>
<td>8</td>
<td>60%</td>
</tr>
</tbody>
</table>

53. **REASONING** Compare each function with its parent function. State whether it contains a horizontal translation, vertical translation, both, or neither. Explain your reasoning.

   a. \(f(x) = 2|x| - 3\)
   
   b. \(f(x) = (x - 8)^2\)
   
   c. \(f(x) = |x + 2| + 4\)
   
   d. \(f(x) = 4x^2\)

54. **CRITICAL THINKING** Use the values \(-1, 0, 1,\) and 2 in the correct box so the graph of each function intersects the \(x\)-axis. Explain your reasoning.

   a. \(f(x) = 3x + \underline{1}\)
   
   b. \(f(x) = |2x - 6| - \underline{3}\)
   
   c. \(f(x) = \underline{x^2} + 1\)
   
   d. \(f(x) = \underline{3}x^2\)

---

### Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Determine whether the ordered pair is a solution of the equation.** *(Skills Review Handbook)*

- 55. \(f(x) = x - 3; (5, 2)\)
- 56. \(f(x) = x - 4; (12, 8)\)
- 57. \(f(x) = 2x + 4; (5, 10)\)
- 58. \(f(x) = 3x + 9; (7, 28)\)

**Find the \(x\)-intercept and the \(y\)-intercept of the graph of the equation.** *(Skills Review Handbook)*

- 59. \(y = x\)
- 60. \(y = x + 2\)
- 61. \(3x + y = 1\)
- 62. \(x - 2y = 8\)
1.3 Transformations of Linear and Absolute Value Functions

**Essential Question** How do the graphs of \( y = f(x) + k \), \( y = f(x - h) \), and \( y = a \cdot f(x) \) compare to the graph of the parent function \( f(x) \)?

**EXPLORATION 1** Transformations of the Absolute Value Function

**Work with a partner.** Compare the graph of the function

\[ y = |x| + k \]  
Transformation

with the graph of the parent function

\[ f(x) = |x| \]  
Parent function

**EXPLORATION 2** Transformations of the Absolute Value Function

**Work with a partner.** Compare the graph of the function

\[ y = |x - h| \]  
Transformation

with the graph of the parent function

\[ f(x) = |x| \]  
Parent function

**EXPLORATION 3** Transformation of the Absolute Value Function

**Work with a partner.** Compare the graph of the function

\[ y = a|x| \]  
Transformation

with the graph of the parent function

\[ f(x) = |x| \]  
Parent function

**Communicate Your Answer**

4. How do the graphs of \( y = f(x) + k \), \( y = f(x - h) \), and \( y = a \cdot f(x) \) compare to the graph of the parent function \( f(x) \)?

5. Compare the graph of each function to the graph of its parent function \( f \). Use a graphing calculator to verify your answers are correct.

a. \( y = x^2 + 1 \)  
b. \( y = (x - 1)^2 \)  
c. \( y = -x^2 \)
**What You Will Learn**

- Write functions representing translations and reflections.
- Write functions representing stretches and shrinks.
- Write functions representing combinations of transformations.

**Translations and Reflections**

You can use function notation to represent transformations of graphs of functions.

**Core Concept**

**Horizontal Translations**

The graph of \( y = f(x - h) \) is a horizontal translation of the graph of \( y = f(x) \), where \( h \neq 0 \).

- Subtraction of \( h \) from the inputs before evaluating the function shifts the graph left when \( h < 0 \) and right when \( h > 0 \).

**Vertical Translations**

The graph of \( y = f(x) + k \) is a vertical translation of the graph of \( y = f(x) \), where \( k \neq 0 \).

- Adding \( k \) to the outputs shifts the graph down when \( k < 0 \) and up when \( k > 0 \).

**EXAMPLE 1**

**Writing Translations of Functions**

Let \( f(x) = 2x + 1 \).

**a.** Write a function \( g \) whose graph is a translation 3 units down of the graph of \( f \).

**b.** Write a function \( h \) whose graph is a translation 2 units to the left of the graph of \( f \).

**SOLUTION**

**a.** A translation 3 units down is a vertical translation that adds \(-3\) to each output value.

\[
\begin{align*}
g(x) &= f(x) + (-3) \\
      &= 2x + 1 + (-3) \\
      &= 2x - 2
\end{align*}
\]

The translated function is \( g(x) = 2x - 2 \).

**b.** A translation 2 units to the left is a horizontal translation that subtracts \(-2\) from each input value.

\[
\begin{align*}
h(x) &= f(x - (-2)) \\
      &= f(x + 2) \\
      &= 2(x + 2) + 1 \\
      &= 2x + 5
\end{align*}
\]

The translated function is \( h(x) = 2x + 5 \).
Section 1.3  Transformations of Linear and Absolute Value Functions

Core Concept

Reflections in the x-axis
The graph of \( y = -f(x) \) is a reflection in the x-axis of the graph of \( y = f(x) \).

\[
\begin{align*}
  &y = f(x) \\
  &y = -f(x)
\end{align*}
\]

Multiplying the outputs by \(-1\) changes their signs.

Reflections in the y-axis
The graph of \( y = f(-x) \) is a reflection in the y-axis of the graph of \( y = f(x) \).

\[
\begin{align*}
  &y = f(x) \\
  &y = f(-x)
\end{align*}
\]

Multiplying the inputs by \(-1\) changes their signs.

EXAMPLE 2  Writing Reflections of Functions

Let \( f(x) = |x + 3| + 1 \).

a. Write a function \( g \) whose graph is a reflection in the x-axis of the graph of \( f \).

b. Write a function \( h \) whose graph is a reflection in the y-axis of the graph of \( f \).

SOLUTION

a. A reflection in the x-axis changes the sign of each output value.

\[
\begin{align*}
g(x) &= -f(x) \\
&= -(|x + 3| + 1) \\
&= -|x + 3| - 1
\end{align*}
\]

The reflected function is \( g(x) = -|x + 3| - 1 \).

b. A reflection in the y-axis changes the sign of each input value.

\[
\begin{align*}
h(x) &= f(-x) \\
&= |-(x + 3)| + 1 \\
&= |-(x - 3)| + 1 \\
&= -1 \cdot |x - 3| + 1 \\
&= |x - 3| + 1
\end{align*}
\]

The reflected function is \( h(x) = |x - 3| + 1 \).

Monitoring Progress

Write a function \( g \) whose graph represents the indicated transformation of the graph of \( f \). Use a graphing calculator to check your answer.

1. \( f(x) = 3x \); translation 5 units up
2. \( f(x) = |x| - 3 \); translation 4 units to the right
3. \( f(x) = -|x + 2| - 1 \); reflection in the x-axis
4. \( f(x) = \frac{1}{2}x + 1 \); reflection in the y-axis
Stretches and Shrinks
In the previous section, you learned that vertical stretches and shrinks transform graphs. You can also use horizontal stretches and shrinks to transform graphs.

Core Concept

Horizontal Stretches and Shrinks
The graph of $y = f(ax)$ is a horizontal stretch or shrink by a factor of $\frac{1}{a}$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the inputs by $a$ before evaluating the function stretches the graph horizontally (away from the y-axis) when $0 < a < 1$, and shrinks the graph horizontally (toward the y-axis) when $a > 1$.

Vertical Stretches and Shrinks
The graph of $y = a \cdot f(x)$ is a vertical stretch or shrink by a factor of $a$ of the graph of $y = f(x)$, where $a > 0$ and $a \neq 1$.

Multiplying the outputs by $a$ stretches the graph vertically (away from the x-axis) when $a > 1$, and shrinks the graph vertically (toward the x-axis) when $0 < a < 1$.

EXAMPLE 3 Writing Stretches and Shrinks of Functions

Let $f(x) = |x - 3| - 5$. Write (a) a function $g$ whose graph is a horizontal shrink of the graph of $f$ by a factor of $\frac{1}{3}$, and (b) a function $h$ whose graph is a vertical stretch of the graph of $f$ by a factor of 2.

SOLUTION

a. A horizontal shrink by a factor of $\frac{1}{3}$ multiplies each input value by 3.

$$g(x) = f(3x)$$

Multiply the input by 3.

$$= |3x - 3| - 5$$

Replace $x$ with $3x$ in $f(x)$.

The transformed function is $g(x) = |3x - 3| - 5$.

b. A vertical stretch by a factor of 2 multiplies each output value by 2.

$$h(x) = 2 \cdot f(x)$$

Multiply the output by 2.

$$= 2 \cdot (|x - 3| - 5)$$

Substitute $|x - 3| - 5$ for $f(x)$.

Distributive Property

$$= 2|x - 3| - 10$$

The transformed function is $h(x) = 2|x - 3| - 10$.

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Write a function $g$ whose graph represents the indicated transformation of the graph of $f$. Use a graphing calculator to check your answer.

5. $f(x) = 4x + 2$; horizontal stretch by a factor of 2
6. $f(x) = |x| - 3$; vertical shrink by a factor of $\frac{1}{3}$
Combinations of Transformations

You can write a function that represents a series of transformations on the graph of another function by applying the transformations one at a time in the stated order.

**EXAMPLE 4** Combining Transformations

Let the graph of $g$ be a vertical shrink by a factor of 0.25 followed by a translation 3 units up of the graph of $f(x) = x$. Write a rule for $g$.

**SOLUTION**

Step 1 First write a function $h$ that represents the vertical shrink of $f$.

$$h(x) = 0.25 \cdot f(x)$$

Multiply the output by 0.25.

$$= 0.25x$$

Substitute $x$ for $f(x)$.

Step 2 Then write a function $g$ that represents the translation of $h$.

$$g(x) = h(x) + 3$$

Add 3 to the output.

$$= 0.25x + 3$$

Substitute 0.25$x$ for $h(x)$.

The transformed function is $g(x) = 0.25x + 3$.

**EXAMPLE 5** Combining Transformations

Write a function $g$ whose graph is a horizontal stretch of the graph of $f(x) = |x|$ by a factor of 3, followed by a reflection in the $y$-axis.

**SOLUTION**

Step 1 First write a function $h$ that represents the horizontal stretch of $f$.

$$h(x) = f\left(\frac{1}{3}x\right)$$

Multiply the input by $\frac{1}{3}$.

$$= \left|\frac{1}{3}x\right|$$

Replace $x$ with $\frac{1}{3}x$ in $f(x)$.

Step 2 Then write a function $g$ that represents the reflection of $h$.

$$g(x) = h(-x)$$

Multiply the input by $-1$.

$$= \left|\frac{1}{3}(-x)\right|$$

Replace $x$ with $-x$ in $h(x)$.

$$= \left|-\frac{1}{3}x\right|$$

Simplify.

The transformed function is $g(x) = \left|-\frac{1}{3}x\right|$. Note that the graph of $g$ is identical to the graph of $h(x) = \left|\frac{1}{3}x\right|$.

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7. Let the graph of $g$ be a translation 6 units down followed by a reflection in the $x$-axis of the graph of $f(x) = |x|$. Write a rule for $g$. Use a graphing calculator to check your answer.

8. Write a function $k$ whose graph is a horizontal stretch of the graph of $f(x) = |x|$ by a factor of 4, followed by a reflection in the $x$-axis.
1. **Vocabulary and Core Concept Check**

   **1. COMPLETE THE SENTENCE** The function \( g(x) = |5x| - 4 \) is a horizontal __________ of the function \( f(x) = |x| - 4 \).

   **2. WHICH ONE DOESN’T BELONG?** Which transformation does not belong with the other three? Explain your reasoning.

   - Translate the graph of \( f(x) = 2x + 3 \) up 2 units.
   - Shrink the graph of \( f(x) = x + 5 \) horizontally by a factor of \( \frac{1}{2} \).
   - Stretch the graph of \( f(x) = x + 3 \) vertically by a factor of 2.
   - Translate the graph of \( f(x) = 2x + 3 \) left 1 unit.

---

**Monitoring Progress and Modeling with Mathematics**

In Exercises 3–8, write a function \( g \) whose graph represents the indicated transformation of the graph of \( f \). Use a graphing calculator to check your answer. (See Example 1.)

3. \( f(x) = x - 5 \); translation 4 units to the left

4. \( f(x) = x + 2 \); translation 2 units to the right

5. \( f(x) = |4x + 3| + 2 \); translation 2 units down

6. \( f(x) = 2x - 9 \); translation 6 units up

7. \( f(x) = 4 - |x + 1| \)

8. \( f(x) = |4x| + 5 \)

9. **WRITING** Describe two different translations of the graph of \( f \) that result in the graph of \( g \).

10. **PROBLEM SOLVING** You open a café. The function \( f(x) = 4000x \) represents your expected net income (in dollars) after being open \( x \) weeks. Before you open, you incur an extra expense of $12,000. What transformation of \( f \) is necessary to model this situation? How many weeks will it take to pay off the extra expense?

---

In Exercises 11–16, write a function \( g \) whose graph represents the indicated transformation of the graph of \( f \). Use a graphing calculator to check your answer. (See Example 2.)

11. \( f(x) = -5x + 2 \); reflection in the \( x \)-axis

12. \( f(x) = \frac{1}{2}x - 3 \); reflection in the \( x \)-axis

13. \( f(x) = |6x| - 2 \); reflection in the \( y \)-axis

14. \( f(x) = |2x - 1| + 3 \); reflection in the \( y \)-axis

15. \( f(x) = -3 + |x - 11| \); reflection in the \( y \)-axis

16. \( f(x) = -x + 1 \); reflection in the \( y \)-axis
In Exercises 17–22, write a function \( g \) whose graph represents the indicated transformation of the graph of \( f \). Use a graphing calculator to check your answer. (See Example 3.)

17. \( f(x) = x + 2 \); vertical stretch by a factor of 5
18. \( f(x) = 2x + 6 \); vertical shrink by a factor of \( \frac{1}{2} \)
19. \( f(x) = |2x| + 4 \); horizontal shrink by a factor of \( \frac{1}{2} \)
20. \( f(x) = |x + 3| \); horizontal stretch by a factor of 4
21. \( f(x) = -2|x - 4| + 2 \)

22. \( f(x) = 6 - x \)

In Exercises 27–32, write a function \( g \) whose graph represents the indicated transformations of the graph of \( f \). (See Examples 4 and 5.)

27. \( f(x) = x \); vertical stretch by a factor of 2 followed by a translation 1 unit up
28. \( f(x) = x \); translation 3 units down followed by a vertical shrink by a factor of \( \frac{1}{3} \)
29. \( f(x) = |x| \); translation 2 units to the left followed by a horizontal stretch by a factor of 2
30. \( f(x) = |x| \); horizontal stretch by a factor of 4 followed by a reflection in the y-axis
31. \( f(x) = |x| \)
32. \( f(x) = |x| \)

ERROR ANALYSIS In Exercises 33 and 34, identify and correct the error in writing the function \( g \) whose graph represents the indicated transformations of the graph of \( f \).

33. \( f(x) = |x| \); translation 3 units to the right followed by a translation 2 units up
\[ g(x) = |x + 3| + 2 \]

34. \( f(x) = x \); translation 6 units down followed by a vertical stretch by a factor of 5
\[ g(x) = 5x - 6 \]

35. MAKING AN ARGUMENT Your friend claims that when writing a function whose graph represents a combination of transformations, the order is not important. Is your friend correct? Justify your answer.
36. MODELING WITH MATHEMATICS During a recent period of time, bookstore sales have been declining. The sales (in billions of dollars) can be modeled by the function $f(t) = -\frac{7}{4}t + 17.2$, where $t$ is the number of years since 2006. Suppose sales decreased at twice the rate. How can you transform the graph of $f$ to model the sales? Explain how the sales in 2010 are affected by this change. (See Example 5.)

MATHEMATICAL CONNECTIONS For Exercises 37–40, describe the transformation of the graph of $f$ to the graph of $g$. Then find the area of the shaded triangle.

37. $f(x) = |x - 3|$
38. $f(x) = -|x| - 2$

39. $f(x) = -x + 4$
40. $f(x) = x - 5$

41. ABSTRACT REASONING The functions $f(x) = mx + b$ and $g(x) = mx + c$ represent two parallel lines.
   a. Write an expression for the vertical translation of the graph of $f$ to the graph of $g$.
   b. Use the definition of slope to write an expression for the horizontal translation of the graph of $f$ to the graph of $g$.

42. HOW DO YOU SEE IT? Consider the graph of $f(x) = mx + b$. Describe the effect each transformation has on the slope of the line and the intercepts of the graph.

43. REASONING The graph of $g(x) = -4|x| + 2$ is a reflection in the $x$-axis, vertical stretch by a factor of 4, and a translation 2 units down of the graph of its parent function. Choose the correct order for the transformations of the graph of the parent function to obtain the graph of $g$. Explain your reasoning.

44. THOUGHT PROVOKING You are planning a cross-country bicycle trip of 4320 miles. Your distance $d$ (in miles) from the halfway point can be modeled by $d = 72|x - 30|$, where $x$ is the time (in days) and $x = 0$ represents June 1. Your plans are altered so that the model is now a right shift of the original model. Give an example of how this can happen. Sketch both the original model and the shifted model.

45. CRITICAL THINKING Use the correct value 0, −2, or 1 with $a$, $b$, and $c$ so the graph of $g(x) = a|x - b| + c$ is a reflection in the $x$-axis followed by a translation one unit to the left and one unit up of the graph of $f(x) = 2|x - 2| + 1$. Explain your reasoning.

---

Maintaining Mathematical Proficiency

Order the values from least to greatest. (Skills Review Handbook)

46. 9, |−4|, −4, 5, 2 |
47. |−32|, 22, −16, −|21|, −10 |
48. −18, |−24|, −19, −|−18|, 22 |
49. −|−3|, |0|, −1, |2|, −2 |

Solve the equation. Check your solution. (Skills Review Handbook)

50. $3y = −18$
51. $6 = 13 - b$
52. $5x + 6 = 41$
53. $19 - 6a = 1$
1.1–1.3 What Did You Learn?

Core Vocabulary

set, p. 4
subset, p. 4
endpoints, p. 4
bounded interval, p. 4
unbounded interval, p. 5
set-builder notation, p. 6
absolute value function, p. 9
parent function, p. 10
transformation, p. 11
translation, p. 11
reflection, p. 11
vertical stretch, p. 12
vertical shrink, p. 12

Core Concepts

Section 1.1
Bounded Intervals on the Real Number Line, p. 4
Set-Builder Notation, p. 6
Unbounded Intervals on the Real Number Line, p. 5

Section 1.2
Parent Functions, p. 10
Describing Transformations, p. 11

Section 1.3
Horizontal Translations, p. 18
Vertical Translations, p. 18
Reflections in the x-axis, p. 19
Reflections in the y-axis, p. 19
Horizontal Stretches and Shrinks, p. 20
Vertical Stretches and Shrinks, p. 20

Mathematical Thinking

1. How can you analyze the values given in the table in Exercise 45 on page 15 to help you determine what type of function models the data?
2. Explain how you would round your answer in Exercise 10 on page 22 if the extra expense is $13,500.

Study Skills

Taking Control of Your Class Time

1. Sit where you can easily see and hear the teacher, and the teacher can see you.
2. Pay attention to what the teacher says about math, not just what is written on the board.
3. Ask a question if the teacher is moving through the material too fast.
4. Try to memorize new information while learning it.
5. Ask for clarification if you do not understand something.
6. Think as intensely as if you were going to take a quiz on the material at the end of class.
7. Volunteer when the teacher asks for someone to go up to the board.
8. At the end of class, identify concepts or problems for which you still need clarification.
9. Use the tutorials at BigIdeasMath.com for additional help.
Write the interval in interval notation. (Section 1.1)

1. The real numbers greater than 8
2. 6 ≤ x < 12
3. x ≥ 4

Write the set of numbers in set-builder notation. (Section 1.1)

4. (−2, 13]
5. (−∞, 5) or [8, ∞)
6. The set of all whole numbers except 100

Identify the function family to which g belongs. Compare the graph of the function to the graph of its parent function. (Section 1.2)

7. \( g(x) = \frac{1}{2}x - 1 \)
8. \( g(x) = 2(x + 1)^2 \)
9. \( g(x) = |x + 1| - 2 \)

Graph the function and its parent function. Then describe the transformation. (Section 1.2)

10. \( f(x) = \frac{3}{2} \)
11. \( f(x) = 3x \)
12. \( f(x) = 2(x - 1)^2 \)
13. \( f(x) = -|x + 2| - 7 \)
14. \( f(x) = \frac{1}{4}x^2 + 1 \)
15. \( f(x) = -\frac{1}{2}x - 4 \)

Write a function g whose graph represents the indicated transformation(s) of the graph of f. (Section 1.3)

16. \( f(x) = 2x + 1; \) translation 3 units up
17. \( f(x) = -3|x - 4|; \) vertical shrink by a factor of \( \frac{1}{2} \)
18. \( f(x) = 3|x + 5|; \) reflection in the x-axis
19. \( f(x) = \frac{1}{3}x - \frac{2}{3}; \) translation 4 units left
20. \( f(x) = x; \) translation 2 units down and a horizontal shrink by a factor of \( \frac{2}{3} \)
21. \( f(x) = x; \) translation 9 units down followed by a reflection in the y-axis
22. \( f(x) = |x|; \) reflection in the x-axis and a vertical stretch by a factor of 4 followed by a translation 7 units down and 1 unit right
23. \( f(x) = |x|; \) translation 1 unit down and 2 units left followed by a vertical shrink by a factor of \( \frac{1}{2} \)

The table shows the total distance a new car travels each month after it is purchased. What type of function can you use to model the data? Estimate the mileage after 1 year. (Section 1.2)

<table>
<thead>
<tr>
<th>Time (months), x</th>
<th>0</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles), y</td>
<td>0</td>
<td>2300</td>
<td>5750</td>
<td>6900</td>
<td>10,350</td>
</tr>
</tbody>
</table>

25. The total cost of an annual pass plus camping for x days in a National Park can be modeled by the function \( f(x) = 20x + 80. \) Senior citizens pay half of this price and receive an additional $30 discount. Describe how to transform the graph of \( f \) to model the total cost for a senior citizen. What is the total cost for a senior citizen to go camping for three days? (Section 1.3)
Section 1.4 Solving Absolute Value Equations

**Essential Question** How can you solve an absolute value equation?

**EXPLORATION 1** Solving an Absolute Value Equation Algebraically

Work with a partner. Consider the absolute value equation

\[ |x + 2| = 3. \]

a. Describe the values of \( x + 2 \) that make the equation true. Use your description to write two linear equations that represent the solutions of the absolute value equation.

b. Use the linear equations you wrote in part (a) to find the solutions of the absolute value equation.

c. How can you use linear equations to solve an absolute value equation?

**EXPLORATION 2** Solving an Absolute Value Equation Graphically

Work with a partner. Consider the absolute value equation

\[ |x + 2| = 3. \]

a. On a real number line, locate the point for which \( x + 2 = 0 \).

b. Locate the points that are 3 units from the point you found in part (a). What do you notice about these points?

c. How can you use a number line to solve an absolute value equation?

**EXPLORATION 3** Solving an Absolute Value Equation Numerically

Work with a partner. Consider the absolute value equation

\[ |x + 2| = 3. \]

a. Use a spreadsheet, as shown, to solve the absolute value equation.

b. Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?

c. How can you use a spreadsheet to solve an absolute value equation?

**Communicate Your Answer**

4. How can you solve an absolute value equation?

5. What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value equation? Give reasons for your answers.
What You Will Learn

- Solve absolute value equations.
- Solve equations involving two absolute values.
- Identify special solutions of absolute value equations.

Solving Absolute Value Equations

An absolute value equation is an equation that contains an absolute value expression. You can solve these types of equations by solving two related linear equations.

Properties of Absolute Value

Let $a$ and $b$ be real numbers. Then the following properties are true.

1. $|a| \geq 0$
2. $|-a| = |a|
3. $|ab| = |a||b|
4. $\frac{|a|}{|b|}$, $b \neq 0$

Solving Absolute Value Equations

To solve $|ax + b| = c$ when $c \geq 0$, solve the related linear equations

$$ax + b = c \quad \text{or} \quad ax + b = -c.$$  

When $c < 0$, the absolute value equation $|ax + b| = c$ has no solution because absolute value always indicates a number that is not negative.

**Example 1**

Solving Absolute Value Equations

Solve each equation. Graph the solutions, if possible.

a. $|x - 4| = 6$

**SOLUTION**

a. Write the two related linear equations for $|x - 4| = 6$. Then solve.

$$x - 4 = 6 \quad \text{or} \quad x - 4 = -6$$  

Write related linear equations.

$$x = 10 \quad \text{or} \quad x = -2$$  

Add 4 to each side.

The solutions are $x = 10$ and $x = -2$.

b. The absolute value of an expression must be greater than or equal to 0. The expression $|3x + 1|$ cannot equal $-5$.

So, the equation has no solution.

**Monitoring Progress**

Solve the equation. Graph the solutions, if possible.

1. $|x| = 10$
2. $|x - 1| = 4$
3. $|3 + x| = -3$
Solving an Absolute Value Equation

Example 2
Solve \( |3x + 9| - 10 = -4 \).

**SOLUTION**
First isolate the absolute value expression on one side of the equation.

\[
|3x + 9| - 10 = -4 \quad \text{Write the equation.}
\]

\[
|3x + 9| = 6 \quad \text{Add 10 to each side.}
\]

Now write two related linear equations for \( |3x + 9| = 6 \). Then solve.

\[
3x + 9 = 6 \quad \text{or} \quad 3x + 9 = -6
\]

\[
3x = -3 \quad 3x = -15
\]

\[
x = -1 \quad x = -5
\]

The solutions are \( x = -1 \) and \( x = -5 \).

Example 3
Writing an Absolute Value Equation

In a cheerleading competition, the minimum length of a routine is 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

**SOLUTION**
1. **Understand the Problem** You know the minimum and maximum lengths. You are asked to write an absolute value equation that represents these lengths.
2. **Make a Plan** Consider the minimum and maximum lengths as solutions to an absolute value equation. Use a number line to find the halfway point between the solutions. Then use the halfway point and the distance to each solution to write an absolute value equation.
3. **Solve the Problem**

\[
|x - 4.5| = 0.5
\]

The equation is \( |x - 4.5| = 0.5 \).
4. **Look Back** To check that your equation is reasonable, substitute the minimum and maximum lengths into the equation and simplify.

\[
|4 - 4.5| = 0.5 \quad \checkmark \quad |5 - 4.5| = 0.5 \quad \checkmark
\]

Monitoring Progress
Solve the equation. Check your solutions.

4. \( |x - 2| + 5 = 9 \) \hspace{1cm} 5. \( 4|2x + 7| = 16 \) \hspace{1cm} 6. \( -2|5x - 1| - 3 = -11 \)

7. For a poetry contest, the minimum length of a poem is 16 lines. The maximum length is 32 lines. Write an absolute value equation that represents the minimum and maximum lengths.
Solving Equations with Two Absolute Values

If the absolute values of two algebraic expressions are equal, then they must either be equal to each other or be opposites of each other.

Core Concept

Solving Equations with Two Absolute Values

To solve $|ax + b| = |cx + d|$, solve the related linear equations $ax + b = cx + d$ or $ax + b = -(cx + d)$.

Example 4

Solving Equations with Two Absolute Values

Solve (a) $|3x - 4| = |x|$ and (b) $|4x - 10| = 2|3x + 1|$.

Solution

a. Write the two related linear equations for $|3x - 4| = |x|$. Then solve.

$3x - 4 = x$ or $3x - 4 = -x$

$-x - x + x + x$  
$2x - 4 = 0$  
$4x - 4 = 0$

$2x = 4$  
$4x = 4$

$x = 2$  
$x = 1$

The solutions are $x = 2$ and $x = 1$.

b. Write the two related linear equations for $|4x - 10| = 2|3x + 1|$. Then solve.

$4x - 10 = 2(3x + 1)$ or $4x - 10 = 2[-(3x + 1)]$

$4x - 10 = 6x + 2$  
$4x - 10 = 2(-3x - 1)$

$-6x - 6x$  
$-2x - 10 = 2$  
$+ 6x + 6x$  
$10x - 10 = -2$

$-2x = 12$  
$10x = 8$

$x = -6$  
$rac{10x}{10} = rac{8}{10}$

$x = 0.8$

The solutions are $x = -6$ and $x = 0.8$.

Monitoring Progress

Solve the equation. Check your solutions.

8. $|x + 8| = |2x + 1|$  
9. $3|x - 4| = |2x + 5|$
Identifying Special Solutions

When you solve an absolute value equation, it is possible for a solution to be *extraneous*. An *extraneous solution* is an apparent solution that must be rejected because it does not satisfy the original equation.

**EXAMPLE 5** Identifying Extraneous Solutions

Solve $|2x + 12| = 4x$. Check your solutions.

**SOLUTION**

Write the two related linear equations for $|2x + 12| = 4x$. Then solve.

$2x + 12 = 4x$ or $2x + 12 = -4x$  
$12 = 2x$ or $12 = -6x$  
$6 = x$ or $-2 = x$

Check the apparent solutions to see if either is extraneous.

- The solution is $x = 6$. Reject $x = -2$ because it is extraneous.

When solving equations of the form $|ax + b| = |cx + d|$, it is possible that one of the related linear equations will not have a solution.

**EXAMPLE 6** Solving an Equation with Two Absolute Values

Solve $|x + 5| = |x + 11|$.

**SOLUTION**

By equating the expression $x + 5$ and the opposite of $x + 11$, you obtain

$x + 5 = -(x + 11)$  
$x + 5 = -x - 11$  
$2x + 5 = -11$  
$2x = -16$  
$x = -8$.

However, by equating the expressions $x + 5$ and $x + 11$, you obtain

$x + 5 = x + 11$  
$x = x + 6$  
$0 = 6$  
which is a false statement. So, the original equation has only one solution.

- The solution is $x = -8$.

**Monitoring Progress**

Solve the equation. Check your solutions.

10. $|x + 6| = 2x$  
11. $|3x - 2| = x$  
12. $|2 + x| = |x - 8|$  
13. $|5x - 2| = |5x + 4|$
Vocabulary and Core Concept Check

1. **VOCABULARY** What is an extraneous solution?

2. **WRITING** Without calculating, how do you know that the equation $|4x - 7| = -1$ has no solution?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression.

3. $|-9|$
4. $-|15|$
5. $|14| - |14|$
6. $|-3| + |3|$
7. $-|-5\cdot(-7)|$
8. $|-0.8\cdot10|$
9. $\frac{27}{-3}$
10. $|-\frac{12}{4}|$

In Exercises 11–24, solve the equation. Graph the solution(s), if possible. (See Examples 1 and 2.)

11. $|w| = 6$
12. $|r| = -2$
13. $|y| = -18$
14. $|x| = 13$
15. $|m + 3| = 7$
16. $|q - 8| = 14$
17. $|-3d| = 15$
18. $\left|\frac{t}{2}\right| = 6$
19. $|4b - 5| = 19$
20. $|x - 1| + 5 = 2$
21. $-4|8 - 5n| = 13$
22. $-3\left|1 - \frac{2}{3}v\right| = -9$
23. $3 = -2\left|\frac{1}{4}s - 5\right| + 3$
24. $9|4p + 2| + 8 = 35$

25. **WRITING EQUATIONS** The minimum distance from Earth to the Sun is 91.4 million miles. The maximum distance is 94.5 million miles. (See Example 3.)

   a. Represent these two distances on a number line.
   b. Write an absolute value equation that represents the minimum and maximum distances.

26. **WRITING EQUATIONS** The shoulder heights of the shortest and tallest miniature poodles are shown.

   a. Represent these two heights on a number line.
   b. Write an absolute value equation that represents these heights.

**USING STRUCTURE** In Exercises 27–30, match the absolute value equation with its graph without solving the equation.

27. $|x + 2| = 4$
28. $|x - 4| = 2$
29. $|x - 2| = 4$
30. $|x + 4| = 2$

   A.
   B.
   C.
   D.
In Exercises 31–34, write an absolute value equation that has the given solutions.

31. \( x = 8 \) and \( x = 18 \)  
32. \( x = -6 \) and \( x = 10 \)  
33. \( x = 2 \) and \( x = 9 \)  
34. \( x = -10 \) and \( x = -5 \)

In Exercises 35–44, solve the equation. Check your solutions. (See Examples 4, 5, and 6.)

35. \( |4n - 15| = |n| \)  
36. \( |2c + 8| = |10c| \)  
37. \( |2b - 9| = |b - 6| \)  
38. \( |3k - 2| = 2|k + 2| \)  
39. \( 4|p - 3| = |2p + 8| \)  
40. \( 2|4w - 1| = 3|4w + 2| \)  
41. \( |3h + 1| = 7h \)  
42. \( |6a - 5| = 4a \)  
43. \( |f - 6| = |f + 8| \)  
44. \( |3x - 4| = |3x - 5| \)

45. MODELING WITH MATHEMATICS Starting from 300 feet away, a car drives toward you. It then passes by you at a speed of 48 feet per second. The distance \( d \) (in feet) of the car from you after \( t \) seconds is given by the equation \( d = |300 - 48t| \). At what times is the car 60 feet from you?

46. MAKING AN ARGUMENT Your friend says that the absolute value equation \( |3x + 8| - 9 = -5 \) has no solution because the constant on the right side of the equation is negative. Is your friend correct? Explain.

47. MODELING WITH MATHEMATICS You randomly survey students about year-round school. The results are shown in the graph.

The error given in the graph means that the actual percent could be 5% more or 5% less than the percent reported by the survey.

a. Write and solve an absolute value equation to find the least and greatest percents of students who could be in favor of year-round school.

b. A classmate claims that \( \frac{1}{3} \) of the student body is actually in favor of year-round school. Does this conflict with the survey data? Explain.

48. MODELING WITH MATHEMATICS The recommended weight of a soccer ball is 430 grams. The actual weight is allowed to vary by up to 20 grams.

a. Write and solve an absolute value equation to find the minimum and maximum acceptable soccer ball weights.

b. A soccer ball weighs 423 grams. Due to wear and tear, the weight of the ball decreases by 16 grams. Is the weight acceptable? Explain.

ERROR ANALYSIS In Exercises 49 and 50, describe and correct the error in solving the equation.

49. \( |2x - 1| = -9 \)

\[ 2x - 1 = -9 \quad \text{or} \quad 2x - 1 = -(-9) \]

\[ 2x = -8 \quad 2x = 10 \]

\[ x = -4 \quad x = 5 \]

The solutions are \( x = -4 \) and \( x = 5 \).

50. \( |5x + 8| = x \)

\[ 5x + 8 = x \quad \text{or} \quad 5x + 8 = -x \]

\[ 4x + 8 = 0 \quad 6x + 8 = 0 \]

\[ 4x = -8 \quad 6x = -8 \]

\[ x = -2 \quad x = \frac{-4}{3} \]

The solutions are \( x = -2 \) and \( x = \frac{-4}{3} \).

51. ANALYZING EQUATIONS Without solving completely, place each equation into one of the three categories.

| \( |x - 2| + 6 = 0 \) | \( |x + 3| - 1 = 0 \) | \( |x + 8| + 2 = 0 \) | \( |x - 1| + 4 = 4 \) | \( |x - 6| - 5 = -9 \) | \( |x + 5| - 8 = -8 \) |

Section 1.4 Solving Absolute Value Equations
52. USING STRUCTURE Fill in the equation $|x - \square| = \square$ with $a$, $b$, $c$, or $d$ so that the equation is graphed correctly.

53. If $x^2 = a^2$, then $|x|$ is ______ equal to $|a|$.

54. If $a$ and $b$ are real numbers, then $|a - b|$ is ______ equal to $|b - a|$.

55. For any real number $p$, the equation $|x - 4| = p$ will ______ have two solutions.

56. For any real number $p$, the equation $|x - p| = 4$ will ______ have two solutions.

57. WRITING Explain why absolute value equations can have no solution, one solution, or two solutions. Give an example of each case.

58. THOUGHT PROVOKING Describe a real-life situation that can be modeled by an absolute value equation with the solutions $x = 62$ and $x = 72$.

59. CRITICAL THINKING Solve the equation shown. Explain how you found your solution(s).

\[8|x + 2| - 6 = 5|x + 2| + 3\]

60. HOW DO YOU SEE IT? The circle graph shows the results of a survey of registered voters the day of an election.

Which Party’s Candidate Will Get Your Vote?

- Democratic: 47%
- Republican: 42%
- Libertarian: 5%
- Green: 2%
- Other: 4%

Error: ±2%

The error given in the graph means that the actual percent could be 2% more or 2% less than the percent reported by the survey.

a. What are the minimum and maximum percents of voters who could vote Republican? Green?

b. How can you use absolute value equations to represent your answers in part (a)?

c. One candidate receives 44% of the vote. Which party does the candidate belong to? Explain.

61. ABSTRACT REASONING How many solutions does the equation $a|x + b| + c = d$ have when $a > 0$ and $c = d$? when $a < 0$ and $c > d$? Explain your reasoning.

62. $x - 5 \leq 7$

63. $x + 4 > 9$

64. $a + 3 > -5$ and $a + 3 < 9$

65. $y - 6 > -12$ and $y - 6 < 5$

66. $2b + 4 \leq -2$ or $2b + 4 \geq 10$

67. $3x + 3 < -12$ or $3x + 3 > 9$

Use a geometric formula to solve the problem. (Skills Review Handbook)

68. A square has an area of 81 square meters. Find the side length.

69. A circle has an area of $36\pi$ square inches. Find the radius.

70. A triangle has a height of 8 feet and an area of 48 square feet. Find the base.

71. A rectangle has a width of 4 centimeters and a perimeter of 26 centimeters. Find the length.
Essential Question  How can you solve an absolute value inequality?

EXPLORATION 1  Solving an Absolute Value Inequality Algebraically

Work with a partner. Consider the absolute value inequality

\[ |x + 2| \leq 3. \]

a. Describe the values of \( x + 2 \) that make the inequality true. Use your description to write two linear inequalities that represent the solutions of the absolute value inequality.

b. Use the linear inequalities you wrote in part (a) to find the solutions of the absolute value inequality.

c. How can you use linear inequalities to solve an absolute value inequality?

EXPLORATION 2  Solving an Absolute Value Inequality Graphically

Work with a partner. Consider the absolute value inequality

\[ |x + 2| \leq 3. \]

a. On a real number line, locate the point for which \( x + 2 = 0 \).

b. Locate the points that are within 3 units from the point you found in part (a). What do you notice about these points?

c. How can you use a number line to solve an absolute value inequality?

EXPLORATION 3  Solving an Absolute Value Inequality Numerically

Work with a partner. Consider the absolute value inequality

\[ |x + 2| \leq 3. \]

a. Use a spreadsheet, as shown, to solve the absolute value inequality.

b. Compare the solutions you found using the spreadsheet with those you found in Explorations 1 and 2. What do you notice?

c. How can you use a spreadsheet to solve an absolute value inequality?

Communicate Your Answer

4. How can you solve an absolute value inequality?

5. What do you like or dislike about the algebraic, graphical, and numerical methods for solving an absolute value inequality? Give reasons for your answers.
Solve each inequality. Graph each solution, if possible.

a. $|x + 7| \leq 2$

**SOLUTION**

1. Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$$x + 7 \geq -2 \quad \text{and} \quad x + 7 \leq 2$$

2. Subtract 7 from each side.

$$x \geq -9 \quad \text{and} \quad x \leq -5$$


$$-9 \leq x \leq -5$$

The solution is $-9 \leq x \leq -5$.

b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

So, the inequality has no solution.
Solve the inequality. Graph the solution, if possible.
1. $|x| \leq 3.5$
2. $|k - 3| < -1$
3. $|2w - 1| < 11$

**Example 2** Solving Absolute Value Inequalities

Solve each inequality. Graph each solution.

**a.** $|c - 1| \geq 5$

**Solution**

Use $|c - 1| \geq 5$ to write a compound inequality. Then solve.

\[
\begin{align*}
c - 1 & \leq -5 \\
+1 & \quad +1 \quad \text{or} \quad c - 1 & \geq 5 \\
& \quad +1 \
\end{align*}
\]

\[
\begin{align*}
c & \leq -4 \\
\quad +1 & \quad +1 \\
& \quad c & \geq 6 \\
\end{align*}
\]

The solution is $c \leq -4$ or $c \geq 6$.

**b.** $|10 - m| \geq -2$

By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|10 - m|$ will always be greater than $-2$.

So, all real numbers are solutions.

**c.** $4|2x - 5| + 1 > 21$

First isolate the absolute value expression on one side of the inequality.

\[
\begin{align*}
4|2x - 5| + 1 & > 21 \\
-1 -1 & \quad \text{Subtract 1 from each side.} \\
4|2x - 5| & > 20 \\
\frac{4}{4} & \quad \text{Divide each side by 4.} \\
|2x - 5| & > 5
\end{align*}
\]

Then use $|2x - 5| > 5$ to write a compound inequality. Then solve.

\[
\begin{align*}
2x - 5 & < -5 \\
+5 & \quad +5 \quad \text{Add 5 to each side.} \\
2x & < 0 \\
\frac{2}{2} & \quad 2x > 10 \\
\frac{2}{2} & \quad \text{Divide each side by 2.} \\
x & < 0 \\
\quad +5 & \quad +5 \\
x & > 5 \\
\frac{2}{2} & \quad \text{Simplify.}
\end{align*}
\]

The solution is $x < 0$ or $x > 5$.

**Monitoring Progress**

Solve the inequality. Graph the solution.

4. $|x + 3| > 8$
5. $|n + 2| - 3 \geq -6$
6. $3|d + 1| - 7 \geq -1$
Solving Real-Life Problems

The absolute deviation of a number $x$ from a given value is the absolute value of the difference of $x$ and the given value.

$$\text{absolute deviation} = |x - \text{given value}|$$

### Example 3: Modeling with Mathematics

You are buying a new computer. The table shows the prices of computers in a store advertisement. You are willing to pay the mean price with an absolute deviation of at most $100. How many of the computer prices meet your condition?

#### SOLUTION

1. **Understand the Problem**
   
   You know the prices of 10 computers. You are asked to find how many computers are at most $100 from the mean price.

2. **Make a Plan**
   
   Calculate the mean price by dividing the sum of the prices by the number of prices, 10. Use the absolute deviation and the mean price to write an absolute value inequality. Then solve the inequality and use it to answer the question.

3. **Solve the Problem**

   The mean price is $\frac{6640}{10} = \$664$. Let $x$ represent a price you are willing to pay.

   $$|x - 664| \leq 100$$

   Write the absolute value inequality.

   $$-100 \leq x - 664 \leq 100$$

   Write a compound inequality.

   $$564 \leq x \leq 764$$

   Add 664 to each expression and simplify.

   The prices you will consider must be at least $564 and at most $764. Six prices meet your condition: $750, $650, $660, $670, $650, and $725.

4. **Look Back**

   You can check that your answer is correct by graphing the computer prices and the mean on a number line. Any point within 100 of 664 represents a price that you will consider.

#### Concept Summary

**Solving Inequalities**

- **One-Step and Multi-Step Inequalities**
  
  - Follow the steps for solving an equation. Reverse the inequality symbol when multiplying or dividing by a negative number.

- **Compound Inequalities**
  
  - If necessary, write the inequality as two separate inequalities. Then solve each inequality separately. Include **and** or **or** in the solution.

- **Absolute Value Inequalities**
  
  - If necessary, isolate the absolute value expression on one side of the inequality. Write the absolute value inequality as a compound inequality. Then solve the compound inequality.
Vocabulary and Core Concept Check

1. **REASONING** Can you determine the solution of \(|4x - 2| \geq -6\) without solving? Explain.

2. **WRITING** Describe how solving \(|w - 9| \leq 2\) is different from solving \(|w - 9| \geq 2\).

Monitoring Progress and Modeling with Mathematics

In Exercises 3–18, solve the inequality. Graph the solution, if possible. (See Examples 1 and 2.)

3. \(|x| < 3

4. \(|y| \geq 4.5

5. \(|d + 9| > 3

6. \(|h - 5| \leq 10

7. \(|2s - 7| \geq -1

8. \(|4c + 5| > 7

9. \(|5p + 2| < -4

10. \(|9 - 4n| < 5

11. \(|6t - 7| - 8 \geq 3

12. \(|3j - 1| + 6 > 0

13. \(3|14 - m| > 18

14. \(|-4b - 8| \leq 12

15. \(2|3w + 8| - 13 \leq -5

16. \(-3|2 - 4u| + 5 < -13

17. \(6|-f + 3| + 7 > 7

18. \(\frac{2}{3}|4y + 6| - 2 \leq 10

19. **MODELING WITH MATHEMATICS** The rules for an essay contest say that entries can have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words. (See Example 3.)

20. **MODELING WITH MATHEMATICS** The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.

21. \(|x - 5| < 20

\begin{align*}
&x - 5 < 20 \\
&x < 25
\end{align*}

22. \(|x + 4| > 13

\begin{align*}
&x + 4 > -13 \quad \text{and} \quad x + 4 < 13 \\
&x > -17 \quad \text{and} \quad x < 9 \\
&-17 < x < 9
\end{align*}

In Exercises 23–26, write the sentence as an absolute value inequality. Then solve the inequality.

23. A number is less than 6 units from 0.

24. A number is more than 9 units from 3.

25. Half of a number is at most 5 units from 14.

26. Twice a number is no less than 10 units from \(-1\).

27. **PROBLEM SOLVING** An auto parts manufacturer throws out gaskets with weights that are not within 0.06 pound of the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gasket(s) should be thrown out?

28. **PROBLEM SOLVING** Six students measure the acceleration (in meters per second per second) of an object in free fall. The measured values are shown. The students want to state that the absolute deviation of each measured value \(x\) from the mean is at most \(d\). Find the value of \(d\).

10.56, 9.52, 9.73, 9.80, 9.78, 10.91
MATHEMATICAL CONNECTIONS In Exercises 29 and 30, write an absolute value inequality that represents the situation. Then solve the inequality.

29. The difference between the areas of the figures is less than 2.

\[ \text{Maintaining Mathematical Proficiency} \]
Evaluate the function for the given value of \( x \).
\[ f(x) = x + 4; \quad x = 3 \]
\[ f(x) = -x + 3; \quad x = 5 \]

Create a scatter plot of the data.
\[ \begin{array}{c|cccccc}
 x & 8 & 10 & 11 & 12 & 15 \\
 \hline
 f(x) & 4 & 9 & 10 & 12 & 12 \\
\end{array} \]

30. The difference between the perimeters of the figures is less than or equal to 3.

\[ \text{REASONING} \]
In Exercises 31–34, tell whether the statement is true or false. If it is false, explain why.

31. If \( a \) is a solution of \( |x + 3| \leq 8 \), then \( a \) is also a solution of \( x + 3 \geq -8 \).

32. If \( a \) is a solution of \( |x + 3| > 8 \), then \( a \) is also a solution of \( x + 3 > 8 \).

33. If \( a \) is a solution of \( |x + 3| \geq 8 \), then \( a \) is also a solution of \( x + 3 \geq -8 \).

34. If \( a \) is a solution of \( x + 3 \leq -8 \), then \( a \) is also a solution of \( |x + 3| \geq 8 \).

35. **MAKING AN ARGUMENT** One of your classmates claims that the solution of \( |n| > 0 \) is all real numbers. Is your classmate correct? Explain your reasoning.

36. **THOUGHT PROVOKING** Draw and label a geometric figure so that the perimeter \( P \) of the figure is a solution of the inequality \( |P - 60| \leq 12 \).

37. **REASONING** What is the solution of the inequality \( |ax + b| < c \), where \( c < 0 \)? What is the solution of the inequality \( |ax + b| > c \), where \( c < 0 \)? Explain.

38. **HOW DO YOU SEE IT?** Write an absolute value inequality for each graph.

39. **WRITING** Explain why the solution set of the inequality \( |x| < 5 \) is the intersection of two sets, while the solution set of the inequality \( |x| > 5 \) is the union of two sets.

40. **PROBLEM SOLVING** Solve the compound inequality below. Describe your steps.
\[ |x - 3| < 4 \text{ and } |x + 2| > 8 \]
Essential Question  How can you use a linear function to model and analyze a real-life situation?

**EXPLORATION 1  Modeling with a Linear Function**

**Work with a partner.** A company purchases a copier for $12,000. The spreadsheet shows how the copier depreciates over an 8-year period.

**a.** Write a linear function to represent the value \( V \) of the copier as a function of the number \( t \) of years.

**b.** Sketch a graph of the function. Explain why this type of depreciation is called **straight line depreciation**.

**c.** Interpret the slope of the graph in the context of the problem.

**EXPLORATION 2  Modeling with Linear Functions**

**Work with a partner.** Match each description of the situation with its corresponding graph. Explain your reasoning.

**a.** A person gives $20 per week to a friend to repay a $200 loan.

**b.** An employee receives $12.50 per hour plus $2 for each unit produced per hour.

**c.** A sales representative receives $30 per day for food plus $0.565 for each mile driven.

**d.** A computer that was purchased for $750 depreciates $100 per year.

**Communicate Your Answer**

3. How can you use a linear function to model and analyze a real-life situation?

4. Use the Internet or some other reference to find a real-life example of straight line depreciation.

   **a.** Use a spreadsheet to show the depreciation.

   **b.** Write a function that models the depreciation.

   **c.** Sketch a graph of the function.
What You Will Learn

- Write equations of linear functions using points and slopes.
- Find lines of fit and lines of best fit.

Writing Linear Equations

**Core Concept**

**Writing an Equation of a Line**

- **Given slope** $m$ and **y-intercept** $b$: Use slope-intercept form:
  \[ y = mx + b \]

- **Given slope** $m$ and a point $(x_1, y_1)$: Use point-slope form:
  \[ y - y_1 = m(x - x_1) \]

- **Given points** $(x_1, y_1)$ and $(x_2, y_2)$: First use the slope formula to find $m$. Then use point-slope form with either given point.

**EXAMPLE 1** Writing a Linear Equation from a Graph

The graph shows the distance Asteroid 2012 DA14 travels in $x$ seconds. Write an equation of the line and interpret the slope. The asteroid came within 17,200 miles of Earth in February, 2013. About how long does it take the asteroid to travel that distance?

**SOLUTION**

From the graph, you can see the slope is $m = \frac{24}{5} = 4.8$ and the y-intercept is $b = 0$. Use slope-intercept form to write an equation of the line.

\[ y = mx + b \]
\[ = 4.8x + 0 \]

Substitute 4.8 for $m$ and 0 for $b$.

The equation is $y = 4.8x$. The slope indicates that the asteroid travels 4.8 miles per second. Use the equation to find how long it takes the asteroid to travel 17,200 miles.

\[ 17,200 = 4.8x \]

Substitute 17,200 for $y$.

\[ 3583 \approx x \]

Divide each side by 4.8.

- Because there are 3600 seconds in 1 hour, it takes the asteroid about 1 hour to travel 17,200 miles.

**Monitoring Progress**

1. The graph shows the remaining balance $y$ on a car loan after making $x$ monthly payments. Write an equation of the line and interpret the slope and y-intercept. What is the remaining balance after 36 payments?

---

**Asteroid 2012 DA14**

Distance (miles) vs. Time (seconds)

**Car Loan**

Balance (thousands of dollars) vs. Number of payments
Modeling with Mathematics

Two prom venues charge a rental fee plus a fee per student. The table shows the total costs for different numbers of students at Lakeside Inn. The total cost \( y \) (in dollars) for \( x \) students at Sunview Resort is represented by the equation

\[
y = 10x + 600.
\]

Which venue charges less per student? How many students must attend for the total costs to be the same?

**SOLUTION**

1. **Understand the Problem** You are given an equation that represents the total cost at one venue and a table of values showing total costs at another venue. You need to compare the costs.

2. **Make a Plan** Write an equation that models the total cost at Lakeside Inn. Then compare the slopes to determine which venue charges less per student. Finally, equate the cost expressions and solve to determine the number of students for which the total costs are equal.

3. **Solve the Problem** First find the slope using any two points from the table. Use \((x_1, y_1) = (100, 1500)\) and \((x_2, y_2) = (125, 1800)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1500}{125 - 100} = \frac{300}{25} = 12
\]

Write an equation that represents the total cost at Lakeside Inn using the slope of 12 and a point from the table. Use \((x_1, y_1) = (100, 1500)\).

\[
y - y_1 = m(x - x_1) \quad \text{Point-slope form}
\]

\[
y - 1500 = 12(x - 100) \quad \text{Substitute for } m, x_1, \text{ and } y_1.
\]

\[
y - 1500 = 12x - 1200 \quad \text{Distributive Property}
\]

\[
y = 12x + 300 \quad \text{Add 1500 to each side.}
\]

Equate the cost expressions and solve.

\[
10x + 600 = 12x + 300 \quad \text{Set cost expressions equal.}
\]

\[
300 = 2x \quad \text{Combine like terms.}
\]

\[
150 = x \quad \text{Divide each side by 2.}
\]

Comparing the slopes of the equations, Sunview Resort charges $10 per student, which is less than the $12 per student that Lakeside Inn charges. The total costs are the same for 150 students.

4. **Look Back** Notice that the table shows the total cost for 150 students at Lakeside Inn is $2100. To check that your solution is correct, verify that the total cost at Sunview Resort is also $2100 for 150 students.

\[
y = 10(150) + 600 \quad \text{Substitute 150 for } x.
\]

\[
= 2100 \quad \text{Simplify.}
\]

**Monitoring Progress**

2. **WHAT IF?** Maple Ridge charges a rental fee plus a $10 fee per student. The total cost is $1900 for 140 students. Describe the number of students that must attend for the total cost at Maple Ridge to be less than the total costs at the other two venues. Use a graph to justify your answer.
Finding Lines of Fit and Lines of Best Fit

Data do not always show an exact linear relationship. When the data in a scatter plot show an approximately linear relationship, you can model the data with a line of fit.

**Core Concept**

**Finding a Line of Fit**

**Step 1** Create a scatter plot of the data.

**Step 2** Sketch the line that most closely appears to follow the trend given by the data points. There should be about as many points above the line as below it.

**Step 3** Choose two points on the line and estimate the coordinates of each point. These points do not have to be original data points.

**Step 4** Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

**EXAMPLE 3** Finding a Line of Fit

The table shows the femur lengths (in centimeters) and heights (in centimeters) of several people. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a person whose femur is 35 centimeters long.

**SOLUTION**

**Step 1** Create a scatter plot of the data.

The data show a linear relationship.

**Step 2** Sketch the line that most closely appears to fit the data. One possibility is shown.

**Step 3** Choose two points on the line and estimate the coordinates of each point.

For the line shown, you might choose (40, 170) and (50, 195).

**Step 4** Write an equation of the line that passes through the two points from Step 3. This equation is a model for the data.

<table>
<thead>
<tr>
<th>Femur length, $x$</th>
<th>Height, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>170</td>
</tr>
<tr>
<td>45</td>
<td>183</td>
</tr>
<tr>
<td>32</td>
<td>151</td>
</tr>
<tr>
<td>50</td>
<td>195</td>
</tr>
<tr>
<td>37</td>
<td>162</td>
</tr>
<tr>
<td>41</td>
<td>174</td>
</tr>
<tr>
<td>30</td>
<td>141</td>
</tr>
<tr>
<td>34</td>
<td>151</td>
</tr>
<tr>
<td>47</td>
<td>185</td>
</tr>
<tr>
<td>45</td>
<td>182</td>
</tr>
</tbody>
</table>

First, find the slope.

$$
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{195 - 170}{50 - 40} = \frac{25}{10} = 2.5
$$

Use point-slope form to write an equation. Use $(x_1, y_1) = (40, 170)$.

$$
y - y_1 = m(x - x_1) \quad \text{Point-slope form}
$$

$$
y - 170 = 2.5(x - 40) \quad \text{Substitute for } m, x_1, \text{and } y_1.
$$

$$
y - 170 = 2.5x - 100 \quad \text{Distributive Property}
$$

$$
y = 2.5x + 70 \quad \text{Add 170 to each side.}
$$

Use the equation to estimate the height of the person.

$$
y = 2.5(35) + 70 \quad \text{Substitute 35 for } x.
$$

$$
= 157.5 \quad \text{Simplify.}
$$

The approximate height of a person with a 35-centimeter femur is 157.5 centimeters.
The **line of best fit** is the line that lies as close as possible to all of the data points. Many technology tools have a **linear regression** feature that you can use to find the line of best fit for a set of data.

The **correlation coefficient**, denoted by \( r \), is a number from \(-1\) to \(1\) that measures how well a line fits a set of data pairs \((x, y)\). When \( r \) is near \(1\), the points lie close to a line with a positive slope. When \( r \) is near \(-1\), the points lie close to a line with a negative slope. When \( r \) is near \(0\), the points do not lie close to any line.

### Example 4 Using a Graphing Calculator

Use the **linear regression** feature on a graphing calculator to find an equation of the line of best fit for the data in Example 3. Estimate the height of a person whose femur is 35 centimeters long. Compare this height to your estimate in Example 3.

#### Solution

**Step 1** Enter the data into two lists.

<table>
<thead>
<tr>
<th>L1</th>
<th>L2</th>
<th>L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>170</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>183</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>195</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>162</td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>174</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>141</td>
<td></td>
</tr>
</tbody>
</table>

\( L3(1)=40 \)

**Step 2** Use the **linear regression** feature. The line of best fit is \( y = 2.6x + 65 \).

**Step 3** Graph the regression equation with the scatter plot.

**Step 4** Use the **trace** feature to find the value of \( y \) when \( x = 35 \).

The approximate height of a person with a 35-centimeter femur is 156 centimeters. This is less than the estimate found in Example 3.

### Monitoring Progress

3. The table shows the humerus lengths (in centimeters) and heights (in centimeters) of several females.

<table>
<thead>
<tr>
<th>Humerus length, ( x )</th>
<th>33</th>
<th>25</th>
<th>22</th>
<th>30</th>
<th>28</th>
<th>32</th>
<th>26</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height, ( y )</td>
<td>166</td>
<td>142</td>
<td>130</td>
<td>154</td>
<td>152</td>
<td>159</td>
<td>141</td>
<td>145</td>
</tr>
</tbody>
</table>

a. Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate the height of a female whose humerus is 40 centimeters long.

b. Use the **linear regression** feature on a graphing calculator to find an equation of the line of best fit for the data. Estimate the height of a female whose femur is 40 centimeters long. Compare this height to your estimate in part (a).
1.6 Exercises

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The linear equation $y = \frac{1}{2}x + 3$ is written in ____________ form.

2. **VOCABULARY** A line of best fit has a correlation coefficient of $-0.98$. What can you conclude about the slope of the line?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, use the graph to write an equation of the line and interpret the slope. (See Example 1.)

3. **Tipping**

<table>
<thead>
<tr>
<th>Cost of meal (dollars)</th>
<th>Tip (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>90</td>
<td>3</td>
</tr>
</tbody>
</table>

4. **Gasoline Tank**

<table>
<thead>
<tr>
<th>Distance (miles)</th>
<th>Fuel (gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>8</td>
</tr>
<tr>
<td>120</td>
<td>4</td>
</tr>
</tbody>
</table>

5. **Savings Account**

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Balance (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>120</td>
</tr>
</tbody>
</table>

6. **Tree Growth**

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>Tree height (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

7. **Typing Speed**

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Words typed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55</td>
</tr>
<tr>
<td>2</td>
<td>165</td>
</tr>
</tbody>
</table>

8. **Swimming Pool**

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>Volume (cubic feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
</tr>
</tbody>
</table>

9. **MODELING WITH MATHEMATICS** Two newspapers charge a fee for placing an advertisement in their paper plus a fee based on the number of lines in the advertisement. The table shows the total costs for different length advertisements at the Daily Times. The total cost $y$ (in dollars) for an advertisement that is $x$ lines long at the Greenville Journal is represented by the equation $y = 2x + 20$. Which newspaper charges less per line? How many lines must be in an advertisement for the total costs to be the same? (See Example 2.)

<table>
<thead>
<tr>
<th>Number of lines, $x$</th>
<th>Total cost, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>6</td>
<td>33</td>
</tr>
<tr>
<td>7</td>
<td>36</td>
</tr>
<tr>
<td>8</td>
<td>39</td>
</tr>
</tbody>
</table>

10. **PROBLEM SOLVING** While on vacation in Canada, you notice that temperatures are reported in degrees Celsius. You know there is a linear relationship between Fahrenheit and Celsius, but you forget the formula. From science class, you remember the freezing point of water is 0°C or 32°F, and its boiling point is 100°C or 212°F.

   a. Write an equation that represents degrees Fahrenheit in terms of degrees Celsius.

   b. The temperature outside is 22°C. What is this temperature in degrees Fahrenheit?

   c. Rewrite your equation in part (a) to represent degrees Celsius in terms of degrees Fahrenheit.

   d. The temperature of the hotel pool water is 83°F. What is this temperature in degrees Celsius?
ERROR ANALYSIS In Exercises 11 and 12, describe and correct the error in interpreting the slope in the context of the situation.

11. Savings Account

The slope of the line is 10, so after 7 years, the balance is $70.

12. Earnings

The slope is 3, so the income is $3 per hour.

In Exercises 13–16, determine whether the data show a linear relationship. If so, write an equation of a line of fit. Estimate \( y \) when \( x = 15 \) and explain its meaning in the context of the situation. (See Example 3.)

13. Minutes walking, \( x \) | 1 | 6 | 11 | 13 | 16
   Calories burned, \( y \)  | 6 | 27 | 50 | 56 | 70

14. Months, \( x \) | 9 | 13 | 18 | 22 | 23
   Hair length (in.), \( y \) | 3 | 5 | 7 | 10 | 11

15. Hours, \( x \) | 3 | 7 | 9 | 17 | 20
   Battery life (%), \( y \) | 86 | 61 | 50 | 26 | 0

16. Shoe size, \( x \) | 6 | 8 | 8.5 | 10 | 13
   Heart rate (bpm), \( y \) | 112 | 94 | 100 | 132 | 87

17. MODELING WITH MATHEMATICS The data pairs \((x, y)\) represent the average annual tuition \( y \) (in dollars) for public colleges in the United States \( x \) years after 2005. Use the linear regression feature on a graphing calculator to find an equation of the line of best fit. Estimate the average annual tuition in 2020. Interpret the slope and \( y \)-intercept in this situation. (See Example 4.)

\([(0, 11,386), (1, 11,731), (2, 11,848)
\[(3, 12,375), (4, 12,804), (5, 13,297)\]

18. MODELING WITH MATHEMATICS The table shows the numbers of tickets sold for a concert when different prices are charged. Write an equation of a line of fit for the data. Does it seem reasonable to use your model to predict the number of tickets sold when the ticket price is $85? Explain.

<table>
<thead>
<tr>
<th>Ticket price (dollars), ( x )</th>
<th>17</th>
<th>20</th>
<th>22</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tickets sold, ( y )</td>
<td>450</td>
<td>423</td>
<td>400</td>
<td>395</td>
</tr>
</tbody>
</table>

USING TOOLS In Exercises 19–24, use the linear regression feature on a graphing calculator to find an equation of the line of best fit for the data. Find and interpret the correlation coefficient.

19.

20.

21.

22.

23.

24.

25. OPEN-ENDED Give two real-life quantities that have (a) a positive correlation, (b) a negative correlation, and (c) approximately no correlation. Explain.
26. **HOW DO YOU SEE IT?** You secure an interest-free loan to purchase a boat. You agree to make equal monthly payments for the next two years. The graph shows the amount of money you still owe.

![Boat Loan Graph]

- a. What is the slope of the line? What does the slope represent?
- b. What is the domain and range of the function? What does each represent?
- c. How much do you still owe after making payments for 12 months?

27. **MAKING AN ARGUMENT** A set of data pairs has a correlation coefficient \( r = 0.3 \). Your friend says that because the correlation coefficient is positive, it is logical to use the line of best fit to make predictions. Is your friend correct? Explain your reasoning.

28. **THOUGHT PROVOKING** Points \( A \) and \( B \) lie on the line \( y = -x + 4 \). Choose coordinates for points \( A \), \( B \), and \( C \) where point \( C \) is the same distance from point \( A \) as it is from point \( B \). Write equations for the lines connecting points \( A \) and \( C \) and points \( B \) and \( C \).

29. **ABSTRACT REASONING** If \( x \) and \( y \) have a positive correlation, and \( y \) and \( z \) have a negative correlation, then what can you conclude about the correlation between \( x \) and \( z \)? Explain.

30. **MATHEMATICAL CONNECTIONS** Which equation has a graph that is a line passing through the point \((8, -5)\) and is perpendicular to the graph of \( y = -4x + 1 \)?

\[
\begin{align*}
\text{A} & : y = \frac{1}{4}x - 5 \\
\text{B} & : y = -4x + 27 \\
\text{C} & : y = -\frac{1}{4}x - 7 \\
\text{D} & : y = \frac{1}{4}x - 7
\end{align*}
\]

31. **PROBLEM SOLVING** You are participating in an orienteering competition. The diagram shows the position of a river that cuts through the woods. You are currently 2 miles east and 1 mile north of your starting point, the origin. What is the shortest distance you must travel to reach the river?

![Orienteering Diagram]

32. **ANALYZING RELATIONSHIPS** Data from North American countries show a positive correlation between the number of personal computers per capita and the average life expectancy in the country.

- a. Does a positive correlation make sense in this situation? Explain.
- b. Is it reasonable to conclude that giving residents of a country personal computers will lengthen their lives? Explain.

### Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the system of linear equations in two variables by elimination or substitution.**

(Skills Review Handbook)

- **33.** \(3x + y = 7\)
  \(-2x - y = 9\)
- **34.** \(4x + 3y = 2\)
  \(2x - 3y = 1\)
- **35.** \(2x + 2y = 3\)
  \(x = 4y - 1\)
- **36.** \(y = 1 + x\)
  \(2x + y = -2\)
- **37.** \(\frac{1}{2}x + 4y = 4\)
  \(2x - y = 1\)
- **38.** \(y = x - 4\)
  \(4x + y = 26\)
1.4–1.6 What Did You Learn?

Core Vocabulary

absolute value equation, p. 28
extraneous solution, p. 31
absolute value inequality, p. 36
absolute deviation, p. 38
line of fit, p. 44
line of best fit, p. 45
correlation coefficient, p. 45

Core Concepts

Section 1.4
Properties of Absolute Value, p. 28
Solving Absolute Value Equations, p. 28
Solving Equations with Two Absolute Values, p. 30
Identifying Special Solutions, p. 31

Section 1.5
Solving Absolute Value Inequalities, p. 36

Section 1.6
Writing an Equation of a Line, p. 42
Finding a Line of Fit, p. 44

Mathematical Thinking

1. How did you decide whether your friend’s argument in Exercise 46 on page 33 made sense?
2. How did you use the structure of the equation in Exercise 59 on page 34 to solve the equation?
3. Describe the given information and the overall goal of Exercise 27 on page 39.
4. Describe how you can write the equation of the line in Exercise 7 on page 46 using only one of the labeled points.

Performance Task

Secret of the Hanging Baskets

A carnival game uses two baskets hanging from springs at different heights. Next to the higher basket is a pile of baseballs. Next to the lower basket is a pile of golf balls. The object of the game is to add the same number of balls to each basket so that the baskets have the same height. But there is a catch—you only get one chance. What is the secret to winning the game?

To explore the answer to this question and more, go to BigIdeasMath.com.
1.1 Interval Notation and Set Notation (pp. 3–8)

a. Write \( x \geq -1 \) in interval notation.

\[ \text{The graph of } x \geq -1 \text{ is the unbounded interval } [-1, \infty). \]

b. Write the set of integers from \(-120\) through \(80\) in set-builder notation.

\[ \{ x \mid -120 \leq x \leq 80, x \in \mathbb{Z} \} \]

Write the interval in interval notation.

1. \( x \leq 48 \)
2. \( x > 6 \)
3. \( -8 < x \leq 16 \)
4. Write the set of integers less than \(-6\) or greater than \(21\) in set-builder notation.

1.2 Parent Functions and Transformations (pp. 9–16)

Graph \( g(x) = (x - 2)^2 + 1 \) and its parent function. Then describe the transformation.

The function \( g \) is a quadratic function.

\[ \text{The graph of } g \text{ is a translation 2 units right and 1 unit up of the graph of the parent quadratic function.} \]

Graph the function and its parent function. Then describe the transformation.

5. \( f(x) = x + 3 \)
6. \( g(x) = |x| - 1 \)
7. \( h(x) = \frac{1}{2}x^2 \)
8. \( h(x) = 4 \)
9. \( f(x) = -|x| - 3 \)
10. \( g(x) = -3(x + 3)^2 \)

1.3 Transformations of Linear and Absolute Value Functions (pp. 17–24)

Let the graph of \( g \) be a translation 2 units to the right followed by a reflection in the \( y \)-axis of the graph of \( f(x) = |x| \). Write a rule for \( g \).

Step 1 First write a function \( h \) that represents the translation of \( f \).

\[ h(x) = f(x - 2) \quad \text{Subtract 2 from the input.} \]
\[ = 2|x - 2| \quad \text{Replace } x \text{ with } x - 2 \text{ in } f(x). \]

Step 2 Then write a function \( g \) that represents the reflection of \( h \).

\[ g(x) = h(-x) \quad \text{Multiply the input by } -1. \]
\[ = |-(x - 2)| \quad \text{Replace } x \text{ with } -x \text{ in } h(x). \]
\[ = |-(x + 2)| \quad \text{Factor out } -1. \]
\[ = |-1| \cdot |x + 2| \quad \text{Product Property of Absolute Value} \]
\[ = |x + 2| \quad \text{Simplify.} \]

\[ \text{The transformed function is } g(x) = |x + 2|. \]
Chapter 1

1.4 Solving Absolute Value Equations  (pp. 27–34)

Solve \(|2x + 6| = 4x\). Check your solutions.

\[
\begin{align*}
2x + 6 &= 4x \\
-2x &= -2x \\
6 &= 2x \\
6 &= -6x \\
2 &= 2x \\
-1 &= x
\end{align*}
\]

Check the apparent solutions to see if either is extraneous.

\[
\begin{align*}
|2x + 6| &= 4x \\
|2(3) + 6| &= ?(3) \\
12 &= 12 \\
12 &= 12 \checkmark
\end{align*}
\]

\[
\begin{align*}
|2x + 6| &= 4x \\
|2(-1) + 6| &= ?(-1) \\
4 &= -4 \\
4 \neq -4 \times
\end{align*}
\]

Check \(|2x + 6| = 4x\) \( \Leftrightarrow \) \(2(3) + 6 = 4(3)\) \( \Leftrightarrow \) \(12 = 12\) \( \checkmark \)

The solution is \(x = 3\). Reject \(x = -1\) because it is extraneous.

Solve the equation. Check your solutions.

14. \(|y + 3| = 17\)  
15. \(-2|5w - 7| + 9 = -7\)  
16. \(|x - 2| = |4 + x|\)

1.5 Solving Absolute Value Inequalities  (pp. 35–40)

Solve \(|x + 11| + 6 > 8\). Graph the solution.

\[
\begin{align*}
|x + 11| + 6 &> 8 \\
6 &> 6 \\
|x + 11| &> 2 \\
x + 11 &< -2 \quad \text{or} \quad x + 11 > 2
\end{align*}
\]

\[
\begin{align*}
x &< -13 \quad \text{or} \quad x > -9
\end{align*}
\]

\[
\text{The solution is } x < -13 \text{ or } x > -9.
\]

Solve the inequality. Graph the solution, if possible.

17. \(|m| \geq 10\)  
18. \(|k - 9| < -4\)  
19. \(4|f - 6| \leq 12\)  
20. \(5|b + 8| - 7 > 13\)  
21. \(|-3g - 2| + 1 < 6\)  
22. \(|9 - 2j| + 10 \geq 2\)
1.6 Modeling with Linear Functions  (pp. 41–48)

The table shows the numbers of ice cream cones sold for different outside temperatures (in degrees Fahrenheit). Do the data show a linear relationship? If so, write an equation of a line of fit and use it to estimate how many ice cream cones are sold when the temperature is 60°F.

<table>
<thead>
<tr>
<th>Temperature, x</th>
<th>53</th>
<th>62</th>
<th>70</th>
<th>82</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cones, y</td>
<td>90</td>
<td>105</td>
<td>117</td>
<td>131</td>
<td>147</td>
</tr>
</tbody>
</table>

**Step 1** Create a scatter plot of the data. The data show a linear relationship.

**Step 2** Sketch the line that appears to most closely fit the data. One possibility is shown.

**Step 3** Choose two points on the line. For the line shown, you might choose (70, 117) and (90, 147).

**Step 4** Write an equation of the line. First, find the slope.

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{147 - 117}{90 - 70} = \frac{30}{20} = 1.5
\]

Use point-slope form to write an equation. Use \((x_1, y_1) = (70, 117)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - 117 = 1.5(x - 70)
\]

**Distributive Property**

\[
y = 1.5x + 12
\]

Add 117 to each side.

Use the equation to estimate the number of ice cream cones sold.

\[
y = 1.5(60) + 12
\]

**Substitute 60 for \(x\).**

\[
y = 102
\]

**Simplify.**

Approximately 102 ice cream cones are sold when the temperature is 60°F.

**Write an equation of the line.**

23. The table shows the total number \(y\) (in billions) of U.S. movie admissions each year for \(x\) years. Use a graphing calculator to find an equation of the line of best fit for the data.

<table>
<thead>
<tr>
<th>Year, (x)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Admissions, (y)</td>
<td>1.24</td>
<td>1.26</td>
<td>1.39</td>
<td>1.47</td>
<td>1.49</td>
<td>1.57</td>
</tr>
</tbody>
</table>

24. You ride your bike and measure how far you travel. After 10 minutes, you travel 3.5 miles. After 30 minutes, you travel 10.5 miles. Write an equation to model your distance. How far can you ride your bike in 45 minutes?
Write an equation of the line and interpret the slope and y-intercept.

1. **Bank Account**
   - Balance (dollars)
   - 0
   - 200
   - 400
   - 600
   - 800
   - Weeks

2. **Shoe Sales**
   - Price of pair of shoes (dollars)
   - 0
   - 10
   - 20
   - 30
   - 40
   - 50
   - Percent discount
   - 6002
   - 0
   - 4
   - 0
   - 8
   - 0

Write the interval in interval notation.

3. \(-4, 3\)

4. \(x > 6\)

Write the set of numbers in set-builder notation.

5. \([-64, -12)\)

6. \((-\infty, 11)\) or \([12, 16)\)

Graph the function and its parent function. Then describe the transformation.

7. \(f(x) = |x - 1|\)

8. \(f(x) = (3x)^2\)

9. \(f(x) = 4\)

Match the transformation of \(f(x) = |x|\) with its graph. Then write a rule for \(g\).

10. \(g(x) = 2f(x + 3)\)

11. \(g(x) = 3f(x) - 2\)

12. \(g(x) = -2f(x) + 3\)

Solve the inequality. Graph the solution, if possible.

13. \(|2q + 8| > 4\)

14. \(-2|y - 3| - 5 \geq -4\)

15. \(4|-3b + 5| - 9 < 7\)

16. A fountain with a depth of 5 feet is drained and then refilled. The water level (in feet) after \(t\) minutes can be modeled by \(f(t) = \frac{1}{4}|t - 20|\). A second fountain with the same depth is drained and filled twice as quickly as the first fountain. Describe how to transform the graph of \(f\) to model the water level in the second fountain after \(t\) minutes. Find the depth of each fountain after 4 minutes. Justify your answers.

17. The graph shows the cost to hire a plumber for \(x\) hours.
   a. What is the plumber’s minimum charge for a service call?
   b. How much will the plumber charge for a 7-hour job?
1. Find the solution(s) of $|3x + 7| = 5x$.  \textit{(TEKS 2A.6.E)}

\[ A \ x = -4, x = -\frac{2}{3} \quad B \ x = -\frac{7}{8}, x = \frac{7}{2} \quad C \ x = \frac{7}{8}, x = \frac{7}{2} \quad D \ x = \frac{7}{2} \]

2. The scatter plot shows the atmospheric temperature at various altitudes. What is the approximate temperature at an altitude of 5 kilometers?  \textit{(TEKS 2A.8.C)}

\[ F \ -32^\circ C \quad G \ -25^\circ C \quad H \ -20^\circ C \quad J \ \text{none of the above} \]

3. Which list shows the functions in order from the widest graph to the narrowest graph?  \textit{(TEKS 2A.6.C)}

\[ A \ y = -5|x|, y = -\frac{2}{3}|x|, y = \frac{5}{6}|x|, y = 8|x| \quad B \ y = -\frac{2}{3}|x|, y = \frac{5}{6}|x|, y = -5|x|, y = 8|x| \quad C \ y = \frac{5}{6}|x|, y = -\frac{2}{3}|x|, y = 8|x|, y = -5|x| \quad D \ y = 8|x|, y = \frac{5}{6}|x|, y = -\frac{2}{3}|x|, y = -5|x| \]

4. Which equation has the graph shown?  \textit{(TEKS 2A.6.C)}

\[ F \ y = |x| + 2 \quad G \ y = |x - 2| \quad H \ y = |x + 2| \quad J \ y = |x| - 2 \]

5. **GRIDDED ANSWER** You are selling sandwiches to raise money for a class field trip. Your daily sales $s$ (in dollars) increase for the first several days and then decrease as given by the function $s(t) = -15|t - 5| + 180$, where $t$ is the time (in days). What is the maximum amount of money you raised in one day?  \textit{(TEKS 2A.6.C)}

6. What is the solution of $|6x - 9| \geq 33$?  \textit{(TEKS 2A.6.F)}

\[ A \ -4 \leq x \leq 7 \quad B \ -7 \leq x \leq 4 \quad C \ x \leq -4 \text{ or } x \geq 7 \quad D \ x \leq -7 \text{ or } x \geq 4 \]
7. The graph shows the value of a comic book over a period of 9 years. What is a reasonable conclusion about the value of a comic book during the time shown? (*TEKS 2A.8.C*)

- **F** It appreciates $2 every year.
- **G** It appreciates $3 every 2 years.
- **H** Its value at 5 years was twice its value at 2 years.
- **J** Its value at 7 years was half its value at 3 years.

---

8. One point on the graph of \( y = -2|x| + c \) is the origin. Which statement is true about \( c \)? (*TEKS 2A.6.C*)

- **A** \( c < 0 \)
- **B** \( c > 0 \)
- **C** \( c = 0 \)
- **D** \( c = -2 \)

---

9. The graph shows a transformation of the graph of \( f(x) = |x| \). Which equation can be used to describe \( g \) in terms of \( f \)? (*TEKS 2A.6.C*)

- **F** \( g(x) = f(x + 4) - 3 \)
- **G** \( g(x) = f(x - 4) + 3 \)
- **H** \( g(x) = f(x - 4) - 3 \)
- **J** \( g(x) = f(x + 4) + 3 \)

---

10. Use the data in the table to find the closest estimate of \( y \) when \( x = 20 \). (*TEKS 2A.8.A, TEKS 2A.8.C*)

<table>
<thead>
<tr>
<th>( x )</th>
<th>12</th>
<th>25</th>
<th>36</th>
<th>50</th>
<th>64</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>100</td>
<td>75</td>
<td>52</td>
<td>26</td>
<td>9</td>
</tr>
</tbody>
</table>

- **A** 83
- **B** 92
- **C** 80
- **D** 56

---

11. Which absolute value inequality is represented by the graph? (*TEKS 2A.6.F*)

- **F** \( |x - 2| < 3 \)
- **G** \( |x + 2| < 3 \)
- **H** \( |x - 2| < 5 \)
- **J** \( -1 < |x| < 5 \)