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2 3 4 5 6 7 8 9 10 WEB 18 17 16 15 14
Ron Larson, Ph.D., is well known as the lead author of a comprehensive program for mathematics that spans middle school, high school, and college courses. He holds the distinction of Professor Emeritus from Penn State Erie, The Behrend College, where he taught for nearly 40 years. He received his Ph.D. in mathematics from the University of Colorado. Dr. Larson’s numerous professional activities keep him actively involved in the mathematics education community and allow him to fully understand the needs of students, teachers, supervisors, and administrators.

Laurie Boswell, Ed.D., is the Head of School and a mathematics teacher at the Riverside School in Lyndonville, Vermont. Dr. Boswell is a recipient of the Presidential Award for Excellence in Mathematics Teaching and has taught mathematics to students at all levels, from elementary through college. Dr. Boswell was a Tandy Technology Scholar and served on the NCTM Board of Directors from 2002 to 2005. She currently serves on the board of NCSM and is a popular national speaker.

Dr. Ron Larson and Dr. Laurie Boswell began writing together in 1992. Since that time, they have authored over two dozen textbooks. In their collaboration, Ron is primarily responsible for the student edition while Laurie is primarily responsible for the teaching edition.
Welcome to Big Ideas Math Geometry. From start to finish, this program was designed with you, the learner, in mind.

As you work through the chapters in your Geometry course, you will be encouraged to think and to make conjectures while you persevere through challenging problems and exercises. You will make errors—and that is ok! Learning and understanding occur when you make errors and push through mental roadblocks to comprehend and solve new and challenging problems.

In this program, you will also be required to explain your thinking and your analysis of diverse problems and exercises. Being actively involved in learning will help you develop mathematical reasoning and use it to solve math problems and work through other everyday challenges.

We wish you the best of luck as you explore Geometry. We are excited to be a part of your preparation for the challenges you will face in the remainder of your high school career and beyond.
Big Ideas Math Algebra 1, Geometry, and Algebra 2 is a research-based program providing a rigorous, focused, and coherent curriculum for high school students. Ron Larson and Laurie Boswell utilized their expertise as well as the body of knowledge collected by additional expert mathematicians and researchers to develop each course. The pedagogical approach to this program follows the best practices outlined in the most prominent and widely-accepted educational research and standards, including:

Achieve, ACT, and The College Board

Adding It Up: Helping Children Learn Mathematics
National Research Council ©2001

Curriculum Focal Points and the Principles and Standards for School Mathematics ©2000
National Council of Teachers of Mathematics (NCTM)

Project Based Learning
The Buck Institute

Rigor/Relevance Framework™
International Center for Leadership in Education

Universal Design for Learning Guidelines
CAST ©2011

We would also like to express our gratitude to the experts who served as consultants for Big Ideas Math Algebra 1, Geometry, and Algebra 2. Their input was an invaluable asset to the development of this program.

Carolyn Briles
Mathematics Teacher
Leesburg, Virginia

Jean Carwin
Math Specialist/TOSA
Snohomish, Washington

Alice Fisher
Instructional Support Specialist, RUSMP
Houston, Texas

Kristen Karbon
Curriculum and Assessment Coordinator
Troy, Michigan

Anne Papakonstantinou, Ed.D.
Project Director, RUSMP
Houston, Texas

Richard Parr
Executive Director, RUSMP
Houston, Texas

Melissa Ruffin
Master of Education
Austin, Texas

Connie Schrock, Ph.D.
Mathematics Professor
Emporia, Kansas

Nancy Siddens
Independent Language Teaching Consultant
Cambridge, Massachusetts

Bonnie Spence
Mathematics Lecturer
Missoula, Montana

Susan Troutman
Associate Director for Secondary Programs, RUSMP
Houston, Texas

Carolyn White
Assoc. Director for Elem. and Int. Programs, RUSMP
Houston, Texas

We would also like to thank all of our reviewers who provided feedback during the final development phases. For a complete list of the Big Ideas Math program reviewers, please visit www.BigIdeasLearning.com.
35. **CONSTRUCTION**  Follow these steps to construct a reflection of \( \triangle ABC \) in line \( m \). Use a compass and straightedge.

**Step 1**  Draw \( \triangle ABC \) and line \( m \).

**Step 2**  Use one compass setting to find two points that are equidistant from \( A \) on line \( m \). Use the same compass setting to find a point on the other side of \( m \) that is the same distance from these two points. Label that point as \( A' \).

**Step 3**  Repeat Step 2 to find points \( B' \) and \( C' \).

Draw \( \triangle A'B'C' \).

36. **USING TOOLS**  Use a reflective device to verify your construction in Exercise 35.

---

**Texas Mathematical Process Standards**

Apply mathematics to problems arising in everyday life, society, and the workplace.

- Real-life scenarios are utilized in *Explorations, Examples, Exercises, and Assessments* so students have opportunities to apply the mathematical concepts they have learned to realistic situations.
- Real-world problems help students use the structure of mathematics to break down and solve more difficult problems.

---

**Example 4  Solving a Real-Life Problem**

A soccer goalie’s position relative to the ball and goalposts forms congruent angles, as shown. Will the goalie have to move farther to block a shot toward the right goalpost \( R \) or the left goalpost \( L \)?

**Solution**

The congruent angles tell you that the goalie is on the bisector of \( \angle LBR \). By the Angle Bisector Theorem, the goalie is equidistant from \( BR \) and \( BL \).

So, the goalie must move the same distance to block either shot.
Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.

- Students are asked to construct arguments, critique the reasoning of others, and evaluate multiple representations of problems in specialized exercises, including *Making an Argument*, *How Do You See It?*, *Drawing Conclusions*, *Reasoning*, *Error Analysis*, *Problem Solving*, and *Writing*.
- Real-life situations are translated into diagrams, tables, equations, and graphs to help students analyze relationships and draw conclusions.

Create and use representations to organize, record, and communicate mathematical ideas.

- *Modeling with Mathematics* exercises allow students to interpret a problem in the context of a real-life situation, while utilizing tables, graphs, visual representations, and formulas.
- Multiple representations are presented to help students move from concrete to representative and into abstract thinking.

Analyze mathematical relationships to connect and communicate mathematical ideas.

- *Using Structure* exercises provide students with the opportunity to explore patterns and structure in mathematics.
- Stepped-out *Examples* encourage students to maintain oversight of their problem-solving process and pay attention to the relevant details in each step.

Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

- *Vocabulary and Core Concept Check* exercises require students to use clear, precise mathematical language in their solutions and explanations.
- *Performance Tasks* for every chapter allow students to apply their skills to comprehensive problems and utilize precise mathematical language when analyzing, interpreting, and communicating their answers.

---

**36. MODELING WITH MATHEMATICS** Three tennis balls are stored in a cylindrical container with a height of 8 inches and a radius of 1.43 inches. The circumference of a tennis ball is 8 inches.

a. Find the volume of a tennis ball.

b. Find the amount of space within the cylinder not taken up by the tennis balls.

---

**32. HOW DO YOU SEE IT?** Name the figure that is represented by each net. Justify your answer.

---

**EXAMPLE 2** Finding a Corresponding Length

In the diagram, \(\triangle DEF \sim \triangle MNP\). Find the value of \(x\).

**SOLUTION**

The triangles are similar, so the corresponding side lengths are proportional.

\[
\frac{MN}{NP} = \frac{DE}{EP}
\]

\[
18 \quad 30
\]

\[
15 \quad x
\]

\[
18x = 450
\]

\[
x = 25
\]

The value of \(x\) is 25.

---

**Circular Motion**

What do the properties of tangents tell us about the forces acting on a satellite orbiting around Earth? How would the path of the satellite change if the force of gravity were removed?

To explore the answers to this question and more, go to [BigIdeasMath.com](http://www.bigideasmath.com).
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See the Big Idea
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See the Big Idea
Learn how to use triangle congruence in a model hang glider challenge.
See the Big Idea
Discover why triangles are used in building for strength.
See the Big Idea
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See the Big Idea
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See the Big Idea
Learn about caring for trees at an arboretum.
Get ready for each chapter by maintaining mathematical proficiency and sharpening your mathematical thinking. Begin each section by working through the explorations to communicate your answer to the essential question. Each lesson will explain what you will learn through examples, core concepts, and core vocabulary.

Answer the monitoring progress questions as you work through each lesson. Look for study tips, common errors, and suggestions for looking at a problem another way throughout the lessons. We will also provide you with guidance for accurate mathematical reading and concept details you should remember.

Sharpen your newly acquired skills with exercises at the end of every section. Halfway through each chapter you will be asked what did you learn? and you can use the mid-chapter quiz to check your progress. You can also use the chapter review and chapter test to review and assess yourself after you have completed a chapter.

Apply what you learned in each chapter to a performance task and build your confidence for taking standardized tests with each chapter’s standards assessment.

For extra practice in any chapter, use your online resources, skills review handbook, or your student journal.
Program Overview

Program Philosophy: Rigor and Balance with Real-Life Applications

The Big Ideas Math® program balances conceptual understanding with procedural fluency. Real-life applications help turn mathematical learning into an engaging and meaningful way to see and explore the real world.

4.2 Lesson

What You Will Learn

- Perform reflections.
- Perform glide reflections.
- Identify lines of symmetry.
- Solve real-life problems involving reflections.

Explorations and guiding Essential Questions encourage conceptual understanding.

Essential Question

How can you reflect a figure in a coordinate plane?

Exploration 1

Reflecting a Triangle Using a Reflective Device

Work with a partner. Use a straightedge to draw any triangle on paper. Label it \( \triangle ABC \).

a. Use the straightedge to draw a line that does not pass through the triangle. Label it \( m \).
b. Place a reflective device on line \( m \).
c. Use the reflective device to plot the images of the vertices of \( \triangle ABC \). Label the images of vertices \( A \), \( B \), and \( C \) as \( A' \), \( B' \), and \( C' \), respectively.
d. Use a straightedge to draw \( \triangle A'B'C' \) by connecting the vertices.

Real-life applications provide students with opportunities to connect classroom lessons to realistic scenarios.

Core Vocabulary

- reflection, p. 186
- line of reflection, p. 186
- glide reflection, p. 188
- line symmetry, p. 189
- line of symmetry, p. 189

Core Concept

Reflections

A reflection is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the line of reflection.

A reflection in a line \( m \) maps every point \( P \) in the plane to a point \( P' \), so that for each point one of the following properties is true.

- If \( P \) is not on \( m \), then \( m \) is the perpendicular bisector of \( PP' \), or
- If \( P \) is on \( m \), then \( P = P' \).

Direct instruction lessons allow for procedural fluency and provide the opportunity to use clear, precise mathematical language.

Solving Real-Life Problems

Example 6

Finding a Minimum Distance

You are going to buy books. Your friend is going to buy CDs. Where should you park to minimize the distance you both will walk?

Solution

Reflect \( B \) in line \( m \) to obtain \( B' \). Then draw \( AB' \). Label the intersection of \( AB' \) and \( m \) as \( C \). Because \( AB' \) is the shortest distance between \( A \) and \( B' \), and \( BC = B'C \), park at point \( C \) to minimize the combined distance, \( AC + BC \), you both have to walk.
Using Dynamic Geometry Software

**Core Concept**

Dynamic geometry software allows you to create geometric drawings, including:

- drawing a point
- drawing a line
- drawing a line segment
- drawing an angle
- measuring an angle
- measuring a line segment
- drawing a circle
- drawing an ellipse
- drawing a perpendicular line
- drawing a polygon
- copying and sliding an object
- reflecting an object in a line

**Example 1** Finding Side Lengths and Angle Measures

Use dynamic geometry software to draw a triangle with vertices at A(−2, 1), B(2, 1), and C(−2, −3). Find the side lengths and angle measures of the triangle.

**Sample**

Using dynamic geometry software, you can create ΔABC, as shown.

![Illustration of ΔABC]

From the display, the side lengths are AB = 4 units, BC = 3 units, and AC = 5 units. The angle measures, rounded to two decimal places, are m∠A ≈ 36.87°, m∠B = 90°, and m∠C ≈ 53.13°.

**Monitoring Progress**

Use dynamic geometry software to draw the polygon with the given vertices. Use the software to find the side lengths and angle measures of the polygon. Round your answers to the nearest hundredth.

1. A(0, 2), B(3, −1), C(4, 3)
2. A(−2, 1), B(−2, −1), C(3, 2)
3. A(1, 1), B(−3, 1), C(−3, −2), D(1, −2)
4. A(1, 1), B(−3, 1), C(−2, −2), D(2, −2)
5. A(−3, 0), B(0, 3), C(3, 0), D(0, −3)
6. A(0, 0), B(4, 0), C(1, 1), D(0, 3)
Personalized Learning

The *Big Ideas Math* program offers teachers and students many ways to personalize and enrich the learning experience of all levels of learners.

**Dynamic Student Edition**

This unique tool, available online or as an eBook App, provides students with embedded 21st century learning resources. Students have the opportunity to interact with the underlying mathematics in a number of ways, including engaging tutorials, interactive manipulatives, flashcards, vocabulary support, and games that enhance the learning experience and promote mathematical understanding.

**Dynamic Assessment & Progress Monitoring Tool**

This tool provides personalized links to remediation at point-of-use on student and teacher reports. Teachers can also create customized practice sets for students struggling with a particular concept. Students will receive immediate feedback on their short-answer and multiple-choice responses.

**Online Lesson Tutorials**

Two- to three-minute lesson tutorial videos provide video and audio support for every example in the textbook. These are valuable for students who miss a class, need a second explanation, or need extra assistance with a homework assignment. Parents can also utilize the tutorials to stay connected or to provide additional help at home.
Through print and digital resources, the *Big Ideas Math* program completely supports the 3-Tier Response to Intervention model. Using research-based strategies, teachers can reach, challenge, and motivate each student with high-quality instruction targeted to individual needs.

**Differentiated Instruction**

![Image of students working in a classroom]

**Customized Learning Intervention**
- *Big Ideas Math*
  Middle School program at BigIdeasMath.com

**Strategic Intervention**
- Lesson Tutorials
- Game Closet
- Skills Review Handbook
- Differentiated Instruction
- Dynamic Assessment and Progress Monitoring Tool

**Daily Intervention**
- Student Journal
- Vocabulary Support
- Lesson Tutorials
- Communicate Your Answer
- Monitoring Progress
- Maintaining Mathematical Proficiency
- Dynamic Assessment and Progress Monitoring Tool
The Dynamic Assessment and Progress Monitoring Tool

The Dynamic Assessment and Progress Monitoring Tool allows teachers to know where their students are and monitor their progress as they guide them to where they need to be. This tool was developed exclusively for Big Ideas Math by Pacific Metrics, the educational technology powerhouse that provides assessment resources to PARCC and the Smarter Balanced Assessment Consortium.

Assessment Creation and Scheduling
Assessments can be created by TEKS Standard or Big Ideas Math content. Features adaptive testing capabilities.

Direct Ties to Remediation
Includes direct links to Lesson Tutorial Videos and Skills Review Handbook resources. Appears at point-of-use for students and teachers.

Customizable Reporting
Facilitates progress monitoring by student or by class. Reports can be generated by assessment or by TEKS Standard.

Assessment Delivery with Embedded Tools
Students have access to scientific calculators, protractors, and other tools when enabled. Short-answer and multiple-choice questions are automatically graded.

Online System with Secure Message Center
Accounts can be accessed from any location. Communicate with one student or an entire class.

Easy-to-Use Drag-and-Drop Interface
Includes a customizable dashboard and scheduling module.
Through the Dynamic Student Edition eBook App, students not only have access to the complete textbook, but they can also explore robust interactive digital resources embedded within each lesson. There is audio support for the text and Lesson Tutorial Videos in both English and Spanish. Interactive investigations, direct links to remediation, and additional resources are linked right to the lesson.

Dynamic Investigations

Dynamic Investigations in the Big Ideas Math program are powered by Desmos® and GeoGebra®. Teachers and students can integrate these investigations into their discovery learning to interact with the explorations in the Student Edition.

Real-Life STEM Videos

Science - Technology - Engineering - Mathematics

Every chapter in the Big Ideas Math program contains a Real-Life STEM Video allowing students to further engage with mathematical concepts. Students learn about the speed of light, natural disasters, solar power, and more!
Lesson Planning Support

Online Lesson Plans
Complete, editable lesson plans are included for every lesson in the program.

Laurie’s Notes
Online teaching support for every chapter is available from master educator Laurie Boswell.

Lesson Presentation Support

Dynamic Classroom
This online tool contains a collection of interactive resources for presenting lessons.

Interactive Whiteboard Lesson Library
Standard or customizable lessons are provided for SMART®, Promethean®, and Mimio® interactive whiteboards.
## Program Resources

### Print

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<th>Student Journal</th>
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</thead>
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<td></td>
<td><em>Available in English and Spanish</em></td>
</tr>
</tbody>
</table>

### Resources by Chapter

- Start Thinking
- Warm Up
- Cumulative Review Warm Up
- Practice A and B
- Enrichment and Extension
- Puzzle Time
- Family Communication Letters  
  *Available in English and Spanish*

### Assessment Book

- Performance Tasks
- Prerequisite Skills Test with Item Analysis
- Quarterly Standards Based Tests
- Quizzes
- Chapter Tests
- Alternative Assessments with Scoring Rubrics
- Pre-Course Test with Item Analysis
- Post-Course Test with Item Analysis

### Technology

#### Student Edition

*With complete English and Spanish audio*

- Dynamic eBook App
- Online Home Edition
- eReader Format

#### Dynamic Classroom

- Interactive Manipulatives
- Multi-Language Glossary
- Vocabulary Flash Cards
- Worked-Out Solutions
- Lesson Tutorial Videos

#### Dynamic Teaching Tools

- Interactive Whiteboard Lesson Library
  - Includes standard and customizable lessons
  - Compatible with SMART®, Promethean®, and Mimio® technology
- Real-Life STEM Videos
- Editable Online Resources
  - Lesson Plans
  - Assessment Book
  - Resources by Chapter
- Answer Presentation Tool

#### Dynamic Assessment and Progress Monitoring Tool

#### Teacher Support for TEKS

- Differentiating the Lesson
- Laurie’s Notes
Teaching English Language Learners

English language learners (ELLs) and their teachers face significant challenges in the mainstream high school mathematics classroom. In addition to linguistic challenges, ELLs are adjusting to a new culture and may have psychosocial issues that affect their ability to acclimate. Teachers may not be aware of all the linguistic and cultural confusion these students encounter. Teaching and learning across the gap of cultural and linguistic differences can be a challenge. The principles for teaching ELLs presented in the following pages are intended to bridge the gap and help ELLs pursue academic achievement.

High school mathematics teachers in Texas must address forty-three English Language Proficiency Standards (ELPS). One such standard is ELPS 4.F.1.

**ELPS 4.F.1** Use visual and contextual support to read grade-appropriate content area text.

The *Big Ideas Math* program provides tools to meet each of the forty-three ELPS required for Geometry.

Language Proficiency

English language learners do not always become proficient in a linear fashion. Great gains one week may be offset by significant challenges later. Many ELLs develop passive language skills (listening, reading) more quickly than active language skills (speaking, writing), and understand more than they can express. Some are adept at speaking and listening, but weak in literacy skills. Still others are comfortable reading and writing but are hesitant to speak, or have difficulty understanding spoken English. A student may be an advanced listener, intermediate speaker, and beginning reader all at the same time.

The Texas English Language Proficiency Assessment System (TELPAS) rates ELLs according to four levels of language proficiency under each of the four language domains of listening, speaking, reading, and writing.

- Beginning
- Intermediate
- Advanced
- Advanced High

See pages xxxiv–xxxv for an abbreviated summary of the ELPS-TELPAS Proficiency Level Descriptors used to rate the proficiency levels of ELLs for the language domains of listening, speaking, and reading. (Note: High school mathematics programs are not required to cover ELPS for the domain of writing.)
The key ELPS support in the *Big Ideas Math* program takes the form of teaching notes entitled SUPPORTING English Language Learners. One such note appears in each section of the book, at the bottom of the appropriate page in the Teaching Edition, to support ELLs while meeting an ELPS.

Notice that student expectations are differentiated according to the language proficiency levels for the language domain of the ELPS addressed.

For example, ELPS 3.D.1 is a speaking proficiency. To meet this standard, a Beginning English speaker is expected to simply repeat the core vocabulary terms, while an Advanced English speaker is expected to read aloud the description of a line.

### Teaching Notes for Supporting English Language Learners

<table>
<thead>
<tr>
<th>ELPS addressed</th>
<th>Teaching instructions</th>
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<tbody>
<tr>
<td><strong>Beginning</strong></td>
<td>Repeat the Core Vocabulary to improve pronunciation.</td>
</tr>
<tr>
<td><strong>Intermediate</strong></td>
<td>Read aloud the description of a point, collinear points, and coplanar points.</td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td>Read aloud the description of a line.</td>
</tr>
<tr>
<td><strong>Advanced High</strong></td>
<td>Read aloud the introduction and the description of a plane.</td>
</tr>
<tr>
<td><strong>ELPS 3.D.1</strong></td>
<td>Speak using grade-level content area vocabulary in context to internalize new English words.</td>
</tr>
</tbody>
</table>

#### Three General Principles for Teaching ELLs

The Center for Applied Linguistics outlines three basic principles for improving the achievement of ELLs in a content area classroom. The three principles are to increase the following as you teach.

1. Comprehension
2. Interaction
3. Thinking and study skills

Practicing methods that promote these principles should result in success not only for your ELLs, but for all students in your classroom.
1. Comprehension
Providing information that ELLs understand is crucial for their access to both language and content. Both verbal and nonverbal support must be considered to make mathematics more accessible to the student. Visual and experiential clues are especially important to Beginning and Intermediate ELLs. Here are some examples.

- illustrations that relate mathematics to everyday life
- charts and graphs that organize information visually
- experiential activities in which students construct models

Having students read information as an introduction to a topic is not very helpful for students with limited language. It is more helpful to immerse them in an experience to help them gain familiarity with the topic and its vocabulary before expecting them to understand the details of the process. For example, students can use visuals or manipulatives as concepts are being introduced.

2. Interaction
Language is learned through communication with others. One negotiates meaning to accomplish real purposes. Students in a discussion will restate, question, explain, and clarify in order to reach a common understanding. This process helps them learn both language and mathematics content. Opportunities for interaction are provided throughout the program in the following ways.

- pair work
- group work
- classroom discussion
- hands on activities

These interactions also allow your students to build relationships that help you to provide a supportive, encouraging environment. This reduces the stress of learning a new language for your ELLs, and promotes real learning.
3. Thinking and Study Skills
Teaching academic skills explicitly helps develop language for ELLs and thinking skills for all students. Students should be given opportunities to perform higher order thinking skills, answer critical-thinking questions, and reinforce study skills. Below are several tools used to support these goals in this program.

- graphic organizers such as charts and tables that help order information
- text features that highlight key terms and concepts
- opportunities for note-taking

The Big Ideas Math program provides comprehensive support for teaching ELLs and all students. By promoting increased comprehension, interaction, and thinking and study skills as outlined above, you can help your ELLs improve their English language skills, and guide all of your students toward academic success.

Reference
ELPS-TELPAS Proficiency Levels

The Texas English Language Proficiency Assessment System (TELPAS) measures the progress of English language learners (ELLs) working toward English Language Proficiency Standards (ELPS). ELPS-TELPAS Proficiency Level Descriptors are used to classify and monitor the language proficiency levels of ELLs as **Beginning**, **Intermediate**, **Advanced**, or **Advanced High** for the language domains of listening, speaking, reading, and writing. (Note: High school mathematics programs are not required to cover ELPS for the domain of writing.) A summary of the descriptors used in the **Big Ideas Math** program follows.

### Summary of ELPS-TELPAS Proficiency Level Descriptors

<table>
<thead>
<tr>
<th></th>
<th><strong>Beginning</strong></th>
<th><strong>Intermediate</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Listening</strong></td>
<td>Have little ability to understand spoken English</td>
<td>Understand simple, high-frequency spoken English</td>
</tr>
<tr>
<td></td>
<td>• struggle to understand simple conversations even with familiar topics</td>
<td>• understand simple conversations and short discussions on familiar topics</td>
</tr>
<tr>
<td></td>
<td>• struggle to distinguish words and phrases</td>
<td>• often distinguish key phrases necessary for general meaning in basic interaction</td>
</tr>
<tr>
<td></td>
<td>• may not seek clarification; frequently remain silent</td>
<td>• able to seek clarification by requesting to repeat, slow down, or rephrase</td>
</tr>
<tr>
<td><strong>Speaking</strong></td>
<td>Have little ability to speak English</td>
<td>Speak in a simple, routine manner</td>
</tr>
<tr>
<td></td>
<td>• hesitate to speak and use only single words or short, memorized phrases to meet immediate needs</td>
<td>• use simple sentences and participate in short conversations; hesitate frequently</td>
</tr>
<tr>
<td></td>
<td>• use a limited bank of concrete vocabulary</td>
<td>• speak using basic vocabulary for everyday interactions; rarely use detail</td>
</tr>
<tr>
<td></td>
<td>• lack the grammar to speak in sentences unless dealing with highly rehearsed or highly familiar material</td>
<td>• emerging awareness of grammar, speak in simple sentences, mostly present tense</td>
</tr>
<tr>
<td></td>
<td>• exhibit errors that hinder overall communication</td>
<td>• errors inhibit communication when using complex or less familiar English</td>
</tr>
<tr>
<td></td>
<td>• pronunciation inhibits communication</td>
<td>• pronunciation usually understandable by people accustomed to ELLs</td>
</tr>
<tr>
<td><strong>Reading</strong></td>
<td>Have little ability to read English</td>
<td>Understand simple, high-frequency English</td>
</tr>
<tr>
<td></td>
<td>• read and understand very limited, familiar English; vocabulary is mostly environmental print, some very high-frequency words, and concrete, visually supported words</td>
<td>• read and understand vocabulary on a wider range of topics, including everyday language, literal meanings, routine academic terms, and very commonly used abstract language</td>
</tr>
<tr>
<td></td>
<td>• read slowly, one word at a time</td>
<td>• read slowly in short phrases; may re-read</td>
</tr>
<tr>
<td></td>
<td>• slight sense of English language structures</td>
<td>• have a growing understanding of basic, routine English language structures</td>
</tr>
<tr>
<td></td>
<td>• comprehend isolated familiar words and phrases, very familiar sentences</td>
<td>• understand simple sentences in short texts, but need visuals, prior knowledge, pretaught vocabulary, and human support</td>
</tr>
<tr>
<td></td>
<td>• depend on visuals and prior knowledge to derive meaning from English text</td>
<td>• struggle to read independently</td>
</tr>
<tr>
<td></td>
<td>• can apply reading comprehension skills only with English text written for this level</td>
<td>• can apply basic comprehension skills when reading linguistically accommodated texts</td>
</tr>
</tbody>
</table>

Complete ELPS-TELPAS Proficiency Level Descriptors are at [www.tea.state.tx.us/student.assessment/ell/telpas](http://www.tea.state.tx.us/student.assessment/ell/telpas).
<table>
<thead>
<tr>
<th><strong>Advanced</strong></th>
<th><strong>Advanced High</strong></th>
</tr>
</thead>
</table>
| **Listening** | Understand grade-appropriate spoken English with support  
  • usually understand longer conversations on familiar and some unfamiliar topics, but sometimes need support  
  • understand most main points, key details, and some implicit information of basic oral language not modified for ELLs  
  • occasionally need the speaker to repeat, slow down, or rephrase | Understand grade-appropriate spoken English with minimal support  
  • understand long, elaborate conversations on familiar and unfamiliar topics with little dependence on support; some exceptions when complex language is used  
  • understand main points, important details, and implicit information at a level nearly comparable to native English speakers  
  • rarely need the speaker to repeat, slow down, or rephrase |
| **Speaking** | Use grade-appropriate English with support  
  • comfortable in most conversations on familiar topics, with some pauses needed  
  • can use content-based terms and common abstract vocabulary to discuss familiar topics; can speak on familiar topics in some detail  
  • grasp basic grammar features, including an ability to use present, past, and future tenses; emerging ability to use complex language structures  
  • errors hinder communication somewhat when using complex grammar, long sentences, and less familiar expressions  
  • pronunciation is usually understandable by people unaccustomed to ELLs | Speak using grade-appropriate English, with minimal support  
  • participate in extended discussions on a variety of topics with only occasional pauses  
  • effective using abstract and content-based vocabulary when low-frequency vocabulary is needed, with some exceptions; comfortable with many native idioms and colloquialisms  
  • use grammar structures and complex sentences to narrate and describe nearly as well as native English speakers  
  • make few errors that hinder communication  
  • mispronounce some words, but rarely interferes with communication |
| **Reading** | Read and understand grade-appropriate English with support  
  • read and understand grade-appropriate vocabulary, such as concrete and abstract terms, phrases beyond literal meaning, and multiple meanings of common words  
  • read longer phrases and simple sentences in familiar material at an appropriate speed  
  • starting to comprehend grade-appropriate text through familiarity with English or by applying reading comprehension skills, with some support needed, especially when material is unfamiliar | Read and understand grade-appropriate English with minimal support  
  • read and understand at a level nearly comparable to native speakers, with some exceptions for specialized vocabulary  
  • read familiar, grade-appropriate text with appropriate rate, intonation, and expression  
  • can comprehend grade-appropriate text by applying reading comprehension skills with minimal support, or from familiarity with English |
# Correlation from Geometry to TEKS

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<tr>
<th>Lesson</th>
<th>TEKS</th>
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<tr>
<td>1.3 Using Midpoint Formulas</td>
<td>G.2.A, G.2.B</td>
</tr>
<tr>
<td>1.4 Perimeter and Area in the Coordinate Plane</td>
<td>Preparing for G.11.B</td>
</tr>
<tr>
<td>1.5 Measuring and Constructing Angles</td>
<td>G.5.B, Preparing for G.5.C</td>
</tr>
<tr>
<td>1.6 Describing Pairs of Angles</td>
<td>Preparing for G.6.A</td>
</tr>
<tr>
<td><strong>Chapter 2: Reasoning and Proofs</strong></td>
<td></td>
</tr>
<tr>
<td>2.1 Conditional Statements</td>
<td>G.4.B</td>
</tr>
<tr>
<td>2.6 Proving Geometric Relationships</td>
<td>G.6.A</td>
</tr>
<tr>
<td><strong>Chapter 3: Parallel and Perpendicular Lines</strong></td>
<td></td>
</tr>
<tr>
<td>3.1 Pairs of Lines and Angles</td>
<td>Preparing for G.5.A</td>
</tr>
<tr>
<td>3.2 Parallel Lines and Transversals</td>
<td>G.5.A, G.6.A</td>
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<td>3.5 Slopes of Lines</td>
<td>G.2.A, G.2.B</td>
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<tr>
<td>3.6 Equations of Parallel and Perpendicular Lines</td>
<td>G.2.B, G.2.C</td>
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<tr>
<td><strong>Chapter 4: Transformations</strong></td>
<td></td>
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<tr>
<td>4.5 Dilations</td>
<td>G.3.A, G.3.C</td>
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<td>Lesson</td>
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<td>-------</td>
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</tr>
<tr>
<td><strong>Chapter 5: Congruent Triangles</strong></td>
<td></td>
</tr>
<tr>
<td>5.1 Angles of Triangles</td>
<td>G.2.B, G.6.D</td>
</tr>
<tr>
<td>5.4 Equilateral and Isosceles Triangles</td>
<td>G.5.C, G.6.D</td>
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<tr>
<td>5.5 Proving Triangle Congruence by SSS</td>
<td>G.5.A, G.6.B</td>
</tr>
<tr>
<td>5.6 Proving Triangle Congruence by ASA and AAS</td>
<td>G.5.A, G.6.B</td>
</tr>
<tr>
<td>5.7 Using Congruent Triangles</td>
<td>G.6.B</td>
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<tr>
<td>5.8 Coordinate Proofs</td>
<td>G.2.B, G.6.D</td>
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</tbody>
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| **Chapter 6: Relationships Within Triangles** |              |
| 6.5 Indirect Proof and Inequalities in One Triangle | G.5.D |
| 6.6 Inequalities in Two Triangles | G.6.D |

| **Chapter 7: Quadrilaterals and Other Polygons** |              |
| 7.1 Angles of Polygons | G.5.A |
| 7.2 Properties of Parallelograms | G.5.A |
| 7.3 Proving that a Quadrilateral is a Parallelogram | G.2.B, G.5.C, G.6.E |

| **Chapter 8: Similarity** |              |
| 8.4 Proportionality Theorems | G.8.A |
Chapter 4 Pacing Guide

Section 1  2 Days
Section 2  2 Days
Section 3  2 Days
Quiz      1 Day
Section 4  2 Days
Section 5  2 Days
Section 6  1 Day
Chapter Review/Chapter Tests 1 Day
Total Chapter 4 13 Days
Year-to-Date 50 Days

Texas Essential Knowledge and Skills Summary

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<tr>
<th>Section</th>
<th>TEKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5</td>
<td>G.3.A, G.3.C</td>
</tr>
</tbody>
</table>

Mathematical Thinking: Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
Maintaining Mathematical Proficiency

Identifying Transformations (8.10.A)

Example 1 Tell whether the red figure is a translation, reflection, rotation, or dilation of the blue figure.

a. The blue figure turns to form the red figure, so it is a rotation.

b. The red figure is a mirror image of the blue figure, so it is a reflection.

Tell whether the red figure is a translation, reflection, rotation, or dilation of the blue figure.

1. 2. 3. 4.

Identifying Similar Figures (8.3.A)

Example 2 Which rectangle is similar to Rectangle A?

Rectangle A

Rectangle B

Rectangle C

Each figure is a rectangle, so corresponding angles are congruent.

Check to see whether corresponding side lengths are proportional.

<table>
<thead>
<tr>
<th>Rectangle A and Rectangle B</th>
<th>Rectangle A and Rectangle C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of A = 8 / 4</td>
<td>Length of A = 8 / 4</td>
</tr>
<tr>
<td>Width of A = 4 / 1</td>
<td>Width of A = 4 / 1</td>
</tr>
<tr>
<td>not proportional</td>
<td>not proportional</td>
</tr>
<tr>
<td>Length of B = 4 / 2</td>
<td>Length of C = 6 / 3</td>
</tr>
<tr>
<td>Width of B = 4 / 1</td>
<td>Width of C = 3 / 1</td>
</tr>
</tbody>
</table>

So, Rectangle C is similar to Rectangle A.

Tell whether the two figures are similar. Explain your reasoning.

5. 6. 7.

8. ABSTRACT REASONING Can you draw two squares that are not similar? Explain your reasoning.

Vocabulary Review

Have students make an Idea and Examples Chart that includes the following types of transformations.

- Translation
- Reflection
- Rotation
- Dilation

Texas Essential Knowledge and Skills

8.10.A The student is expected to generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane.

8.3.A The student is expected to generalize that the ratio of corresponding sides of similar shapes are proportional, including a shape and its dilation.

ANSWERS

1. reflection
2. rotation
3. dilation
4. translation
5. no; \( \frac{12}{14} \neq \frac{5}{7} \), The sides are not proportional.
6. yes; The corresponding angles are congruent and the corresponding side lengths are proportional.
7. yes; The corresponding angles are congruent and the corresponding side lengths are proportional.
8. no; Squares have four right angles, so the corresponding angles are always congruent. Because all four sides are congruent, the corresponding sides will always be proportional.
Mathematical Thinking

Using Dynamic Geometry Software

Core Concept

Using Dynamic Geometry Software

Dynamic geometry software allows you to create geometric drawings, including:

- drawing a point
- drawing a line
- drawing a line segment
- drawing an angle
- measuring an angle
- measuring a line segment
- drawing a circle
- drawing an ellipse
- drawing a perpendicular line
- drawing a polygon
- copying and sliding an object
- reflecting an object in a line

EXAMPLE 1 Finding Side Lengths and Angle Measures

Use dynamic geometry software to draw a triangle with vertices at \(A(-2, -1), B(2, 1),\) and \(C(2, -2)\). Find the side lengths and angle measures of the triangle.

SOLUTION

Using dynamic geometry software, you can create \(\triangle ABC\), as shown.

Sample

Points
\(A(-2, 1)\)
\(B(2, 1)\)
\(C(2, -2)\)

Segments
\(AB = 4\)
\(BC = 3\)
\(AC = 5\)

Angles
\(m\angle A = 36.87^\circ\)
\(m\angle B = 90^\circ\)
\(m\angle C = 53.13^\circ\)

From the display, the side lengths are \(AB \approx 4\) units, \(BC \approx 3\) units, and \(AC \approx 5\) units. The angle measures, rounded to two decimal places, are \(m\angle A \approx 36.87^\circ\), \(m\angle B = 90^\circ\), and \(m\angle C \approx 53.13^\circ\).

Monitoring Progress

Use dynamic geometry software to draw the polygon with the given vertices. Use the software to find the side lengths and angle measures of the polygon. Round your answers to the nearest hundredth.

1. \(A(0, 2), B(3, -1), C(4, 3)\)
2. \(A(-2, 1), B(-2, -1), C(3, 2)\)
3. \(A(1, 1), B(-3, 1), C(-3, -2), D(1, -2)\)
4. \(A(1, 1), B(-3, 1), C(-2, -2), D(2, -2)\)
5. \(A(-3, 0), B(0, 3), C(3, 0), D(0, -3)\)
6. \(A(0, 0), B(4, 0), C(1, 1), D(0, 3)\)

If students need help...

Skills Review Handbook
- Maintaining Mathematical Proficiency
Lesson Tutorials
- Start the next Section
Game Closet at BigIdeasMath.com

If students got it...

Student Journal
- Game Closet at BigIdeasMath.com
Lesson Tutorials
- Start the next Section
Skills Review Handbook
4.1 Translations

**Essential Question** How can you translate a figure in a coordinate plane?

**EXPLORATION 1** Translating a Triangle in a Coordinate Plane

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

b. Copy the triangle and translate (or slide) it to form a new figure, called an image, \( \triangle A'B'C' \) (read as “triangle A prime, B prime, C prime”).

c. What is the relationship between the coordinates of the vertices of \( \triangle ABC \) and those of \( \triangle A'B'C' \)?

d. What do you observe about the side lengths and angle measures of the two triangles?

**EXPLORATION 2** Translating a Triangle in a Coordinate Plane

Work with a partner.

a. The point \((x, y)\) is translated \(a\) units horizontally and \(b\) units vertically. Write a rule to determine the coordinates of the image of \((x, y)\).

\[
(x, y) \rightarrow (x+a, y+b)
\]

b. Use the rule you wrote in part (a) to translate \( \triangle ABC \) 4 units left and 3 units down.

What are the coordinates of the vertices of the image, \( \triangle A'B'C' \)?

c. Draw \( \triangle A'B'C' \). Are its side lengths the same as those of \( \triangle ABC \)? Justify your answer.

**EXPLORATION 3** Comparing Angles of Translations

Work with a partner.

a. In Exploration 2, is \( \triangle ABC \) a right triangle? Justify your answer.

b. In Exploration 2, is \( \triangle A'B'C' \) a right triangle? Justify your answer.

c. Do you think translations always preserve angle measures? Explain your reasoning.

**Communicate Your Answer**

4. How can you translate a figure in a coordinate plane?

5. In Exploration 2, translate \( \triangle A'B'C' \) 3 units right and 4 units up. What are the coordinates related to the coordinates of the vertices of the original triangle, \( \triangle ABC \)?

**ANSWERS**

1. a. Check students’ work.

b. Check students’ work.

c. The x-values of each of the three vertices in the image can be attained by adding the same amount (positive or negative) to the corresponding x-values of the vertices in the original figure. The same is true for the y-values.

d. The side lengths and angle measures of the original figure are equal to the corresponding side lengths and angle measures of the image.

2. a. \(x + a; y + b\)

b. \(A'(-4, 0), B'(0, 2), C'(-1, -6)\)

c. yes; Use the Distance Formula to find the lengths.

3. a. yes; \((AB)^2 + (AC)^2 = (BC)^2\)

b. yes; The side lengths of the image are the same as the original figure.

3c–5. See Additional Answers.
In the diagram, name the vector and write in component form.

\[ \overrightarrow{PQ}; (-4, 5) \]

In the diagram, name the vector and write its component form.

**SOLUTION**

The vector is \( \overrightarrow{JK} \). To move from the initial point \( J \) to the terminal point \( K \), you move 3 units right and 4 units up. So, the component form is \( \langle 3, 4 \rangle \).

A transformation is a function that moves or changes a figure in some way to produce a new figure called an image. Another name for the original figure is the preimage. The points on the preimage are the inputs for the transformation, and the points on the image are the outputs.

**Core Concept**

**Translations**

A translation moves every point of a figure the same distance in the same direction. More specifically, a translation maps, or moves, the points \( P \) and \( Q \) of a plane figure along a vector \( \langle a, b \rangle \) to the points \( P' \) and \( Q' \), so that one of the following statements is true.

- \( PP' = QQ' \) and \( PP' \parallel QQ' \), or
- \( PP' = QQ' \) and \( PP' \) and \( QQ' \) are collinear.

Translations map lines to parallel lines and segments to parallel segments. For instance, in the figure above, \( \overrightarrow{PQ} \parallel \overrightarrow{P'O'} \).
**Example 2** Translating a Figure Using a Vector

The vertices of \( \triangle ABC \) are \( A(0, 3), B(2, 4), \) and \( C(1, 0) \). Translate \( \triangle ABC \) using the vector \( (5, -1) \).

**Solution**

First, graph \( \triangle ABC \). Use \( (5, -1) \) to move each vertex 5 units right and 1 unit down. Label the image vertices. Draw \( \triangle A'B'C' \). Notice that the vectors drawn from preimage vertices to image vertices are parallel.

You can also express translation along the vector \((a, b)\) using a rule, which has the notation \((x, y) \rightarrow (x + a, y + b)\).

**Example 3** Writing a Translation Rule

Write a rule for the translation of \( \triangle ABC \) to \( \triangle A'B'C' \).

**Solution**

To go from \( A \) to \( A' \), you move 4 units left and 1 unit up, so you move along the vector \((-4, 1)\).

So, a rule for the translation is \((x, y) \rightarrow (x - 4, y + 1)\).

**Example 4** Translating a Figure in the Coordinate Plane

Graph quadrilateral \( ABCD \) with vertices \( A(-1, 2), B(-1, 5), C(4, 6), \) and \( D(4, 2) \) and its image after the translation \((x, y) \rightarrow (x + 3, y - 1)\).

**Solution**

Graph quadrilateral \( ABCD \). To find the coordinates of the vertices of the image, add 3 to the \( x \)-coordinates and subtract 1 from the \( y \)-coordinates of the vertices of the preimage. Then graph the image, as shown at the left.

\[
(x, y) \rightarrow (x + 3, y - 1)
\]

\[
A(-1, 2) \rightarrow A'(2, 1)
\]

\[
B(-1, 5) \rightarrow B'(2, 4)
\]

\[
C(4, 6) \rightarrow C'(7, 5)
\]

\[
D(4, 2) \rightarrow D'(7, 1)
\]

**Monitoring Progress** Help in English and Spanish at BigIdeasMath.com

1. Name the vector and write its component form.
2. The vertices of \( \triangle LMN \) are \( L(2, 2), M(5, 3), \) and \( N(9, 1) \). Translate \( \triangle LMN \) using the vector \((-2, 6)\).
3. In Example 3, write a rule to translate \( \triangle A'B'C' \) back to \( \triangle ABC \).
4. Graph \( \triangle RST \) with vertices \( R(2, 2), S(5, 2), \) and \( T(3, 5) \) and its image after the translation \((x, y) \rightarrow (x + 1, y + 2)\).

Section 4.1 Translations 179

**English Language Learners**

**Illustrate**

Create a sheet with two columns. In the first column, show the graphs of four translations. In the second column, show the rules for the four translations—out of order. Have students match each graph with its rule. Ask students to explain their choices.
Performing Compositions

A rigid motion is a transformation that preserves length and angle measure. Another name for a rigid motion is an isometry. A rigid motion maps lines to lines, rays to rays, and segments to segments.

**Postulate 4.1 Translation Postulate**
A translation is a rigid motion.

Because a translation is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the translation shown.

- \( DE = D'E' \), \( EF = E'F' \), \( FD = F'D' \)
- \( m \angle D = m \angle D' \), \( m \angle E = m \angle E' \), \( m \angle F = m \angle F' \)

When two or more transformations are combined to form a single transformation, the result is a composition of transformations.

**Theorem 4.1 Composition Theorem**
The composition of two (or more) rigid motions is a rigid motion.

**Proof** Ex. 35, p. 184

The theorem above is important because it states that no matter how many rigid motions you perform, lengths and angle measures will be preserved in the final image. For instance, the composition of two or more translations is a translation, as shown.

**Example 5** Performing a Composition

Graph \( \overline{RS} \) with endpoints \( R(-8, 5) \) and \( S(-6, 8) \) and its image after the composition.

**Translation:** \( (x, y) \rightarrow (x - 1, y + 4) \)

**Translation:** \( (x, y) \rightarrow (x + 4, y - 6) \)
Solving Real-Life Problems

**EXAMPLE 6** Modeling with Mathematics

You are designing a favicon for a golf website. In an image-editing program, you move the red rectangle 2 units left and 3 units down. Then you move the red rectangle 1 unit right and 1 unit up. Rewrite the composition as a single translation.

**SOLUTION**

1. **Understand the Problem** You are given two translations. You need to rewrite the result of the composition of the two translations as a single translation.

2. **Make a Plan** You can choose an arbitrary point \((x, y)\) in the red rectangle and determine the horizontal and vertical shift in the coordinates of the point after both translations. This tells you how much you need to shift each coordinate to map the original figure to the final image.

3. **Solve the Problem** Let \(A(x, y)\) be an arbitrary point in the red rectangle. After the first translation, the coordinates of its image are 
   \[ A'(x - 2, y - 3). \]
   The second translation maps \(A'(x - 2, y - 3)\) to 
   \[ A''(x - 2 + 1, y - 3 + 1) = A''(x - 1, y - 2). \]
   The composition of translations uses the original point \((x, y)\) as the input and returns the point \((x - 1, y - 2)\) as the output.
   - So, the single translation rule for the composition is \((x, y) \rightarrow (x - 1, y - 2)\).

4. **Look Back** Check that the rule is correct by testing a point. For instance, \((10, 12)\) is a point in the red rectangle. Apply the two translations to \((10, 12)\).

   \[
   (10, 12) \rightarrow (8, 9) \rightarrow (9, 10)
   
   \[
   (10, 12) \rightarrow (9, 10)
   
   \]

   Does the final result match the rule you found in Step 3?

   \[
   (10, 12) \rightarrow (10 - 1, 12 - 2) = (9, 10)
   
   \]

**MONITORING PROGRESS**

5. **Graph** \(TU\) with endpoints \(T(1, 2)\) and \(U(4, 6)\) and its image after the composition.
   - **Translation:** \((x, y) \rightarrow (x - 2, y - 3)\)
   - **Translation:** \((x, y) \rightarrow (x - 4, y + 5)\)

6. **Graph** \(VW\) with endpoints \(V(-6, -4)\) and \(W(-3, 1)\) and its image after the composition.
   - **Translation:** \((x, y) \rightarrow (x + 3, y + 1)\)
   - **Translation:** \((x, y) \rightarrow (x - 6, y - 4)\)

7. **In Example 6,** you move the gray square 2 units right and 3 units up. Then you move the gray square 1 unit left and 1 unit down. Rewrite the composition as a single transformation.

\[(x, y) \rightarrow (x + 2, y + 3)\]

**Extra Example 6**

Another graphic artist is designing an alternate icon for the one in the graph in Example 6. She moves the red rectangle 3 units right and 1 unit down. Then she moves the red rectangle 1 unit left and 4 units up. Rewrite the composition as a single transformation.

\[(x, y) \rightarrow (x + 2, y + 3)\]

**MONITORING PROGRESS**

5. **Graph** \(TU\) with endpoints \(T(1, 2)\) and \(U(4, 6)\) and its image after the composition.

6. **Graph** \(VW\) with endpoints \(V(-6, -4)\) and \(W(-3, 1)\) and its image after the composition.

7. **In Example 6,** you move the gray square 2 units right and 3 units up. Then you move the gray square 1 unit left and 1 unit down. Rewrite the composition as a single transformation.

\[(x, y) \rightarrow (x + 1, y + 2)\]

**Closure**

- Draw a triangle in Quadrant I. Have students identify a translation that would map the triangle entirely to Quadrant III. Write the translation using coordinate notation and vector notation.
1. **VOCABULARY** Name the preimage and image of the transformation \( \triangle ABC \to \triangle A'B'C' \).

2. **COMPLETE THE SENTENCE** A ______ moves every point of a figure the same distance in the same direction.

### Monitoring Progress and Modeling with Mathematics

**In Exercises 3 and 4, name the vector and write its component form. (See Example 1.)**

3. 
![Vector Diagram](image1)

4. 
![Vector Diagram](image2)

**In Exercises 5–8, the vertices of \( \triangle DEF \) are \( D(2, 5), E(6, 3), \) and \( F(4, 0) \). Translate \( \triangle DEF \) using the given vector. Graph \( \triangle DEF \) and its image. (See Example 2.)**

5. \((6, 0)\)

6. \((-5, -1)\)

7. \((-3, -7)\)

8. \((-2, -4)\)

**In Exercises 9 and 10, find the component form of the vector that translates \( P(-3, 6) \) to \( P' \).**

9. \(P'(0, 1)\)

10. \(P'(-4, 8)\)

**In Exercises 11 and 12, write a rule for the translation of \( \triangle LMN \) to \( \triangle L'M'N' \). (See Example 3.)**

11. 
![Translation Diagram](image3)

12. 
![Translation Diagram](image4)

**In Exercises 13–16, use the translation.**

13. \((x, y) \to (x - 8, y + 4)\)

14. \((x, y) \to (x - 1, y - 5)\)

15. \((x, y) \to (x + 3, y + 1)\)

16. \((x, y) \to (x + 3, y + 1)\)

**In Exercises 17–20, graph \( \triangle PQR \) with vertices \( P(-2, 3), Q(1, 2), \) and \( R(3, -1) \) and its image after the translation. (See Example 4.)**

17. \((x, y) \to (x + 4, y + 6)\)

18. \((x, y) \to (x + 9, y - 2)\)

19. \((x, y) \to (x - 2, y - 5)\)

20. \((x, y) \to (x - 1, y + 3)\)

**In Exercises 21 and 22, graph \( \triangle XYZ \) with vertices \( X(2, 4), Y(6, 0), \) and \( Z(7, 2) \) and its image after the composition. (See Example 5.)**

21. **Translation:** \((x, y) \to (x + 2, y + 4)\)

   **Translation:** \((x, y) \to (x - 5, y - 9)\)

22. **Translation:** \((x, y) \to (x - 6, y)\)

   **Translation:** \((x, y) \to (x + 2, y + 7)\)

18–22. See Additional Answers.
In Exercises 23 and 24, describe the composition of translations.

23. \[ A \rightarrow A' \rightarrow B \rightarrow C \rightarrow D \]

24. \[ D \rightarrow E \rightarrow F \rightarrow G \rightarrow H \]

27. **PROBLEM SOLVING** You are studying an amoeba through a microscope. Suppose the amoeba moves on a grid-indexed microscope slide in a straight line from square B3 to square G7.

\[ \text{Translation: } (x, y) \rightarrow (x - 1, y - 2). \]

a. Describe the translation.
b. The side length of each grid square is 2 millimeters. How far does the amoeba travel?
c. The amoeba moves from square B3 to square G7 in 24.5 seconds. What is its speed in millimeters per second?

28. **MATHEMATICAL CONNECTIONS** Translation A maps \((x, y)\) to \((x + n, y + t)\). Translation B maps \((x, y)\) to \((x + s, y + m)\).

a. Translate a point using Translation A, followed by Translation B. Write an algebraic rule for the final image of the point after this composition.
b. Translate a point using Translation B, followed by Translation A. Write an algebraic rule for the final image of the point after this composition.
c. Compare the rules you wrote for parts (a) and (b). Does it matter which translation you do first? Explain your reasoning.

29. a. The amoeba moves right 5 and down 4.
   b. about 12.8 mm
   c. about 0.52 mm/sec

28. a. \((x, y) \rightarrow (x + n + s, y + t + m)\)
   b. \((x, y) \rightarrow (x + s + n, y + m + t)\)
   c. no; Each image will end up in the same place.

29. \[ r = 100, s = 8, t = 5, w = 54 \]
30. \[ a = 35, b = 14, c = 5 \]
ANSWERS
31. \( E’(-3, -4), F’(-2, -5), G’(0, -1) \)
32. figures 5 and 7; To go from figure 5 to figure 7, you move 4 units right and 8 units up.
33. \((x, y) \rightarrow (x - m, y - n); \) You must go back the same number of units in the opposite direction.
34–50. See Additional Answers.

31. USING STRUCTURE Quadrilateral \( \text{DEFG} \) has vertices \( D(-1, 2), E(-2, 0), F(-1, -1), \) and \( G(1, 3) \). A translation maps quadrilateral \( \text{DEFG} \) to quadrilateral \( D’E’F’G’ \). The image of \( D \) is \( D’(-2, -2) \). What are the coordinates of \( E’, F’, \) and \( G’ \)?

32. HOW DO YOU SEE IT? Which two figures represent a translation? Describe the translation.

33. REASONING The translation \((x, y) \rightarrow (x + m, y + n)\) maps \( P \) to \( P’ \). Write a rule for the translation of \( PQ \) to \( P’Q’ \). Explain your reasoning.

34. DRAWING CONCLUSIONS The vertices of a rectangle are \( Q(-2, -3), R(2, -4), S(5, 4), \) and \( T(5, -3) \).
   a. Translate rectangle \( QRST \) 3 units left and 3 units down to produce rectangle \( Q’R’S’T’ \). Find the area of rectangle \( QRST \) and the area of rectangle \( Q’R’S’T’ \).
   b. Compare the areas. Make a conjecture about the areas of a preimage and its image after a translation.

35. PROVING A THEOREM Prove the Composition Theorem (Theorem 4.1).
36. PROVING A THEOREM Use properties of translations to prove each theorem.
   a. Corresponding Angles Theorem (Theorem 3.1)
   b. Corresponding Angles Converse (Theorem 3.5)
37. WRITING Explain how to use translations to draw a rectangular prism.
38. MATHEMATICAL CONNECTIONS The vector \( \overrightarrow{PQ} = (4, 1) \) describes the translation of \( A(-1, w) \) onto \( A’(2x + 1, 4) \) and \( B(8y - 1, 1) \) onto \( B’(3, 3c) \). Find the values of \( w, x, y, \) and \( z \).
39. MAKING AN ARGUMENT A translation maps \( GH \) to \( G’H’ \). Your friend claims that if you draw segments connecting \( G \) to \( G’ \) and \( H \) to \( H’ \), then the resulting quadrilateral is a parallelogram. Is your friend correct? Explain your reasoning.

40. THOUGHT PROVOKING You are a graphic designer for a company that manufactures floor tiles. Design a floor tile in a coordinate plane. Then use translations to show how the tiles cover an entire floor. Describe the translations that map the original tile to four other tiles.

41. REASONING The vertices of \( \Delta ABC \) are \( A(2, 2), B(4, 2), \) and \( C(3, 4) \). Graph the image of \( \Delta ABC \) after the transformation \((x, y) \rightarrow (x + y, y)\). Is this transformation a translation? Explain your reasoning.
42. PROOF \( MN \) is perpendicular to line \( \ell \). \( M’N’ \) is the translation of \( MN \) 2 units to the left. Prove that \( M’N’ \) is perpendicular to \( \ell \).

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons.

43. Tell whether the figure can be folded in half so that one side matches the other.
   (Skills Review Handbook)
44. Simplify the expression. (Skills Review Handbook)
45. \(-(-x)\)
46. \(-(x + 3)\)
47. \(-(-x)\)
48. \(-(x + 3)\)
49. \(x - (12 - 5x)\)
50. \(x - (-2x + 4)\)

If students need help...

Resources by Chapter
• Practice A and Practice B
• Puzzle Time

If students got it...

Resources by Chapter
• Enrichment and Extension
• Cumulative Review

Student Journal
• Practice

Differentiating the Lesson
Skills Review Handbook

Start the next Section
4.2 Reflections

**Essential Question**
How can you reflect a figure in a coordinate plane?

**EXPLORATION 1** Reflecting a Triangle Using a Reflective Device

*Work with a partner.* Use a straightedge to draw any triangle on paper. Label it \( \triangle ABC \).

a. Use the straightedge to draw a line that does not pass through the triangle. Label it \( m \).
b. Place a reflective device on line \( m \).
c. Use the reflective device to plot the images of the vertices of \( \triangle ABC \). Label the images of vertices \( A \), \( B \), and \( C \) as \( A' \), \( B' \), and \( C' \), respectively.
d. Use a straightedge to draw \( \triangle A'B'C' \) by connecting the vertices.

**EXPLORATION 2** Reflecting a Triangle in a Coordinate Plane

*Work with a partner.* Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

a. Reflect \( \triangle ABC \) in the \( y \)-axis to form \( \triangle A'B'C' \).
b. What is the relationship between the coordinates of the vertices of \( \triangle ABC \) and those of \( \triangle A'B'C' \)?
c. What do you observe about the side lengths and angle measures of the two triangles?
d. Reflect \( \triangle ABC \) in the \( x \)-axis to form \( \triangle A''B''C'' \). Then repeat parts (b) and (c).

**Communicate Your Answer**

3. How can you reflect a figure in a coordinate plane?

**Texas Essential Knowledge and Skills**

G.3.A The student is expected to describe and perform transformations of figures in a plane using coordinate notation.

G.3.B The student is expected to determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

G.3.C The student is expected to identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

G.3.D The student is expected to identify and distinguish between reflectional and rotational symmetry in a plane figure.

**ANSWERS**

1. **a.** Check students’ work.
2. **b.** Check students’ work.
3. **c.** *Sample answer:*

4. **d.** *Sample answer:*

2–3. See Additional Answers.

**Supporting English Language Learners**

Present the Essential Question. Point out that the visuals in the explorations help you understand what to do. Have students work in pairs of mixed language abilities to complete the explorations and Communicate Your Answer. Explain that the activities on this page help them prepare for what they will learn in Section 4.2.

**Beginning/Advanced** Beginning students complete the actions required as Advanced students explain them. Advanced students state the answers.

**Intermediate/Advanced High** Intermediate students complete the actions required and provide any specific answers. Advanced High students provide answers that require elaboration.

**ELPS 4.D** Use prereading supports such as graphic organizers, illustrations, and pretaught topic-related vocabulary and other prereading activities to enhance comprehension of written text.
4.2 Lesson

What You Will Learn

- Perform reflections.
- Perform glide reflections.
- Identify lines of symmetry.
- Solve real-life problems involving reflections.

Core Vocabulary

- Reflection, p. 186
- Line of reflection, p. 186
- Glide reflection, p. 188
- Line symmetry, p. 189
- Line of symmetry, p. 189

Performing Reflections

Core Concept

Reflections

A reflection is a transformation that uses a line like a mirror to reflect a figure. The mirror line is called the line of reflection.

A reflection in a line \( m \) maps every point \( P \) in the plane to a point \( P' \), so that for each point one of the following properties is true.

- If \( P \) is not on \( m \), then \( m \) is the perpendicular bisector of \( PP' \), or
- If \( P \) is on \( m \), then \( P = P' \).

Example 1 Reflecting in Horizontal and Vertical Lines

Graph \( \triangle ABC \) with vertices \( A(1, 3) \), \( B(5, 2) \), and \( C(2, 1) \) and its image after the reflection described.

a. In the line \( n: x = -1 \)

b. In the line \( m: y = 3 \)

Monitoring Progress

Answers

1. See Additional Answers.

2. See Additional Answers.

3. 4. See Additional Answers.

Extra Example 1

Graph \( \triangle ABC \) with vertices \( A(1, 3) \), \( B(5, 2) \), and \( C(2, 1) \) and its image after the reflection described.

a. In the line \( n: x = -1 \)

b. In the line \( m: y = 3 \)

SOLUTION

a. Point \( A \) is 2 units left of line \( n \), so its reflection \( A' \) is 2 units right of line \( n \) at \((5, 3)\). Also, \( B' \) is 2 units left of line \( n \) at \((1, 2)\), and \( C' \) is 1 unit right of line \( n \) at \((4, 1)\).

b. Point \( A \) is 2 units above line \( m \), so \( A' \) is 2 units below line \( m \) at \((1, -1)\). Also, \( B' \) is 1 unit below line \( m \) at \((5, 0)\). Because point \( C \) is on line \( m \), you know that \( C = C' \).
**EXAMPLE 2** Reflecting in the Line \(y = x\)

Graph \(FG\) with endpoints \(F(-1, 2)\) and \(G(1, 2)\) and its image after a reflection in the line \(y = x\).

**SOLUTION**

The slope of \(y = x\) is 1. The segment from \(F\) to its image, \(FF'\), is perpendicular to the line of reflection \(y = x\), so the slope of \(FF'\) will be \(-1\) (because \(1(-1) = -1\)). From \(F\), move 1.5 units right and 1.5 units down to \(F'(2, -1)\).

The slope of \(GG'\) will also be \(-1\). From \(G\), move 0.5 unit right and 0.5 unit down to \(y = x\). Then move 0.5 unit right and 0.5 unit down to locate \(G'(2, 1)\).

You can use coordinate rules to find the images of points reflected in four special lines.

**Core Concept**

**Coordinate Rules for Reflections**

- If \((a, b)\) is reflected in the \(x\)-axis, then its image is the point \((a, -b)\).
- If \((a, b)\) is reflected in the \(y\)-axis, then its image is the point \((-a, b)\).
- If \((a, b)\) is reflected in the line \(y = x\), then its image is the point \((b, a)\).
- If \((a, b)\) is reflected in the line \(y = -x\), then its image is the point \((-b, -a)\).

**EXAMPLE 3** Reflecting in the Line \(y = -x\)

Graph \(FG\) from Example 2 and its image after a reflection in the line \(y = -x\).

**SOLUTION**

Use the coordinate rule for reflecting in the line \(y = -x\) to find the coordinates of the endpoints of the image. Then graph \(FG\) and its image.

\[
\begin{align*}
(a, b) &\rightarrow (-b, -a) \\
F(-1, 2) &\rightarrow F'(-2, 1) \\
G(1, 2) &\rightarrow G'(2, -1)
\end{align*}
\]

**Monitoring Progress**

Help in English and Spanish at BigIdeasMath.com

The vertices of \(\triangle JKL\) are \(J(1, 3), K(4, 4),\) and \(L(3, 1)\).

5. Graph \(\triangle JKL\) and its image after a reflection in the \(x\)-axis.
6. Graph \(\triangle JKL\) and its image after a reflection in the \(y\)-axis.
7. Graph \(\triangle JKL\) and its image after a reflection in the line \(y = x\).
8. Graph \(\triangle JKL\) and its image after a reflection in the line \(y = -x\).
9. In Example 3, verify that \(FF'\) is perpendicular to \(y = -x\).

9. \(m_{FF'} = \frac{1 - 2}{2 - (-1)} = 1;\) The slope of \(y = -x\) is \(-1;\) Because \(1(-1) = -1\), the lines are perpendicular by Slopes of Perpendicular Lines (Thm. 3.12).
Extra Example 4
Graph \( \triangle ABC \) with vertices \( A(3, 2) \), \( B(6, 3) \), and \( C(7, 1) \) and its image after the glide reflection.

**Translation:** \((x, y) \rightarrow (x, y - 6)\)

**Reflection:** in the \(-y\)-axis

**MONITORING PROGRESS**

**ANSWERS**

10. 

11. translation: \((x, y) \rightarrow (x + 12, y)\), reflection: in the \(x\)-axis

**Performing Glide Reflections**

**Postulate**

**Postulate 4.2 Reflection Postulate**

A reflection is a rigid motion.

Because a reflection is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the reflection shown.

- \( DE = D'E' \), \( EF = E'F' \), \( FD = F'D' \)
- \( m\angle D = m\angle D' \), \( m\angle E = m\angle E' \), \( m\angle F = m\angle F' \)

Because a reflection is a rigid motion, the Composition Theorem (Theorem 4.1) guarantees that any composition of reflections and translations is a rigid motion.

**A glide reflection** is a transformation involving a translation followed by a reflection in which every point \( P \) is mapped to a point \( P' \) by the following steps.

**Step 1** First, a translation maps \( P \) to \( P' \).

**Step 2** Then, a reflection in a line \( k \) parallel to the direction of the translation maps \( P' \) to \( P'' \).

**EXAMPLE 4** Performing a Glide Reflection

Graph \( \triangle ABC \) with vertices \( A(3, 2) \), \( B(6, 3) \), and \( C(7, 1) \) and its image after the glide reflection.

**Translation:** \((x, y) \rightarrow (x - 12, y)\)

**Reflection:** in the \(-x\)-axis

**SOLUTION**

Begin by graphing \( \triangle ABC \). Then graph \( \triangle A'B'C' \) after a translation 12 units left. Finally, graph \( \triangle A''B''C'' \) after a reflection in the \(-x\)-axis.

**Monitoring Progress**

10. **WHAT IF?** In Example 4, \( \triangle ABC \) is translated 4 units down and then reflected in the \(-y\)-axis. Graph \( \triangle ABC \) and its image after the glide reflection.

11. In Example 4, describe a glide reflection from \( \triangle A''B''C'' \) to \( \triangle ABC \).

**Differentiated Instruction**

**Kinesthetic**

Another way to find a line of symmetry is to trace the figure on tracing paper, and then try to fold the paper so that one half of the figure matches up with the other half. When this happens, the fold crease is a line of symmetry. Have students try this method with the figures in Example 5.
Identifying Lines of Symmetry
A figure in the plane has line symmetry when the figure can be mapped onto itself by a reflection in a line. This line of reflection is a line of symmetry, such as line \( m \) at the left. A figure can have more than one line of symmetry.

**Example 5**: Identifying Lines of Symmetry

How many lines of symmetry does each hexagon have?

- a.  
- b.  
- c.  

**SOLUTION**

- a.  
- b.  
- c.  

**Monitoring Progress**

Determine the number of lines of symmetry for the figure.

- 12.  
- 13.  
- 14.  

- 15. Draw a hexagon with no lines of symmetry.

**Solving Real-Life Problems**

**Example 6**: Finding a Minimum Distance

You are going to buy books. Your friend is going to buy CDs. Where should you park to minimize the distance you both will walk?

**SOLUTION**

Reflect \( B \) in line \( m \) to obtain \( B' \). Then draw \( AB' \). Label the intersection of \( AB' \) and \( m \) as \( C \). Because \( AB' \) is the shortest distance between \( A \) and \( B' \) and \( BC = B'C \), park at point \( C \) to minimize the combined distance, \( AC + BC \), you both have to walk.

**Monitoring Progress**

16. Look back at Example 6. Answer the question by using a reflection of point \( A \) instead of point \( B \).
4.2 Exercises

Vocabulary and Core Concept Check

1. VOCABULARY A glide reflection is a combination of which two transformations?

2. WHICH ONE DOESN'T BELONG? Which transformation does not belong with the other three? Explain your reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, determine whether the coordinate plane shows a reflection in the x-axis, y-axis, or neither.

3. 

4. 

5. 

6. 

In Exercises 7–12, graph \( \triangle JKL \) and its image after a reflection in the given line. (See Example 1.)

7. \( J(2, -4), K(3, 7), L(6, -1); \) x-axis

8. \( J(5, 3), K(1, -2), L(-3, 4); \) y-axis

9. \( J(2, -1), K(4, -5), L(3, 1); x = -1 \)

10. \( J(-1, -1), K(3, 0), L(0, -4); x = 2 \)

11. \( J(2, 4), K(-4, -2), L(-1, 0); y = 1 \)

12. \( J(3, -5), K(4, -1), L(0, -3); y = -3 \)

In Exercises 13–16, graph the polygon and its image after a reflection in the given line. (See Examples 2 and 3.)

13. \( y = x \)

14. \( y = x \)

15. \( y = -x \)

16. \( y = -x \)

In Exercises 7–12, graph \( \triangle JKL \) and its image after a reflection in the given line. (See Example 1.)

7. \( J(2, -4), K(3, 7), L(6, -1); \) x-axis

8. \( J(5, 3), K(1, -2), L(-3, 4); \) y-axis

9. \( J(2, -1), K(4, -5), L(3, 1); x = -1 \)

10. \( J(-1, -1), K(3, 0), L(0, -4); x = 2 \)

11. \( J(2, 4), K(-4, -2), L(-1, 0); y = 1 \)

12. \( J(3, -5), K(4, -1), L(0, -3); y = -3 \)

13–16. See Additional Answers.
In Exercises 17–20, graph \( \triangle RST \) with vertices \( R(4, 1) \), \( S(7, 3) \), and \( T(6, 4) \) and its image after the glide reflection. (See Example 4.)

17. Translation: \((x, y) \rightarrow (x, y - 1)\)
   Reflection: in the y-axis
18. Translation: \((x, y) \rightarrow (x - 3, y)\)
   Reflection: in the line \( y = -1 \)
19. Translation: \((x, y) \rightarrow (x, y + 4)\)
   Reflection: in the line \( x = 3 \)
20. Translation: \((x, y) \rightarrow (x + 2, y + 2)\)
   Reflection: in the line \( y = x \)

In Exercises 21–24, determine the number of lines of symmetry for the figure. (See Example 5.)

21.

22.

23.

24.

25. USING STRUCTURE Identify the line symmetry (if any) of each word.
   a. LOOK
   b. MOM
   c. OX
   d. DAD

26. ERROR ANALYSIS Describe and correct the error in describing the transformation.

27. MODELING WITH MATHEMATICS You park at some point \( K \) on line \( n \). You deliver a pizza to House \( H \), go back to your car, and deliver a pizza to House \( J \). Assuming that you can cut across both lawns, how can you determine the parking location \( K \) that minimizes the distance \( HK + KJ \) ? (See Example 6.)

28. ATTENDING TO PRECISION Use the numbers and symbols to create the glide reflection resulting in the image shown.

29–32. Find point \( C \) on the \( x \)-axis so \( AC + BC \) is a minimum.

29. \( A(1, 4), B(6, 1) \)
30. \( A(-4, -5), B(12, 3) \)
31. \( A(-8, 4), B(-1, 3) \)
32. \( A(-1, 7), B(5, -4) \)

33. MATHEMATICAL CONNECTIONS The line \( y = 3x + 2 \) is reflected in the line \( y = x \). What is the equation of the image?

\[ y = -3x - 4 \]

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ANSWERS
34. See Additional Answers.
35. 

36. Line up the reflective device on line 
$m$ to verify that 
$\triangle ABC$ reflects onto 
$\triangle A'B'C'$ and that 
$\triangle ABC \cong \triangle A'B'C'$.
37. 

38–49. See Additional Answers.

Mini-Assessment

$\triangle ABC$ has vertices $A(1, 3), B(4, 1),$ and 
$C(1, 1)$.
1. Graph $\triangle ABC$ and its image after a 
reflection in the line $y = x$.

2. Graph $\triangle ABC$ and its image after the 
glide reflection. 
Translation: $(x, y) \rightarrow (x - 4, y)$ 
Reflection: in the x-axis

3. How many lines of symmetry does the figure have?

34. **HOW DO YOU SEE IT?** Use Figure A.

35. **CONSTRUCTION** Follow these steps to construct a 
reflection of $\triangle ABC$ in line $m$. Use a compass 
and straightedge.

   **Step 1** Draw $\triangle ABC$ and line $m$.

   **Step 2** Use one compass setting 
to find two points that are 
equidistant from $A$ on line 
m. Use the same compass 
setting to find a point on 
the other side of $m$ that is 
the same distance from 
these two points. Label 
that point as $A'$.

   **Step 3** Repeat Step 2 to find points $B'$ and $C'$.

36. **USING TOOLS** Use a reflective device to verify your 
construction in Exercise 35.

37. **MATHEMATICAL CONNECTIONS** Reflect $\triangle MNQ$ in 
the line $y = -2x$.

**THOUGHT PROVOKING** Is the composition of a 
translation and a reflection commutative? (In other 
words, do you obtain the same image regardless of 
the order in which you perform the transformations?) 
Justify your answer.

38. **MATHEMATICAL CONNECTIONS** Point $B'(1, 4)$ is the 
image of $B(3, 2)$ after a reflection in line $c$. Write an 
equation for line $c$.

**Maintaining Mathematical Proficiency**

Reviewing what you learned in previous grades and lessons

40. $m\angle AOC$ 
41. $m\angle AOD$

42. $m\angle BOE$ 
43. $m\angle AOE$

44. $m\angle COD$ 
45. $m\angle EOD$

46. $m\angle COE$ 
47. $m\angle AOB$

48. $m\angle COB$ 
49. $m\angle BOD$

192 Chapter 4 Transformations

If students need help... If students got it...

Resources by Chapter
- Practice A and Practice B
- Puzzle Time

Resources by Chapter
- Enrichment and Extension
- Cumulative Review

Student Journal
- Practice

Start the next Section

Differentiating the Lesson
Skills Review Handbook
**4.3 Rotations**

**Essential Question** How can you rotate a figure in a coordinate plane?

**EXPLORATION 1 Rotating a Triangle in a Coordinate Plane**

Work with a partner.

a. Use dynamic geometry software to draw any triangle and label it \( \triangle ABC \).

b. Rotate the triangle 90° counterclockwise about the origin to form \( \triangle A'B'C' \).

c. What is the relationship between the coordinates of the vertices of \( \triangle ABC \) and those of \( \triangle A'B'C' \)?

d. What do you observe about the side lengths and angle measures of the two triangles?

![Diagram of triangle ABC and its rotation]

**EXPLORATION 2 Rotating a Triangle in a Coordinate Plane**

Work with a partner.

a. The point \((x, y)\) is rotated 90° counterclockwise about the origin. Write a rule to determine the coordinates of the image of \((x, y)\).

b. Use the rule you wrote in part (a) to rotate \( \triangle ABC \) 90° counterclockwise about the origin. What are the coordinates of the vertices of the image, \( \triangle A'B'C' \)?

c. Draw \( \triangle A'B'C' \). Are its side lengths the same as those of \( \triangle ABC \)? Justify your answer.

**EXPLORATION 3 Rotating a Triangle in a Coordinate Plane**

Work with a partner.

a. The point \((x, y)\) is rotated 180° counterclockwise about the origin. Write a rule to determine the coordinates of the image of \((x, y)\). Explain how you found the rule.

b. Use the rule you wrote in part (a) to rotate \( \triangle ABC \) (from Exploration 2) 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, \( \triangle A'B'C' \)?

**Communicate Your Answer**

4. How can you rotate a figure in a coordinate plane?

5. In Exploration 3, rotate \( \triangle A'B'C' \) 180° counterclockwise about the origin. What are the coordinates of the vertices of the image, \( \triangle A''B''C'' \)? How are these coordinates related to the coordinates of the vertices of the original triangle, \( \triangle ABC \)?

**SUPPORTING English Language Learners**

Read aloud the instructions for Exploration 1. Point out that students are being asked to draw, label, and observe triangles. Have them demonstrate understanding of this basic vocabulary by working in groups of mixed language abilities to do the exploration.

**Beginning** Complete part (a).

**Intermediate** Complete part (b).

**Advanced** Complete part (c).

**Advanced High** Complete part (d).

**ELPS 2.C.3** Learn basic vocabulary heard during classroom instruction and interactions.

---

**Texas Essential Knowledge and Skills**

**G.3.A** The student is expected to describe and perform transformations of figures in a plane using coordinate notation.

**G.3.B** The student is expected to determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

**G.3.C** The student is expected to identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

**G.3.D** The student is expected to identify and distinguish between reflectional and rotational symmetry in a plane figure.

**ANSWERS**

1. a. Check students’ work.

b. Check students’ work.

c. The \( x \)-value of each vertex of \( \triangle A'B'C' \) is the opposite of the \( y \)-value of its corresponding vertex in \( \triangle ABC \). The \( y \)-value of each vertex of \( \triangle A'B'C' \) is equal to the \( x \)-value of its corresponding vertex in \( \triangle ABC \).

d. The side lengths and angle measures of the original figure are equal to the corresponding side lengths and angle measures of the image. For example, \( AB = A'B' \) and \( m\angle A = m\angle A' \).

2. a. \((x, y) \rightarrow (-y, x)\)

b. \(A'(-3, 0), B'(-5, 4), C'(3, 3)\)

2c–5. See Additional Answers.
Extra Example 1
Use the diagram in Example 1. Draw a 60° rotation of \( \triangle ABC \) about point \( P \).

Core Vocabulary
- rotation, p. 194
- center of rotation, p. 194
- angle of rotation, p. 194
- rotational symmetry, p. 197
- center of symmetry, p. 197

What You Will Learn
- Perform rotations.
- Perform compositions with rotations.
- Identify rotational symmetry.

Performing Rotations

A rotation is a transformation in which a figure is turned about a fixed point called the center of rotation. Rays drawn from the center of rotation to a point and its image form the angle of rotation.

A rotation about a point \( P \) through an angle of \( x^\circ \) maps every point \( Q \) in the plane to a point \( Q' \) so that one of the following properties is true.

- If \( Q \) is not the center of rotation \( P \), then \( PQ = Q'P \) and \( m\angle PQQ' = x^\circ \), or
- If \( Q \) is the center of rotation \( P \), then \( Q = Q' \).

The figure above shows a 40° counterclockwise rotation. Rotations can be clockwise or counterclockwise. In this chapter, all rotations are counterclockwise unless otherwise noted.

**EXAMPLE 1** Drawing a Rotation
Draw a 120° rotation of \( \triangle ABC \) about point \( P \).

**SOLUTION**

**Step 1** Draw a segment from \( P \) to \( A \).

**Step 2** Draw a ray to form a 120° angle with \( PA \).

**Step 3** Draw \( A' \) so that \( PA' = PA \).

**Step 4** Repeat Steps 1–3 for each vertex. Draw \( \triangle A'B'C' \).

English Language Learners

**Visual Aid**
Students often confuse the meaning of the terms clockwise and counterclockwise. Tell them they can look at the movement of the hands of a clock for a visual reminder of the meaning of clockwise.
You can rotate a figure more than 180°. The diagram shows rotations of point A 130°, 220°, and 310° about the origin. Notice that point A and its images all lie on the same circle. A rotation of 360° maps a figure onto itself.

You can use coordinate rules to find the coordinates of a point after a rotation of 90°, 180°, or 270° about the origin.

Core Concept

Coordinate Rules for Rotations about the Origin

When a point \((a, b)\) is rotated counterclockwise about the origin, the following are true.

- For a rotation of 90°,
  \((a, b) \rightarrow (−b, a)\).
- For a rotation of 180°,
  \((a, b) \rightarrow (−a, −b)\).
- For a rotation of 270°,
  \((a, b) \rightarrow (b, −a)\).

**EXAMPLE 2** Rotating a Figure in the Coordinate Plane

Graph quadrilateral \(RSTU\) with vertices \(R(3, 1), S(5, 1), T(5, −3), \) and \(U(2, −1)\) and its image after a 270° rotation about the origin.

**SOLUTION**

Use the coordinate rule for a 270° rotation to find the coordinates of the vertices of the image. Then graph quadrilateral \(RSTU\) and its image.

\[
\begin{align*}
(a, b) & \rightarrow (b, −a) \\
R(3, 1) & \rightarrow R′(1, −3) \\
S(5, 1) & \rightarrow S′(1, −5) \\
T(5, −3) & \rightarrow T′(−3, −5) \\
U(2, −1) & \rightarrow U′(−1, −2)
\end{align*}
\]

Monitoring Progress

1. Trace \(\triangle DEF\) and point \(P\). Then draw a 50° rotation of \(\triangle DEF\) about point \(P\).

2. Graph \(\triangle JKL\) with vertices \(J(3, 0), K(4, 3), \) and \(L(6, 0)\) and its image after a 90° rotation about the origin.
**Performing Compositions with Rotations**

### Postulate

**Postulate 4.3 Rotation Postulate**

A rotation is a rigid motion.

---

Because a rotation is a rigid motion, and a rigid motion preserves length and angle measure, the following statements are true for the rotation shown.

- \( DE = D'E' \), \( EF = E'F' \), \( FD = F'D' \)
- \( m\angle D = m\angle D' \), \( m\angle E = m\angle E' \), \( m\angle F = m\angle F' \)

Because a rotation is a rigid motion, the Composition Theorem (Theorem 4.1) guarantees that compositions of rotations and other rigid motions, such as translations and reflections, are rigid motions.

---

**Example 3** Performing a Composition

Graph \( RS \) with endpoints \( R(1, -3) \) and \( S(2, -6) \) and its image after the composition.

**Reflection:** in the \( y \)-axis

**Rotation:** 90° about the origin

**Solution**

1. **Step 1** Graph \( RS \).
2. **Step 2** Reflect \( RS \) in the \( y \)-axis.
   
   \( R'(-1, -3) \) and \( S'(-2, -6) \).
3. **Step 3** Rotate \( R'S' \) 90° about the origin.
   
   \( R''(3, -1) \) and \( S''(6, -2) \).

---

**Common Error**

Unless you are told otherwise, perform the transformations in the order given.

---

**Monitoring Progress**

1. Graph \( RS \) from Example 3. Perform the rotation first, followed by the reflection. Does the order of the transformations matter? Explain.
2. **Extra Example 3** Graph \( RS \) with endpoints \( R(1, -3) \) and \( S(2, -6) \) and its image after the composition.
   
   **Rotation:** 180° about the origin
   
   **Reflection:** in the \( y \)-axis

**Solutions**

3. Yes; The image is in Quadrant I, not Quadrant IV.
4. Graph \( AB \) with endpoints \( A(-4, 4) \) and \( B(-1, 7) \) and its image after the composition.
   
   **Translation:** \((x, y) \rightarrow (x - 2, y - 1)\)
   
   **Rotation:** 90° about the origin
5. Graph \( \triangle TUV \) with vertices \( T(1, 2), U(3, 5), \) and \( V(6, 3) \) and its image after the composition.
   
   **Rotation:** 180° about the origin
   
   **Reflection:** in the \( x \)-axis
**Identifying Rotational Symmetry**

A figure in the plane has **rotational symmetry** when the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure. This point is the **center of symmetry**. Note that the rotation can be either clockwise or counterclockwise.

For example, the regular octagon at the left has rotational symmetry. The center is the intersection of the diagonals. Rotations of 45°, 90°, 135°, or 180° about the center all map the octagon onto itself. The regular octagon also has **point symmetry**, which is 180° rotational symmetry.

**Example 4**

**Identifying Rotational Symmetry**

Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a. parallelogram  
   
   ![Parallelogram](image1.png)

**Solution**

a. The parallelogram has rotational symmetry. The center is the intersection of the diagonals. A 180° rotation about the center maps the parallelogram onto itself.

b. trapezoid  
   
   ![Trapezoid](image2.png)

**Solution**

b. The trapezoid does not have rotational symmetry because no rotation of 180° or less maps the trapezoid onto itself.

**Example 5**

**Distinguishing Between Types of Symmetry**

Identify the line symmetry and rotational symmetry of the equilateral triangle.

**Solution**

The triangle has line symmetry. Three lines of symmetry can be drawn for the figure.

For a figure with $s$ lines of symmetry, the smallest rotation that maps the figure onto itself has the measure $\frac{360°}{s}$. So, the equilateral triangle has $\frac{360°}{3}$ or 120° rotational symmetry.

**Monitoring Progress**

Help in English and Spanish at BigIdeasMath.com

Determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

7. rhombus  
   
   ![Rhombus](image3.png)

8. octagon  
   
   ![Octagon](image4.png)

9. right triangle  
   
   ![Right Triangle](image5.png)

10. Identify the line symmetry and rotational symmetry of a non-equilateral isosceles triangle.

---

**Extra Example 4**

Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

a.  
   
   ![Figure A](image6.png)

**Yes, 120°**

b.  
   
   ![Figure B](image7.png)

**No**

**Extra Example 5**

Identify the line symmetry and rotational symmetry for the rectangle.

- **Two lines of symmetry and 180° rotational symmetry**

**Monitoring Progress Answers**

7. yes; The center is the intersection of the diagonals. A rotation of 180° about the center maps the rhombus onto itself.

8. yes; The center is the intersection of the diagonals. Rotations of 90° and 180° about the center map the octagon onto itself.

9. no

10. the altitude, no rotational symmetry

---

**Closure**

- **Muddiest Point:** Ask students to identify, aloud or on a paper to be collected, the muddiest point(s) about the lesson. What was difficult to understand?
4.3 Exercises

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** When a point \((a, b)\) is rotated counterclockwise about the origin, \((a, b) \rightarrow (b, -a)\) is the result of a rotation of _____.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

   - What are the coordinates of the vertices of the image after a 90° counterclockwise rotation about the origin?
   - What are the coordinates of the vertices of the image after a 270° clockwise rotation about the origin?
   - What are the coordinates of the vertices of the image after turning the figure 90° to the left about the origin?
   - What are the coordinates of the vertices of the image after a 270° counterclockwise rotation about the origin?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, trace the polygon and point \(P\). Then draw a rotation of the polygon about point \(P\) using the given number of degrees. (See Example 1.)

3. 30°
4. 80°

5. 150°
6. 130°

In Exercises 7–10, graph the polygon and its image after a rotation of the given number of degrees about the origin. (See Example 2.)

7. 90°

8. 180°
9. 180°
10. 270°

In Exercises 11–14, graph \(\overline{XY}\) with endpoints \(X(-3, 1)\) and \(Y(4, -5)\) and its image after the composition. (See Example 3.)

11. **Translation**: \((x, y) \rightarrow (x, y + 2)\)
    **Rotation**: 90° about the origin

12. **Rotation**: 180° about the origin
    **Translation**: \((x, y) \rightarrow (x - 1, y + 1)\)

13. **Rotation**: 270° about the origin
    **Reflection**: in the y-axis

14. **Reflection**: in the line \(y = x\)
    **Rotation**: 180° about the origin

10–14. See Additional Answers.
In Exercises 15 and 16, graph \( \triangle LMN \) with vertices \( L(1, 6), M(−2, 4), \) and \( N(3, 2) \) and its image after the composition. (See Example 3.)

15. Rotation: 90° about the origin
   Translation: \((x, y) \rightarrow (x − 3, y + 2)\)

16. Reflection: in the \( x \)-axis
   Rotation: 270° about the origin

In Exercises 17–20, determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself. (See Example 4.)

17.  *
18.  *
19.  *
20.  *

In Exercises 21–24, identify the line symmetry and rotational symmetry of the figure. (See Example 5.)

21.  *
22.  *
23.  *
24.  *

ERROR ANALYSIS In Exercises 25 and 26, the endpoints of \( \overline{CD} \) are \( C(−1, 1) \) and \( D(2, 3) \). Describe and correct the error in finding the coordinates of the vertices of the image after a rotation of 270° about the origin.

25. \[ C(−1, 1) \rightarrow C′(−1, −1) \]
    \[ D(2, 3) \rightarrow D′(2, −3) \]

26. \[ C(−1, 1) \rightarrow C′(1, −1) \]
    \[ D(2, 3) \rightarrow D′(5, 2) \]

MATHEMATICAL CONNECTIONS Use the graph of \( y = 2x − 3 \).

27. CONSTRUCTION Follow these steps to construct a rotation of \( \triangle ABC \) by angle \( D \) around a point \( O \). Use a compass and straightedge.

   Step 1 Draw \( \triangle ABC \), \( L \), and \( O \), the center of rotation.
   Step 2 Draw \( OA \). Use the construction for copying an angle to copy \( \angle D \) at \( O \), as shown. Then use distance \( OA \) and center \( O \) to find \( A′ \).
   Step 3 Repeat Step 2 to find points \( B′ \) and \( C′ \). Draw \( \triangle A′B′C′ \).

28. REASONING You enter the revolving door at a hotel.
   a. You rotate the door 180°.
      What does this mean in the context of the situation?
   b. You rotate the door 360°.
      What does this mean in the context of the situation?

29. The rule for a 270° rotation, \((x, y) \rightarrow (y, −x)\), should have been used instead of the rule for a reflection in the \( x \)-axis;
   \[ C(−1, 1) \rightarrow C′(1, 1), D(2, 3) \rightarrow D′(3, −2) \]

30. MAKING AN ARGUMENT Your friend claims that rotating a figure by 180° is the same as reflecting a figure in the \( y \)-axis and then reflecting it in the \( x \)-axis. Is your friend correct? Explain your reasoning.

ANSWERS

15. yes, yes; A 180° rotation about the center maps the figure onto itself.
16. yes, yes; a 90° rotation about the center maps the figure onto itself.
17. yes; Rotations of 90° and 180° about the center map the figure onto itself.
18. yes; Rotations of 72° and 144° about the center map the figure onto itself.
19. yes; Rotations of 45°, 90°, 135°, and 180° about the center map the figure onto itself.
20. yes; A 180° rotation about the center maps the rectangle onto itself.
21. no, yes; 90°, 180°
22. yes, yes; four lines of symmetry
23. yes, no; one line of symmetry
24. yes, yes; two lines of symmetry
25. 180°
26. The rule for a 270° rotation, \((x, y) \rightarrow (y, −x)\), should have been used instead of the rule for a reflection in the \( x \)-axis;
   \[ C(−1, 1) \rightarrow C′(1, 1), D(2, 3) \rightarrow D′(3, −2) \]

27–30. See Additional Answers.
If students got it...

31. **DRAWING CONCLUSIONS** A figure only has point symmetry. How many times can you rotate the figure before it is back where it started?

32. **ANALYZING RELATIONSHIPS** Is it possible for a figure to have 90° rotational symmetry but not 180° rotational symmetry? Explain your reasoning.

33. **ANALYZING RELATIONSHIPS** Is it possible for a figure to have 180° rotational symmetry but not 90° rotational symmetry? Explain your reasoning.

34. **THOUGHT PROVOKING** Can rotations of 90°, 180°, 270°, and 360° be written as the composition of two reflections? Justify your answer.

35. **USING AN EQUATION** Inside a kaleidoscope, two mirrors are placed next to each other to form a V. The angle between the mirrors determines the number of lines of symmetry in the image. Use the formula \( n(m\angle 1) = 180° \) to find the measure of \( \angle 1 \), the angle between the mirrors, for the number \( n \) of lines of symmetry.
   a. \( n = 1 \)
   b. \( n = 2 \)
   c. \( n = 3 \)

36. **REASONING** Use the coordinate rules for clockwise rotations of 90°, 180°, or 270° about the origin.

37. **USING STRUCTURE** \( \triangle XYZ \) has vertices \( X(2, 5) \), \( Y(3, 1) \), and \( Z(0, 2) \). Rotate \( \triangle XYZ \) 90° about the point \( P(−2, −1) \).

**If students need help...**

Resources by Chapter
- Practice A and Practice B
- Puzzle Time

**If students got it...**

Resources by Chapter
- Enrichment and Extension
- Cumulative Review

Student Journal
- Practice

Differentiating the Lesson
Skills Review Handbook

**Mini-Assessment**

1. Draw a 135° rotation of \( \triangle ABC \) about point \( P \).

2. \( \overline{RS} \) has endpoints \( R(−2, 1) \) and \( S(1, 2) \). Graph \( \overline{RS} \) and its image after the composition.

   Rotation: 90° about the origin
   Reflection: in the x-axis

3. Does the figure have rotational symmetry? If so, describe any rotations that map the figure onto itself.

   yes; 90°, 180°

**ANSWERS**

31. twice

32. no; Because the figure has 90° rotational symmetry, the image will still be symmetrical to the preimage after two 90° rotations, which is the equivalent of a 180° rotation.

33. yes; Sample answer: A rectangle (that is not a square) is one example of a figure that has 180° rotational symmetry, but not 90° rotational symmetry.

34–41. See Additional Answers.
4.1–4.3 What Did You Learn?

Core Vocabulary

- vector, p. 178
- initial point, p. 178
- terminal point, p. 178
- horizontal component, p. 178
- vertical component, p. 178
- component form, p. 178
- transformation, p. 178
- image, p. 178
- preimage, p. 178
- translation, p. 178
- rigid motion, p. 180
- composition of transformations, p. 180
- reflection, p. 186
- line of reflection, p. 186
- glide reflection, p. 188
- line symmetry, p. 189
- line of symmetry, p. 189
- rotation, p. 194
- center of rotation, p. 194
- angle of rotation, p. 194
- rotational symmetry, p. 197
- center of symmetry, p. 197

Core Concepts

Section 4.1

Vectors, p. 178
Translations, p. 178

Postulate 4.1 Translation Postulate, p. 180
Theorem 4.1 Composition Theorem, p. 180

Section 4.2

Reflections, p. 186
Coordinate Rules for Reflections, p. 187

Postulate 4.2 Reflection Postulate, p. 188
Line Symmetry, p. 189

Section 4.3

Rotations, p. 194
Coordinate Rules for Rotations about the Origin, p. 195

Postulate 4.3 Rotation Postulate, p. 196
Rotational Symmetry, p. 197

Mathematical Thinking

1. How could you determine whether your results make sense in Exercise 26 on page 183?
2. State the meaning of the numbers and symbols you chose in Exercise 28 on page 191.
3. Describe the steps you would take to arrive at the answer to Exercise 29 part (a) on page 199.

ANSWERS

1. Recreate the chess board on a coordinate plane and substitute the coordinates into your rule to verify both the composition and the single translation yield the same result.
2. $x + 3$ means that the figure will slide 3 units to the right, and $y + 3$ means the figure will slide 3 units up.
3. Find two points on the line $y = 2x - 3$, their images after the rotation, and use the images to find the equation of the new line.

Study Skills

Keeping a Positive Attitude

Ever feel frustrated or overwhelmed by math? You're not alone. Just take a deep breath and assess the situation. Try to find a productive study environment, review your notes and examples in the textbook, and ask your teacher or peers for help.
ANSWERS

1. Graph quadrilateral \(ABCD\) with vertices \(A(-4, 1), B(-3, 3), C(0, 1), \) and \(D(-2, 0)\) and its image after the translation. (Section 4.1)
   \[1. \quad (x, y) \rightarrow (x + 4, y - 2) \quad 2. \quad (x, y) \rightarrow (x - 1, y + 5) \quad 3. \quad (x, y) \rightarrow (x + 3, y + 6)\]

2. Graph the polygon with the given vertices and its image after a reflection in the given line. (Section 4.2)
   \[4. \quad A(-5, 6), B(-7, 8), C(-3, 11); \text{ } x \text{-axis} \quad 5. \quad D(-5, -1), E(-2, 1), F(-1, -3); \text{ } y = x \quad 6. \quad J(-1, 4), K(2, 5), L(5, 2), M(4, -1); \text{ } x = 3\]

Graph \(\triangle ABC\) with vertices \(A(2, -1), B(5, 2), \) and \(C(8, -2)\) and its image after the glide reflection. (Section 4.2)
   \[8. \quad \text{Translation: } (x, y) \rightarrow (x, y + 6) \quad \text{Reflection: in the } y\text{-axis} \quad 9. \quad \text{Translation: } (x, y) \rightarrow (x - 9, y) \quad \text{Reflection: in the line } y = 1\]

Determine the number of lines of symmetry for the figure. (Section 4.2)

10. 

Graph the polygon and its image after a rotation of the given number of degrees about the origin. (Section 4.3)

14. \(90°\) 

15. \(270°\) 

16. \(180°\)

Graph \(\triangle L MN\) with vertices \(L(-3, -2), M(-1, 1), \) and \(N(2, -3)\) and its image after the composition. (Sections 4.1–4.3)

17. \(\text{Translation: } (x, y) \rightarrow (x - 4, y + 3)\)  
   \(\text{Rotation: } 180° \text{ about the origin}\)

18. \(\text{Rotation: } 90° \text{ about the origin} \quad \text{Reflection: in the } y\text{-axis}\)

19. The figure shows a game in which the object is to create solid rows using the pieces given. Using only translations and rotations, describe the transformations for each piece at the top that will form two solid rows at the bottom. (Section 4.1 and Section 4.3)

20. Game Instructions

See Additional Answers.
**4.4 Congruence and Transformations**

**Essential Question** What conjectures can you make about a figure reflected in two lines?

**EXPLORATION 1** Reflections in Parallel Lines

**Work with a partner:** Use dynamic geometry software to draw any scalene triangle and label it \( \triangle ABC \).

a. Draw any line \( \overline{DE} \). Reflect \( \triangle ABC \) in \( \overline{DE} \) to form \( \triangle A'B'C' \).

b. Draw a line parallel to \( \overline{DE} \). Reflect \( \triangle A'B'C' \) in the new line to form \( \triangle A''B''C'' \).

c. Draw the line through point \( A \) that is perpendicular to \( \overline{DE} \). What do you notice?

d. Find the distance between points \( A \) and \( A'' \). Find the distance between the two parallel lines. What do you notice?

e. Hide \( \triangle A'B'C' \). Is there a single transformation that maps \( \triangle ABC \) to \( \triangle A''B''C'' \)? Explain.

f. Make conjectures based on your answers in parts (c)–(e). Test your conjectures by changing \( \triangle ABC \) and the parallel lines.

**EXPLORATION 2** Reflections in Intersecting Lines

**Work with a partner:** Use dynamic geometry software to draw any scalene triangle and label it \( \triangle ABC \).

a. Draw any line \( \overline{DE} \). Reflect \( \triangle ABC \) in \( \overline{DE} \) to form \( \triangle A'B'C' \).

b. Draw another line \( \overline{DF} \) so that angle \( \angle EDF \) is less than or equal to 90°. Reflect \( \triangle A'B'C' \) in \( \overline{DF} \) to form \( \triangle A''B''C'' \).

c. Find the measure of \( \angle EDF \). Rotate \( \triangle ABC \) counterclockwise about point \( D \) twice using the measure of \( \angle EDF \).

d. Make a conjecture about a figure reflected in two intersecting lines. Test your conjecture by changing \( \triangle ABC \) and the lines.

**Communicate Your Answer**

3. What conjectures can you make about a figure reflected in two lines?

4. Point \( Q \) is reflected in two parallel lines, \( GH \) and \( \overline{JK} \), to form \( Q' \) and \( Q'' \). The distance from \( GH \) to \( \overline{JK} \) is 3.2 inches. What is the distance \( QQ''? \)

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**SUPPORTING English Language Learners**

Read aloud the Essential Question. Have students work in pairs of mixed language abilities to complete the explorations and Communicate Your Answer.

**Beginning/Advanced** Beginning students state the parts of the explorations as Advanced students complete them. Beginning students state any one-word answers. Advanced students state answers requiring phrases or sentences.

**Intermediate/Advanced High** Intermediate students read aloud the parts of the explorations and state any short answers. Advanced High students provide explanations that require more complex language.

**ELPS 3.D.2** Speak using grade-level content area vocabulary in context to build academic language proficiency.

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**Texas Essential Knowledge and Skills**

**G.3.B** The student is expected to determine the image or pre-image of a given two-dimensional figure under a composition of rigid transformations, a composition of non-rigid transformations, and a composition of both, including dilations where the center can be any point in the plane.

**G.3.C** The student is expected to identify the sequence of transformations that will carry a given pre-image onto an image on and off the coordinate plane.

**G.6.C** The student is expected to apply the definition of congruence, in terms of rigid transformations, to identify congruent figures and their corresponding sides and angles.

**ANSWERS**

1. a. Check student’s work.

b. Check student’s work.

c. Sample answer:

The line passes through \( A' \) and \( A'' \).

d. The distance between \( A \) and \( A'' \) is twice the distance between the parallel lines.

e. Yes; \( \triangle A''B''C'' \) is a translation of \( \triangle ABC \).

1f–4. See Additional Answers.
**What You Will Learn**

- Identify congruent figures.
- Describe congruence transformations.
- Use theorems about congruence transformations.

### Identifying Congruent Figures

Two geometric figures are congruent figures if and only if there is a rigid motion or a composition of rigid motions that maps one of the figures onto the other. Congruent figures have the same size and shape.

**Core Vocabulary**

- Congruent figures, p. 204
- Congruence transformation, p. 205

#### Identifying Congruent Figures

You can identify congruent figures in the coordinate plane by identifying the rigid motion or composition of rigid motions that maps one of the figures onto the other. Recall from Postulates 4.1–4.3 and Theorem 4.1 that translations, reflections, rotations, and compositions of these transformations are rigid motions.

**Example 1**

Identify any congruent figures in the coordinate plane. Explain.

**Solution**

- **\( \triangle DEF \cong \triangle ABC \)**: \( \triangle ABC \) is a translation of \( \triangle DEF \) 5 units right and 1 unit up.
- **\( \square GHJI \cong \square NPQR \)**: \( \triangle ABC \) is a rotation of \( \triangle DEF \) 90° about the origin.
- **\( \triangle JKL \cong \triangle MNP \)**: \( \triangle JKL \) is a reflection of \( \triangle MNP \) in the y-axis.

**Monitoring Progress**

1. Identify any congruent figures in the coordinate plane. Explain.

**Differentiated Instruction**

**Kinesthetic**

Students may not fully understand that only rigid transformations produce congruent figures. Have students take a sheet of paper and transform it in various ways. Students should not only slide, spin, and flip the paper, but they should also cut, fold, or crumple the paper into a ball. After each transformation, ask students to explain how they decide whether the transformed paper is congruent to its original shape. Encourage students to use the terms *sides*, *vertices*, *length*, *distance*, or *angles* in their responses.
Congruence Transformations

Another name for a rigid motion or a combination of rigid motions is a congruence transformation because the preimage and image are congruent. The terms “rigid motion” and “congruence transformation” are interchangeable.

**Example 2: Describing a Congruence Transformation**

Describe a congruence transformation that maps \(\square ABCD\) to \(\square EFGH\).

**SOLUTION**

The two vertical sides of \(\square ABCD\) rise from left to right, and the two vertical sides of \(\square EFGH\) fall from left to right. If you reflect \(\square ABCD\) in the \(y\)-axis, as shown, then the image, \(\square A'B'C'D'\), will have the same orientation as \(\square EFGH\).

Then you can map \(\square A'B'C'D'\) to \(\square EFGH\) using a translation of 4 units down.

So, a congruence transformation that maps \(\square ABCD\) to \(\square EFGH\) is a reflection in the \(y\)-axis followed by a translation of 4 units down.

**Monitoring Progress**

2. In Example 2, describe another congruence transformation that maps \(\square ABCD\) to \(\square EFGH\).

3. Describe a congruence transformation that maps \(\triangle KLM\) to \(\triangle MNP\).
Extra Example 3
In the diagram, a reflection in line $a$ maps $PQ$ to $P'Q'$. A reflection in line $b$ maps $P'Q'$ to $P''Q''$. Also, $PJ = 3$ and $LP'' = 8$.

- a. Name any segments congruent to each segment: $PQ$, $PJ$, and $QK$.
  $PQ \cong P'Q' \cong P''Q''$; $PJ \cong P'J$; $QK \cong Q'K$
  Yes, $JKLM$ is a rectangle, and opposite sides of a rectangle are congruent.
- c. What is the length of $PP''$?
  22 units

**MONITORING PROGRESS ANSWERS**

4. translation 3.2 cm right
5. They are perpendicular by Reflections in Parallel Lines (Thm. 4.2).
6. 3.2 cm

**Using Theorems about Congruence Transformations**
Compositions of two reflections result in either a translation or a rotation. A composition of two reflections in parallel lines results in a translation, as described in the following theorem.

**Theorem**

**Reflections in Parallel Lines Theorem**

If lines $k$ and $m$ are parallel, then a reflection in line $k$ followed by a reflection in line $m$ is the same as a translation.

If $A'$ is the image of $A$, then

1. $AA'$ is perpendicular to $k$ and $m$, and
2. $AA' = 2d$, where $d$ is the distance between $k$ and $m$.

**Proof** Ex. 31, p. 210

**Example 3 Using the Reflections in Parallel Lines Theorem**

In the diagram, a reflection in line $k$ maps $GH$ to $G'H'$. A reflection in line $m$ maps $G'H'$ to $G''H''$. Also, $HB = 9$ and $DH'' = 4$.

- a. Name any segments congruent to each segment: $GH$, $HB$, and $GA$.
  - $GH = G'H'$, and $GH = G'H'$.
  - $HB = H'B$. $GA = G'A$
  - Yes, $AC = BD$ because $G'G''$ and $HH''$ are perpendicular to both $k$ and $m$.
  - So, $BD$ and $AC$ are opposite sides of a rectangle.
- c. What is the length of $GG''$?
  - By the properties of reflections, $H'B = 9$ and $H'D = 4$. The Reflections in Parallel Lines Theorem implies that $GG'' = HH'' = 2 \cdot BD$, so the length of $GG''$ is $2(9 + 4) = 26$ units.
A composition of two reflections in intersecting lines results in a rotation, as described in the following theorem.

**Theorem 4.3  Reflections in Intersecting Lines Theorem**

If lines $k$ and $m$ intersect at point $P$, then a reflection in line $k$ followed by a reflection in line $m$ is the same as a rotation about point $P$.

The angle of rotation is $2x^\circ$, where $x^\circ$ is the measure of the acute or right angle formed by lines $k$ and $m$.

**Proof**  Ex. 31, p. 254

Using the Reflections in Intersecting Lines Theorem

In the diagram, the figure is reflected in line $k$. The image is then reflected in line $m$. Describe a single transformation that maps $F$ to $F''$.

**SOLUTION**

By the Reflections in Intersecting Lines Theorem, a reflection in line $k$ followed by a reflection in line $m$ is the same as a rotation about point $P$. The measure of the acute angle formed between lines $k$ and $m$ is $70^\circ$. So, by the Reflections in Intersecting Lines Theorem, the angle of rotation is $2(70^\circ) = 140^\circ$. A single transformation that maps $F$ to $F''$ is a $140^\circ$ rotation about point $P$.

You can check that this is correct by tracing lines $k$ and $m$ and point $F$, then rotating the point $140^\circ$.

**Monitoring Progress**

7. In the diagram, the preimage is reflected in line $k$, then in line $m$. Describe a single transformation that maps the blue figure onto the green figure.

8. A rotation of $76^\circ$ maps $C$ to $C'$. To map $C$ to $C'$ using two reflections, what is the measure of the angle formed by the intersecting lines of reflection?

**Closure**

- **Writing Prompt:** A congruence transformation is … a rigid motion where the preimage and image are congruent.
- **Writing Prompt:** The composition of two reflections is equivalent to … a translation when the lines are parallel and is equivalent to a rotation when the lines are intersecting.
**Vocabulary and Core Concept Check**

1. **COMPLETE THE SENTENCE** Two geometric figures are _________ if and only if there is a rigid motion or a composition of rigid motions that moves one of the figures onto the other.

2. **VOCABULARY** Why is the term congruence transformation used to refer to a rigid motion?

**Monitoring Progress and Modeling with Mathematics**

**In Exercises 3 and 4, identify any congruent figures in the coordinate plane. Explain. (See Example 1.)**

3. 

4. 

**In Exercises 5 and 6, describe a congruence transformation that maps the blue preimage to the green image. (See Example 2.)**

5. 

6. 

**In Exercises 7–10, determine whether the polygons with the given vertices are congruent. Use transformations to explain your reasoning.**

7. 

8. 

9. 

10. 

**In Exercises 11–14, let \( k \parallel m \), \( \triangle ABC \) is reflected in line \( k \), and \( \triangle A'B'C' \) is reflected in line \( m \). (See Example 3.)**

11. 

12. 

13. 

14. 

**ANSWERS**

1. congruent

2. The preimage and image are congruent in a rigid transformation.

3. \( \triangle HJK \cong \triangle QRS \), \( \triangle DEF \cong \triangle LMNP \). \( \triangle HJK \) is a 90° rotation of \( \triangle QRS \). \( \triangle DEF \) is translation 7 units right and 3 units down of \( \triangle LMNP \).

4. \( \triangle MNP \cong \triangle TUV \), \( \triangle EFG \cong \triangle QRS \), \( \triangle HJK \cong \triangle ABCD \). \( \triangle MNP \) is a 90° rotation of \( \triangle TUV \). \( \triangle EFG \) is a 180° rotation of \( \triangle QRS \). \( \triangle HJK \) is a translation 4 units down and 7 units right of \( \triangle ABCD \).

5. Sample answer: 180° rotation about the origin followed by a translation 5 units left and 1 unit down

6. Sample answer: 180° rotation about the origin

7. yes; \( \triangle TUV \) is a translation 4 units right of \( \triangle QRS \). So, \( \triangle TUV \cong \triangle QRS \).

8. yes; \( \triangle CDEF \) is a 90° rotation of \( \triangle WXYZ \). So, \( \triangle CDEF \cong \triangle WXYZ \).

9. no; \( M \) and \( N \) are translated 2 units right of their corresponding vertices, \( L \) and \( K \), but \( P \) is translated only 1 unit right of its corresponding vertex, \( J \). So, this is not a rigid motion.

10. yes; A congruence transformation that maps \( \triangle ABCD \) to \( \triangle GHEF \) is a translation 5 units down, followed by a reflection in the y-axis. So, \( \triangle ABCD \cong \triangle GHEF \).

11. \( A'B'C' \)

12. line \( k \) and line \( m \)

13. 5.2 in.

14. yes; Because \( \triangle A'B'C' \) is a reflection of \( \triangle A'B'C' \) in the line \( m \), each vertex in the image is the same distance from the line of reflection as its preimage.
In Exercises 15 and 16, find the angle of rotation that maps A onto A′. (See Example 4.)

**15.**

\[ \begin{align*}
A' &\quad m \\
55° &\quad k
\end{align*} \]

**16.**

\[ \begin{align*}
A' &\quad m \\
15° &\quad k
\end{align*} \]

**17.** ERROR ANALYSIS Describe and correct the error in describing the congruence transformation.

\[ \triangle ABC \text{ is mapped to } \triangle A'B'C' \text{ by a translation 3 units down and a reflection in the } y\text{-axis}. \]

**18.** ERROR ANALYSIS Describe and correct the error in using the Reflections in Intersecting Lines Theorem (Theorem 4.3).

\[ \text{A 72° rotation about point } P \text{ maps the blue image to the green image.} \]

In Exercises 19–22, find the measure of the acute or right angle formed by intersecting lines so that C can be mapped to C′ using two reflections.

**19.** A rotation of 84° maps C to C′.

**20.** A rotation of 24° maps C to C′.

**21.** The rotation \( (x, y) \rightarrow (-x, -y) \) maps C to C′.

**22.** The rotation \( (x, y) \rightarrow (y, -x) \) maps C to C′.

**23.** REASONING Use the Reflections in Parallel Lines Theorem (Theorem 4.2) to explain how you can make a glide reflection using three reflections. How are the lines of reflection related?

**24.** DRAWING CONCLUSIONS The pattern shown is called a tessellation.

\[ \text{a. What transformations did the artist use when creating this tessellation?} \]

\[ \text{b. Are the individual figures in the tessellation congruent? Explain your reasoning.} \]

**CRITICAL THINKING** In Exercises 25–28, tell whether the statement is always, sometimes, or never true. Explain your reasoning.

**25.** A congruence transformation changes the size of a figure.

**26.** If two figures are congruent, then there is a rigid motion or a composition of rigid motions that maps one figure onto the other.

**27.** The composition of two reflections results in the same image as a rotation.

**28.** A translation results in the same image as the composition of two reflections.

**29.** REASONING During a presentation, a marketing representative uses a projector so everyone in the auditorium can view the advertisement. Is this projection a congruence transformation? Explain your reasoning.

**ANSWERS**

15. 110°

16. 30°

17. A translation 5 units right and a reflection in the x-axis should have been used; \( \triangle ABC \) is mapped to \( \triangle A'B'C' \) by a translation 5 units right, followed by a reflection in the x-axis.

18. If \( x^\circ \) is the measure of the acute angle formed by the intersecting lines, an angle of \( 2x^\circ \) should be used to describe the angle of rotation; a 144° rotation about point \( P \) maps the blue image to the green image.

19. 42°

20. 12°

21. 90°

22. 45°

23. Reflect the figure in two parallel lines instead of translating the figure; The third line of reflection is perpendicular to the parallel lines.

24. a. rotations and translations

25. yes; All of the figures could be created using one or more rigid transformations of an original shape.

26. always; Every figure can be mapped onto a congruent figure using transformations.

27. sometimes; Reflecting in \( y = x \) then \( y = x \) is not a rotation. Reflecting in the y-axis then \( x \)-axis is a rotation of 180°.

28. sometimes; It would depend on the translations.

29. no; The image on the screen is larger.
ANSWERS
30–43. See Additional Answers.

Mini-Assessment

1. Identify any congruent figures in the coordinate plane. Explain.


See Additional Answers.

2. Describe a congruence transformation that maps \( \triangle JKL \) to \( \triangle MNP \).

![Diagram of points K, L, M, N, P, Q, R, S, T, U, V]

See Additional Answers.

3. \( \triangle TUV \) is reflected over line \( k \) and then that image is reflected over line \( m \). The distance between lines \( k \) and \( m \) is 2.8 cm. What is the distance between \( T \) and \( T' \)?


5.6 cm

4. Lines \( k \) and \( m \) intersect. Figure A is reflected over line \( k \) and then that image is reflected over line \( m \) to produce figure \( A' \). A 24° rotation about the intersection of lines \( m \) and \( k \) maps \( A \) to \( A'' \). What is the measure of the acute angle formed by the lines? 12°

30. HOW DO YOU SEE IT? What type of congruence transformation can be used to verify each statement about the stained glass window?

![Stained glass window diagram]

a. Triangle 5 is congruent to Triangle 8.

b. Triangle 1 is congruent to Triangle 4.

c. Triangle 2 is congruent to Triangle 7.

d. Pentagon 3 is congruent to Pentagon 6.

31. PROVING A THEOREM Prove the Reflections in Parallel Lines Theorem (Theorem 4.2).


Given: A reflection in line \( \ell \) maps \( JK \) to \( J'K' \), a reflection in line \( m \) maps \( J'K' \) to \( J''K'' \), and \( \ell \parallel m \).

Prove: a. \( KK'' \) is perpendicular to \( \ell \) and \( m \).

b. \( KK'' = 2d \), where \( d \) is the distance between \( \ell \) and \( m \).

32. THOUGHT PROVOKING A tessellation is the covering of a plane with congruent figures so that there are no gaps or overlaps (see Exercise 24). Draw a tessellation that involves two or more types of transformations. Describe the transformations that are used to create the tessellation.

33. MAKING AN ARGUMENT \( \overline{PQ} \), with endpoints \( P(1, 3) \) and \( Q(3, 2) \), is reflected in the y-axis. The image \( \overline{P'Q'} \) is then reflected in the x-axis to produce the image \( \overline{P''Q''} \). One classmate says that \( \overline{PQ} \) is mapped to \( \overline{P''Q''} \) by the translation \( (x, y) \to (x - 4, y - 5) \). Another classmate says that \( \overline{PQ} \) is mapped to \( \overline{P''Q''} \) by \( (2 \cdot 90)° \), or \( 180° \), rotation about the origin. Which classmate is correct? Explain your reasoning.

34. CRITICAL THINKING Does the order of reflections for a composition of two reflections in parallel lines matter? For example, is reflecting \( \triangle ABC \) in line \( \ell \) and then its image in line \( m \) the same as reflecting \( \triangle ABC \) in line \( m \) and then its image in line \( \ell \)?

CONSTRUCTION In Exercises 35 and 36, copy the figure. Then use a compass and straightedge to construct two lines of reflection that produce a composition of reflections resulting in the same image as the given transformation.

35. Translation: \( \triangle ABC \to \triangle A'B'C' \)

![Diagram of points A, B, C, A', B', C']

36. Rotation about \( P \): \( \triangle XYZ \to \triangle X'Y'Z' \)

![Diagram of points X, Y, Z, X', Y', Z']

Maintaining Mathematical Proficiency

Solve the equation. Check your solution. (Skills Review Handbook)

37. \( 5x + 16 = -3x \)

38. \( 12 + 6m = 2m \)

39. \( 4b + 8 = 6b - 4 \)

40. \( 7w - 9 = 13 - 4w \)

41. \( 7(2m + 11) = 4n \)

42. \( -2(8 - y) = -6y \)

43. Last year, the track team’s yard sale earned $500. This year, the yard sale earned $625. What is the percent of increase? (Skills Review Handbook)

210 Chapter 4 Transformations

If students need help... If students got it...

Resources by Chapter
- Practice A and Practice B
- Puzzle Time

Resources by Chapter
- Enrichment and Extension
- Cumulative Review

Student Journal
- Practice

Start the next Section

Differentiating the Lesson
Skills Review Handbook
Essential Question  What does it mean to dilate a figure?

Exploration 1  Dilating a Triangle in a Coordinate Plane

Work with a partner. Use dynamic geometry software to draw any triangle and label it △ABC.

a. Dilate △ABC using a scale factor of 2 and a center of dilation at the origin to form △A'B'C'. Compare the coordinates, side lengths, and angle measures of △ABC and △A'B'C'.

b. Repeat part (a) using a scale factor of 1/2.

c. What do the results of parts (a) and (b) suggest about the coordinates, side lengths, and angle measures of the image of △ABC after a dilation with a scale factor of k?

Exploration 2  Dilating Lines in a Coordinate Plane

Work with a partner. Use dynamic geometry software to draw AB that passes through the origin and AC that does not pass through the origin.

a. Dilate AB using a scale factor of 3 and a center of dilation at the origin. Describe the image.

b. Dilate AC using a scale factor of 3 and a center of dilation at the origin. Describe the image.

c. Repeat parts (a) and (b) using a scale factor of 1/3.

d. What do you notice about dilations of lines passing through the center of dilation and dilations of lines not passing through the center of dilation?

Communicate Your Answer

3. What does it mean to dilate a figure?

4. Repeat Exploration 1 using a center of dilation at a point other than the origin.

1. a. The x-value of each vertex of △A'B'C' is k times the x-value of its corresponding vertex of △ABC, and the y-value of each vertex of △A'B'C' is k times the y-value of its corresponding vertex of △ABC. Each side of △A'B'C' is k times as long as its corresponding side of △ABC. Each angle of △A'B'C' is congruent to its corresponding angle of △ABC.

b. The image is a line that is parallel to AC. The x- and y-intercepts of the image are each three times the x- and y-intercepts of AC.

c. The image of AB is a line that coincides with AB. The image of AC is a line that is parallel to AC. The x- and y-intercepts of the image are each one-fourth of the x- and y-intercepts of AC.

2d–4. See Additional Answers.

ANSWERS

1. a. Check students’ work. The x-value of each vertex of △A'B'C' is twice the x-value of its corresponding vertex of △ABC, and the y-value of each vertex of △A'B'C' is twice the y-value of its corresponding vertex of △ABC. Each side of △A'B'C' is twice as long as its corresponding side of △ABC. Each angle of △A'B'C' is congruent to its corresponding angle of △ABC.

b. Sample answer:
4.5 Lesson

What You Will Learn
- Identify and perform dilations.
- Solve real-life problems involving scale factors and dilations.

Core Vocabulary
- dilation, p. 212
- center of dilation, p. 212
- scale factor, p. 212
- enlargement, p. 212
- reduction, p. 212

Core Concept
Dilations
A dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point \( C \) called the center of dilation and a scale factor \( k \), which is the ratio of the lengths of the corresponding sides of the image and the preimage.

A dilation with center of dilation \( C \) and scale factor \( k \) maps every point \( P \) in a figure to a point \( P' \) so that the following are true.
- If \( P \) is the center point \( C \), then \( P = P' \).
- If \( P \) is not the center point \( C \), then the image point \( P' \) lies on \( \overrightarrow{CP} \). The scale factor \( k \) is a positive number such that \( k = \frac{CP'}{CP} \).
- Angle measures are preserved.

A dilation does not change any line that passes through the center of dilation. A dilation maps a line that does not pass through the center of dilation to a parallel line. In the figure above, \( \overline{PR} \parallel \overline{PR'} \), \( \overline{PQ} \parallel \overline{P'Q'} \), and \( \overline{QR} \parallel \overline{Q'R'} \).

When the scale factor \( k > 1 \), a dilation is an enlargement. When \( 0 < k < 1 \), a dilation is a reduction.

Example 1: Identifying Dilations
Find the scale factor of the dilation. Then tell whether the dilation is a reduction or an enlargement.

a. \( \frac{12}{8} \); reduction

b. \( \frac{18}{30} \); reduction

Solutions
a. Because \( \frac{CP'}{CP} = \frac{12}{8} \), the scale factor is \( k = \frac{3}{2} \). So, the dilation is an enlargement.

b. Because \( \frac{CP'}{CP} = \frac{18}{30} \), the scale factor is \( k = \frac{3}{5} \). So, the dilation is a reduction.

Monitoring Progress
1. In a dilation, \( CP' = 3 \) and \( CP = 12 \). Find the scale factor. Then tell whether the dilation is a reduction or an enlargement.

Reading
The scale factor of a dilation can be written as a fraction, decimal, or percent.

Supporting English Language Learners
Read aloud What You Will Learn and present the Core Concept. Have students read portions of the Core Concept aloud to check their reading ability.

Beginning State examples of words that they frequently see and automatically know. \( A, \ is, \ and, \ in, \ or, \ with, \) and \( to \) would all be possibilities.

Intermediate Read one of the bulleted statements aloud.

Advanced Read the introduction to the Core Concept Dilations.

Advanced High Read the introduction to the Core Concept Dilations and restate it using their own words.

ELPS 4.C.1 Develop basic sight vocabulary used routinely in written classroom materials.
**Core Concept**

**Coordinate Rules for Dilations**

If \( P(x, y) \) is the preimage of a point, then its image \( P' \) after a dilation centered at \( C \) with scale factor \( k \) is shown below.

<table>
<thead>
<tr>
<th>Center ((x, y))</th>
<th>Image (P'(kx, ky))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0))</td>
<td>(P'(kx, ky))</td>
</tr>
<tr>
<td>((a, b))</td>
<td>(P'(ka - a, kb - b))</td>
</tr>
</tbody>
</table>

**EXAMPLE 2**

**Dilating a Figure in the Coordinate Plane**

Graph \( \triangle ABC \) with vertices \( A(2, 1), B(4, 1) \), and \( C(4, -1) \) and its image after a dilation centered at \((0, 0)\) with a scale factor of 2.

**SOLUTION**

Use the coordinate rule for a dilation centered at \((0, 0)\) with \( k = 2 \) to find the coordinates of the vertices of the image. Then graph \( \triangle ABC \) and its image.

\[
\begin{align*}
(x, y) &\rightarrow (2x, 2y) \\
A(2, 1) &\rightarrow A'(4, 2) \\
B(4, 1) &\rightarrow B'(8, 2) \\
C(4, -1) &\rightarrow C'(8, -2)
\end{align*}
\]

**EXAMPLE 3**

**Dilating a Figure in the Coordinate Plane**

Graph quadrilateral \( KLMN \) with vertices \( K(-2, 8), L(1, 8), M(4, 5) \), and \( N(-2, -1) \) and its image after a dilation centered at \((1, 2)\) with a scale factor of \( \frac{3}{2} \).

**SOLUTION**

Use the coordinate rule for a dilation centered at \((a, b)\) with \( k = \frac{3}{2} \) to find the coordinates of the vertices of the image. Then graph quadrilateral \( KLMN \) and its image.

\[
\begin{align*}
(x, y) &\rightarrow \left(\frac{3}{2}(x - 1) + 1, \frac{3}{2}(y - 2) + 2\right) \\
K(-2, 8) &\rightarrow K'(0, 4) \\
L(1, 8) &\rightarrow L'(1, 4) \\
M(4, 5) &\rightarrow M'(2, 3) \\
N(-2, -1) &\rightarrow N'(0, 1)
\end{align*}
\]

**Monitoring Progress**

Graph \( \triangle PQR \) and its image after a dilation centered at \((0, 0)\) with scale factor \( k \).

2. \( P(-2, -1), Q(-1, 0), R(0, -1) \): \( C(0, 0) \), \( k = 4 \)
3. \( P(5, -5), Q(10, -5), R(10, 5) \): \( C(0, 0) \), \( k = 0.4 \)
4. \( P(-4, 6), Q(-2, 3), R(2, 8) \): \( C(-1, -3) \), \( k = 3 \)
5. \( P(-1, -2), Q(-1, 8), R(9, -2) \): \( C(-1, -8) \), \( k = 0.1 \)

**Differentiated Instruction**

**Auditory**

Some students may have difficulty distinguishing dilations from rigid transformations. Ask students for examples of dilations in the real world, and have them explain why these are dilations. Answers may include enlarging a photo, changing the size of a font on a computer, and zooming in on a digital image. Ensure student explanations mention that the size has changed, but relative positions in a figure have not.
Chapter 4

Transformations

In the coordinate plane, you can have scale factors that are negative numbers. When this occurs, the figure rotates 180°. So, when \(k > 0\), a dilation with a scale factor of \(-k\) is the same as the composition of a dilation with a scale factor of \(k\) followed by a rotation of 180° about the center of dilation.

Using the coordinate rules for a dilation centered at (0, 0) and a rotation of 180°, you can think of the notation as \((x, y) \rightarrow (kx, ky) \rightarrow (-kx, -ky)\).

Using a Negative Scale Factor

Graph \(\triangle FGH\) with vertices \(F(-4, -2)\), \(G(-2, 4)\), and \(H(-2, -2)\) and its image after a dilation centered at (0, 0) with a scale factor of \(-\frac{1}{2}\).

SOLUTION

Use the coordinate rule for a dilation with center (0, 0) and \(k = -\frac{1}{2}\) to find the coordinates of the vertices of the image. Then graph \(\triangle FGH\) and its image.

\[(x, y) \rightarrow \left(-\frac{1}{2}x, -\frac{1}{2}y\right)\]

\(F(-4, -2) \rightarrow F'(2, 1)\)

\(G(-2, 4) \rightarrow G'(1, -2)\)

\(H(-2, -2) \rightarrow H'(1, 1)\)

Monitoring Progress

6. Graph \(\triangle PQR\) with vertices \(P(1, 2)\), \(Q(3, 1)\), and \(R(1, -3)\) and its image after a dilation centered at (0, 0) with a scale factor of \(-2\).

7. Suppose a figure containing the origin is dilated with center of dilation \((0, 0)\). Explain why the corresponding point in the image of the figure is also the origin.
Solving Real-Life Problems

**EXAMPLE 5** Finding a Scale Factor

You are making your own photo stickers. Your photo is 4 inches by 4 inches. The image on the stickers is 1.1 inches by 1.1 inches. What is the scale factor of this dilation?

**SOLUTION**
The scale factor is the ratio of a side length of the sticker image to a side length of the original photo, or \( \frac{1.1 \text{ in.}}{4 \text{ in.}} \).

So, in simplest form, the scale factor is \( \frac{11}{40} \).

**EXAMPLE 6** Finding the Length of an Image

You are using a magnifying glass that shows the image of an object that is six times the object’s actual size. Determine the length of the image of the spider seen through the magnifying glass.

**SOLUTION**

\[
\frac{\text{image length}}{\text{actual length}} = k
\]

\[
\frac{x}{1.5} = 6
\]

\[
x = 9
\]

So, the image length through the magnifying glass is 9 centimeters.

**Monitoring Progress**

8. An optometrist dilates the pupils of a patient’s eyes to get a better look at the back of the eyes. A pupil dilates from 4.5 millimeters to 8 millimeters. What is the scale factor of this dilation?

9. The image of a spider seen through the magnifying glass in Example 6 is shown at the left. Find the actual length of the spider.

**Closure**

**Writing Prompt:** How are dilations alike/different from rigid transformations? **Sample answer:** They are alike because they both preserve angle measure. They are different because the lengths of the sides are congruent for rigid transformations and proportional for dilations.
ANSWERS
1. $P'(kx, ky)$
2. 60%; Because $0.6 < 1$, 60% is a scale factor for a reduction. The other three are scale factors for enlargements.
3. $\frac{3}{2}$; reduction
4. $\frac{5}{3}$; enlargement
5. $\frac{3}{2}$; reduction
6. $\frac{2}{3}$; enlargement
7–14. See Additional Answers.

15. 11. Center $C, k = 3$ 12. Center $P, k = \frac{1}{3}$ 13. Center $R, k = 0.25$ 14. Center $C, k = 75$

In Exercises 15–18, graph the polygon and its image after a dilation centered at $C$ with scale factor $k$.

16. 15. $X(6, –1), Y(–2, –4), Z(1, 2); C(0, 0), k = 3$
16. $A(0, 5), R(–10, –5), C(5, –5); C(0, 0), k = 120$
17. $T(7, 1), U(4, 4), V(1, 13), W(–2, 4); C(–2, 4), k = \frac{4}{3}$
18. $J(3, 1), K(5, –3), L(5, 5), M(3, 7); C(1, 1), k = 0.5$

In Exercises 19–22, graph the polygon and its image after a dilation centered at $0, 0$ with scale factor $k$.

19. $R(–5, –10), C(–10, 15), D(0, 5); k = \frac{1}{2}$
20. $L(0, 0), M(–4, 1), N(–3, –6); k = 3$
21. $R(–7, –1), S(2, 5), T(–2, –3), U(–3, –3); k = –4$
22. $W(8, –2), X(6, 0), Y(–6, 4), Z(–2, 2); k = –0.5$

20–22. See Additional Answers.

216 Chapter 4 Transformations
In Exercises 23 and 24, describe and correct the error in finding the scale factor of the dilation.

23. 

![Diagram]

In Exercises 25–28, the red figure is the image of the blue figure after a dilation with center C. Find the scale factor of the dilation. Then find the value of the variable.

25. 

![Diagram]

26. 

![Diagram]

27. 

![Diagram]

28. 

![Diagram]

In Exercises 31–34, you are using a magnifying glass. Use the length of the insect and the magnification level to determine the length of the image seen through the magnifying glass. (See Example 6.)

31. emperor moth  
   Magnification: 5×  
   Actual: 2 in.  
   Magnified: 15 in.  

32. ladybug  
   Magnification: 10×  
   Actual: 0.6 in.  
   Magnified: 6 in.

33. dragonfly  
   Magnification: 20×  
   Actual: 4.2 in.  
   Magnified: 84 in.

34. carpenter ant  
   Magnification: 15×  
   Actual: 1.5 in.  
   Magnified: 22.5 in.

35. ANALYZING RELATIONSHIPS Use the given actual and magnified lengths to determine which of the following insects were looked at using the same magnifying glass. Explain your reasoning.

   - grasshopper  
     Actual: 2 in.  
     Magnified: 15 in.

   - black beetle  
     Actual: 0.6 in.  
     Magnified: 4.2 in.

   - honeybee  
     Actual: 1/2 in.  
     Magnified: 15/2 in.

   - monarch butterfly  
     Actual: 3.9 in.  
     Magnified: 29.25 in.

In Exercises 31–34, you are using a magnifying glass. Use the length of the insect and the magnification level to determine the length of the image seen through the magnifying glass. (See Example 6.)

31. emperor moth  
   Magnification: 5×  
   Actual: 2 in.  
   Magnified: 15 in.

32. ladybug  
   Magnification: 10×  
   Actual: 0.6 in.  
   Magnified: 6 in.

33. dragonfly  
   Magnification: 20×  
   Actual: 4.2 in.  
   Magnified: 84 in.

34. carpenter ant  
   Magnification: 15×  
   Actual: 1.5 in.  
   Magnified: 22.5 in.

36. THOUGHT PROVOKING Draw ΔABC and ΔA′B′C′ so that ΔA′B′C′ is a dilation of ΔABC. Find the center of dilation and explain how you found it.

37. REASONING Your friend prints a 4-inch by 6-inch photo for you from the school dance. All you have is an 8-inch by 10-inch frame. Can you dilate the photo to fit the frame? Explain your reasoning.

ANSWERS

23. The scale factor should be calculated by finding $\frac{CP'}{CP}$, not $\frac{CP}{CP'}$; $k = \frac{3}{12} = \frac{1}{4}$

24. The scale factor should be calculated by finding the ratio of the length of a side of the image to the length of the corresponding side of the preimage; $k = \frac{5}{3}$, $x = 21$

25. $k = \frac{5}{3}$, $x = 21$

26. $k = 2$, $n = 6$

27. $k = \frac{2}{3}$, $y = 3$

28. $k = \frac{1}{4}$, $m = 16$

29. $k = 2$

30. $k = \frac{29}{17}$

31. 300 mm

32. 45 mm

33. 940 mm

34. 180 mm

35. grasshopper, honey bee, and monarch butterfly; The scale factor for these three is $k = \frac{15}{2}$. The scale factor for the black beetle is $k = 7$.

36. Sample answer:

![Graph]

37. no; The scale factor for the shorter sides is $\frac{3}{2} = 2$, but the scale factor for the longer sides is $\frac{10}{6} = \frac{5}{3}$. The scale factor for both sides has to be the same or the picture will be distorted.
ANSWERS

38. larger star; smaller star; Because the scale factor is between 0 and 1, the dilation is a reduction.

39. $x = 5, y = 25$

40. A figure that is 200% larger than the preimage will be twice as large.

41. original

42. dilated

43. original

44. dilated

45–50. See Additional Answers.

51. $A'(2, -5), B'(0, 0), C'(-3, 1)$

52. $A'(1, 2), B'(-1, 7), C'(-4, 8)$

53. $A'(-5, 2), B'(3, 3), C'(0, 4)$

54. $A'(0, 1), B'(-2, 4), C'(-5, 5)$

55. $A'(3, -3), B'(1, 2), C'(-2, 3)$

56. $A'(-1, 0), B'(-3, 5), C'(-6, 6)$

Mini-Assessment

1. Find the scale factor of the dilation. Then tell whether it is a reduction or an enlargement.

$k = \frac{5}{12}$; reduction

2. Graph $\triangle DEF$ with vertices $D(2, 6), E(2, 2)$, and $F(4, 2)$ and its image after a dilation centered at $(0, 0)$ with scale factor $\frac{1}{2}$.

3. A photographer enlarges a 4 inch $\times$ 5 inch photo to an 8 inch $\times$ 10 inch photo. What is the scale factor of the dilation? 2

45. ANALYZING RELATIONSHIPS Dilate the line through $O(0, 0)$ and $A(1, 2)$ using center $(0, 0)$ and a scale factor of 2.
   a. What do you notice about the lengths of $O'A'$ and $OA$?
   b. What do you notice about $O'A'$ and $OA$?

46. ANALYZING RELATIONSHIPS Dilate the line through $A(0, 1)$ and $B(1, 2)$ using center $(0, 0)$ and a scale factor of $\frac{1}{3}$.
   a. What do you notice about the lengths of $A'B'$ and $AB$?
   b. What do you notice about $A'B'$ and $AB$?

47. ATTENDING TO PRECISION You are making a blueprint of your house. You measure the lengths of the walls of your room to be 11 feet by 12 feet. When you draw your room on the blueprint, the lengths of the walls are 8.25 inches by 9 inches. What scale factor dilates your room to the blueprint?

48. MAKING AN ARGUMENT Your friend claims that dilating a figure by 1 is the same as dilating a figure by $-1$ because the original figure will not be enlarged or reduced. Is your friend correct? Explain your reasoning.

49. USING STRUCTURE Rectangle $WXYZ$ has vertices $W(-3, -1), X(-3, 3), Y(5, 3)$, and $Z(5, -1)$.
   a. Find the perimeter and area of the rectangle.
   b. Dilate the rectangle using center $(0, 0)$ and a scale factor of 3. Find the perimeter and area of the dilated rectangle. Compare with the original rectangle. What do you notice?
   c. Repeat part (b) using a scale factor of $\frac{1}{3}$.
   d. Make a conjecture for how the perimeter and area change when a figure is dilated.

50. REASONING You put a reduction of a page on the original page. Explain why there is a point that is in the same place on both pages.

Maintaining Mathematical Proficiency

The vertices of $\triangle ABC$ are $A(2, -1), B(0, 4)$, and $C(-3, 5)$. Find the coordinates of the vertices of the image after the translation. (Section 4.1)

51. $(x, y) \rightarrow (x - 1, y + 3)$

52. $(x, y) \rightarrow (x - 2, y)$

53. $(x, y) \rightarrow (x + 3, y - 1)$

54. $(x, y) \rightarrow (x + 1, y - 2)$

55. $(x, y) \rightarrow (x - 3, y + 1)$

56. $(x, y) \rightarrow (x - 3, y - 1)$

If students need help...

- Practice A and Practice B
- Puzzle Time

If students got it...

- Enrichment and Extension
- Cumulative Review

Student Journal

- Practice

Differentiating the Lesson

- Skills Review Handbook

Start the next Section
Essential Question: When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure?

Two figures are similar figures when they have the same shape but not necessarily the same size.

EXPLORATION 1  Dilations and Similarity

**Work with a partner.**

a. Use dynamic geometry software to draw any triangle and label it \(\triangle ABC\).

b. Dilate the triangle using center \((0, 0)\) and a scale factor of 3. Is the image similar to the original triangle? Justify your answer.

Sample

<table>
<thead>
<tr>
<th>Points</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(A(-2, 1))</td>
<td></td>
</tr>
<tr>
<td>(B(-1, -1))</td>
<td></td>
</tr>
<tr>
<td>(C(1, 0))</td>
<td></td>
</tr>
<tr>
<td>(D(0, 0))</td>
<td></td>
</tr>
<tr>
<td>Segments</td>
<td></td>
</tr>
<tr>
<td>(AB = 2.24)</td>
<td></td>
</tr>
<tr>
<td>(BC = 2.24)</td>
<td></td>
</tr>
<tr>
<td>(AC = 3.16)</td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td></td>
</tr>
<tr>
<td>(m\angle A = 45^\circ)</td>
<td></td>
</tr>
<tr>
<td>(m\angle B = 90^\circ)</td>
<td></td>
</tr>
<tr>
<td>(m\angle C = 45^\circ)</td>
<td></td>
</tr>
</tbody>
</table>

EXPLORATION 2  Rigid Motions and Similarity

**Work with a partner.**

a. Use dynamic geometry software to draw any triangle.

b. Copy the triangle and translate it 3 units left and 4 units up. Is the image similar to the original triangle? Justify your answer.

c. Reflect the triangle in the y-axis. Is the image similar to the original triangle? Justify your answer.

d. Rotate the original triangle 90° counterclockwise about the origin. Is the image similar to the original triangle? Justify your answer.

Communicate Your Answer

3. When a figure is translated, reflected, rotated, or dilated in the plane, is the image always similar to the original figure? Explain your reasoning.

4. A figure undergoes a composition of transformations, which includes translations, reflections, rotations, and dilations. Is the image similar to the original figure? Explain your reasoning.

4. Yes; According to Composition Theorem (Thm. 4.2), the composition of two or more rigid motions is a rigid motion. Also, a dilation preserves angle measures and results in an image with lengths proportional to the preimage lengths. So, a composition of rigid motions or dilations will result in an image that has angle measures congruent to the corresponding angle measures of the original figure, and sides that are either congruent or proportional to the corresponding sides of the original figure.

ANSWERS

1. a. Check students’ work.

b. Check students’ work; yes; Each side of \(\triangle A'B'C'\) is three times as long as its corresponding side of \(\triangle ABC\). The corresponding angles are congruent. Because the corresponding sides are proportional and the corresponding angles are congruent, the image is similar to the original triangle.

2. See Additional Answers.

3. Yes; The corresponding sides are always congruent or proportional, and the corresponding angles are always congruent.
Extra Example 1
Graph \( \overline{AB} \) with endpoints \( A(12, -6) \) and \( B(0, -3) \) and its image after the similarity transformation.

Reflection: in the \( y \)-axis
Dilation: center \((0, 0)\) and \( k = \frac{1}{3} \)

**MONITORING PROGRESS ANSWERS**

1. [Diagram of a graph showing points and axes]

2. [Diagram of a graph showing points and axes]

**EXAMPLE 1** Performing a Similarity Transformation

Graph \( \triangle ABC \) with vertices \( A(-4,1), B(-2,2), \) and \( C(-2,1) \) and its image after the similarity transformation.

**Translation:** \( (x, y) \rightarrow (x + 5, y + 1) \)

**Dilation:** center \((2, -1)\) and \( k = 2 \)

**SOLUTION**

**Step 1** Graph \( \triangle ABC \).

**Step 2** Translate \( \triangle ABC \) 5 units right and 1 unit up. \( \triangle A'B'C' \) has vertices \( A'(1,2), B'(3,3), \) and \( C'(3,2) \).

**Step 3** Dilate \( \triangle A'B'C' \) using center \((2, -1)\) and a scale factor of \( 2 \). \( \triangle A''B''C'' \) has endpoints \( A''(0,5), B''(4,7), \) and \( C''(4,5) \).

**Monitoring Progress**

1. Graph \( \overline{CD} \) with endpoints \( C(-2,2) \) and \( D(2,2) \) and its image after the similarity transformation.

   - **Rotation:** \( 90^\circ \) about the origin
   - **Dilation:** center \((0,0)\) and \( k = \frac{1}{2} \)

2. Graph \( \triangle FGH \) with vertices \( F(2,1), G(5,3), \) and \( H(3,-1) \) and its image after the similarity transformation.

   - **Reflection:** in the \( x \)-axis
   - **Dilation:** center \((1,-1)\) and \( k = 1.5 \)

**SUPPORTING English Language Learners**

Have students use the following Core Vocabulary as they read the topic Performing Similarity Transformations: *similarity transformation, similar figures*.

- **Beginning** Repeat the Core Vocabulary to improve pronunciation.
- **Intermediate** Read aloud one sentence with Core Vocabulary.
- **Advanced** Read a paragraph aloud.
- **Advanced High** Explain a sentence with Core Vocabulary using their own words.

**ELPS 3.D.1** Speak using grade-level content area vocabulary in context to internalize new English words.
Performing a Composition of Dilations

Graph \(\triangle MNP\) with vertices \(M(-6, 4), N(-4, 4),\) and \(P(2, 6)\) and its image after the similarity transformation.

**Dilation:**
- Center: \((2, 2)\) and \(k = \frac{1}{2}\)
- Center: \((1, 3)\) and \(k = 3\)

**SOLUTION**

**Step 1**
Graph \(\triangle MNP\).

**Step 2**
Dilate \(\triangle MNP\) using center \((2, 2)\) and \(k = \frac{1}{2}\).

\[
(x, y) \rightarrow \left(\frac{1}{2}x - 2\right) + 2, \frac{1}{2}(y - 2) + 2
\]

\(M(-6, 4) \rightarrow M'(2, 3)\)

\(N(-4, 4) \rightarrow N'(1, 3)\)

\(P(2, 6) \rightarrow P'(2, 4)\)

The vertices of \(\triangle M'N'P'\) are \(M'(2, 3), N'(1, 3),\) and \(P'(2, 4)\).

**Step 3**
Dilate \(\triangle M'N'P'\) using center \((1, 5)\) and \(k = 3\).

\[
(x, y) \rightarrow (3x - 1) + 1, 3(y - 5) + 5
\]

\(M'(2, 3) \rightarrow M''(-8, -1)\)

\(N'(1, 3) \rightarrow N''(-5, -1)\)

\(P'(2, 4) \rightarrow P''(4, 2)\)

The vertices of \(\triangle M''N''P''\) are \(M''(-8, -1), N''(-5, -1),\) and \(P''(4, 2)\).

---

**Extra Example 2**
Graph \(\triangle ABC\) with vertices \(A(-5, 0), B(-3, 0),\) and \(C(-3, 4)\) and its image after the similarity transformation.

**Dilation:**
- Center: \((-1, 1)\) and \(k = 2\)
- Center: \((-5, 7)\) and \(k = \frac{1}{4}\)

The vertices of \(\triangle A'B'C'\) are \(A'(-9, -1), B'(-5, -1),\) and \(C'(-5, 7)\).

---

**Monitoring Progress Answers**

3. Graph \(\triangle QRS\) with vertices \(Q(2, 4), R(4, 6),\) and \(S(-2, 2)\) and its image after the similarity transformation.

- **Dilation:**
  - Center: \((0, 0)\) and \(k = \frac{1}{2}\)
  - Center: \((0, 0)\) and \(k = 2\)

4. Graph \(\triangle TUV\) with vertices \(T(6, 8),\) \(U(8, 10),\) and \(V(12, 4)\) and its image after the similarity transformation.

- **Dilation:**
  - Center: \((-2, 1)\) and \(k = \frac{1}{2}\)
  - Center: \((3, 1)\) and \(k = 3\)

5. Graph \(WXYZ\) with vertices \(W(-6, -4), X(-2, -4), Y(-2, 2),\) and \(Z(-6, 2)\) and its image after the similarity transformation.

- **Dilation:**
  - Center: \((1, 0)\) and \(k = 2\)
  - Center: \((0, -1)\) and \(k = 3\)

---

**Differentiated Instruction**

**Organization**
Have students create a chart to compare the properties of congruence transformations and similarity transformations.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Congruence</th>
<th>Similarity ((k \neq 1, -1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preserves Angle Congruence</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preserves Length</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preserves Shape</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Preserves Size</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Describing Similarity Transformations

**Example 3** Describing a Similarity Transformation

Describe a similarity transformation that maps trapezoid PQRS to trapezoid WXYZ.

**SOLUTION**

QR falls from left to right, and XW rises from left to right. If you reflect trapezoid PQRS in the y-axis as shown, then the image, trapezoid \(P'Q'R'S'\), will have the same orientation as trapezoid WXYZ.

Trapezoid WXYZ appears to be about one-third as large as trapezoid \(P'Q'R'S'\). Dilate trapezoid \(P'Q'R'S'\) using center \((0, 0)\) and a scale factor of \(\frac{1}{3}\).

\[
(x, y) \rightarrow \left(\frac{1}{3}x, \frac{1}{3}y\right)
\]

\(P'(6, 3) \rightarrow P''(2, 1)\)

\(Q'(3, 3) \rightarrow Q''(1, 1)\)

\(R'(0, -3) \rightarrow R''(0, -1)\)

\(S'(6, -3) \rightarrow S''(2, -1)\)

The vertices of trapezoid \(P''Q''R''S''\) match the vertices of trapezoid WXYZ.

So, a similarity transformation that maps trapezoid PQRS to trapezoid WXYZ is a reflection in the y-axis followed by a dilation centered at \((0, 0)\) with a scale factor of \(\frac{1}{3}\).

**Monitoring Progress Answers**

6. Sample answer: reflection in the x-axis followed by a dilation with a scale factor of \(-\frac{1}{3}\)

7. Sample answer: dilation with a scale factor of \(\frac{1}{2}\) followed by a 180° rotation about the origin

**Closure**

- 3-2-1: Distribute a printed 3-2-1 reflection sheet and give time for students to reflect on their learning and to write.
- 3 new things (concepts, skills, procedures, …) I learned in this chapter
- 2 things (concepts, skills, procedures, …) I am still struggling with
- 1 thing that will help me tomorrow
Vocabulary and Core Concept Check

1. **VOCABULARY** What is the difference between similar figures and congruent figures?
2. **COMPLETE THE SENTENCE** A transformation that produces a similar figure, such as a dilation, is called a __________.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–14, graph \(\triangle FGH\) with vertices \(F(-2, 2), G(-2, -4),\) and \(H(-4, -4)\) and its image after the similarity transformation. (See Examples 1 and 2.)

3. Translation: \((x, y) \to (x + 3, y + 1)\)
   Dilation: center \((0, 0)\) and \(k = 2\)
4. Translation: \((x, y) \to (x - 2, y - 1)\)
   Dilation: center \((-1, 2)\) and \(k = 3\)
5. Reflection: in the y-axis
   Dilation: center \((2, 3)\) and \(k = \frac{1}{2}\)
6. Dilation: center \((0, 0)\) and \(k = \frac{1}{2}\)
   Reflection: in the y-axis
7. Dilation: center \((0, 0)\) and \(k = \frac{3}{4}\)
   Reflection: in the x-axis
8. Rotation: \(90^\circ\) about the origin
   Dilation: center \((0, 0)\) and \(k = 3\)
9. Rotation: \(180^\circ\) about the origin
   Dilation: center \((2, -2)\) and \(k = 2\)
10. Dilation: center \((0, 0)\) and \(k = \frac{1}{2}\)
    Dilation: center \((0, 0)\) and \(k = 3\)
11. Dilation: center \((0, 0)\) and \(k = 3\)
    Dilation: center \((0, 0)\) and \(k = 3\)
12. Dilation: center \((-2, 1)\) and \(k = \frac{3}{4}\)
    Dilation: center \((-1, -1)\) and \(k = 4\)
13. Dilation: center \((1, 3)\) and \(k = 2\)
    Dilation: center \((3, 2)\) and \(k = \frac{1}{2}\)

In Exercises 15 and 16, describe a similarity transformation that maps the blue preimage to the green image. (See Example 3.)

15.

![Similarity Transformation](image)

16.

![Similarity Transformation](image)

In Exercises 17–20, determine whether the polygons with the given vertices are similar. Use transformations to explain your reasoning.
17. \(A(6, 0), B(9, 6), C(12, 6)\) and \(D(0, 3), E(1, 5), F(2, 5)\)
18. \(Q(-1, 0), R(-2, 2), S(1, 3), T(2, 1)\) and \(W(0, 2), X(4, 4), Y(6, -2), Z(2, -4)\)
19. \(G(-2, 3), H(4, 3), J(4, 0)\) and \(K(1, 0), L(6, -2), M(1, -2)\)
20. \(P(-4, 3), Q(-4, 1), R(-1, 1), G(-4, 1)\) and \(L(1, -1), M(3, -1), N(6, -3), P(1, -3)\)

Section 4.6 Similarity and Transformations 223

**ANSWERS**

1. Congruent figures have the same size and shape. Similar figures have the same shape, but not necessarily the same size.
2. similarity transformation
3. ...
4. ...
5. ...
8–20. See Additional Answers.
21—32. See Additional Answers.

Mini-Assessment

1. \(AB\) has endpoints \(A(-8, 6)\) and \(B(6, 0)\). Find the endpoints of its image after the similarity transformation.
   Translation: \((x, y) \mapsto (x, y - 6)\)
   Dilation: center \((0, 0)\) and \(k = \frac{1}{2}\)

2. Graph \(\triangle PQR\) with vertices \(P(3, 2), Q(4, 0),\) and \(R(1, 0)\) and its image after the similarity transformation.
   Dilation: center \((0, 0)\) and \(k = 2\)
   Dilation: center \((6, 2)\) and \(k = \frac{1}{2}\)

3. Describe a similarity transformation that maps quadrilateral \(STUV\) to quadrilateral \(DEFG\).

Sample answer: A reflection in the \(x\)-axis followed by a dilation centered at \((0, 0)\) with a scale factor of 2

22. ERROR ANALYSIS Describe and correct the error in comparing the figures.

25. ANALYZING RELATIONSHIPS Graph a polygon in a coordinate plane. Use a similarity transformation involving a dilation (where \(k\) is a whole number) and a translation to graph a second polygon. Then, describe a similarity transformation that maps the second polygon onto the first.

26. THOUGHT PROVOKING Is the composition of a rotation and a dilation commutative? (In other words, do you obtain the same image regardless of the order in which you perform the transformations?) Justify your answer.

27. MATHEMATICAL CONNECTIONS Quadrilateral \(JKLM\) is mapped to quadrilateral \(J'K'L'M'\) using the dilation \((x, y) \mapsto \left(\frac{1}{3}x, \frac{1}{3}y\right)\). Then quadrilateral \(J'K'L'M'\) is mapped to quadrilateral \(J''K''L''M''\) using the translation \((x, y) \mapsto (x + 3, y - 4)\). The vertices of quadrilateral \(J'K'L'M'\) are \(J(-12, 0), K(-12, 18), L(-6, 18),\) and \(M(-6, 0)\). Find the coordinates of the vertices of quadrilateral \(JKLM\) and quadrilateral \(J''K''L''M''\) similar? Explain.

28. REPEATED REASONING Use the diagram.

a. Connect the midpoints of the sides of \(\triangle QRS\) to make another triangle. Is this triangle similar to \(\triangle QRS\)? Use transformations to support your answer.

b. Repeat part (a) for two other triangles. What conjecture can you make?

29. Classify the angle as acute, obtuse, right, or straight. (Section 1.5)

30. 

31. 

32. 

If students need help... | If students got it...
--- | ---
Resources by Chapter • Practice A and Practice B | Resources by Chapter • Enrichment and Extension • Cumulative Review
Student Journal • Practice | Start the next Section
Differentiating the Lesson Skills Review Handbook
4.4–4.6 What Did You Learn?

Core Vocabulary
- congruent figures, p. 204
- congruence transformation, p. 205
- dilation, p. 212
- center of dilation, p. 212
- scale factor, p. 212
- enlargement, p. 212
- reduction, p. 212
- similarity transformation, p. 220
- similar figures, p. 220

Core Concepts

Section 4.4
Identifying Congruent Figures, p. 204
Describing a Congruence Transformation, p. 205
Theorem 4.2 Reflections in Parallel Lines Theorem, p. 206
Theorem 4.3 Reflections in Intersecting Lines Theorem, p. 207

Section 4.5
Dilations and Scale Factor, p. 212
Coordinate Rules for Dilations, p. 213
Negative Scale Factors, p. 214

Section 4.6
Similarity Transformations, p. 220

Mathematical Thinking
1. Revisit Exercise 33 on page 210. Try to recall the process you used to reach the solution. Did you have to change course at all? If so, how did you approach the situation?

2. Describe a real-life situation that can be modeled by Exercise 28 on page 217.

Performance Task

The Magic of Optics

Look at yourself in a shiny spoon. What happened to your reflection? Can you describe this mathematically? Now turn the spoon over and look at your reflection on the back of the spoon. What happened? Why?

To explore the answers to these questions and more, go to BigIdeasMath.com.

ANSWERS
1. Sample answer: Draw a picture and label the given information. Then look at the results and try to figure out what needs to be proven in order to get there; yes; When unsure, look back at related definitions, postulates, and theorems to see which ones might be helpful. Points $L$ and $M$ must be identified, so use the Ruler Postulate (Post. 1.1) and Segment Addition Postulate (Post. 1.2). Then the rest will start falling into place.

2. Sample answer: This drawing could represent the reduction of a $16 \times 28$ painting into a $4 \times 7$ photograph or computer graphic.
4.1 Translations (pp. 177–184)

Graph quadrilateral $ABCD$ with vertices $A(1, -2)$, $B(3, -1)$, $C(0, 3)$, and $D(-4, 1)$ and its image after the translation $(x, y) \rightarrow (x + 2, y - 2)$.

Graph quadrilateral $ABCD$. To find the coordinates of the vertices of the image, add 2 to the $x$-coordinates and subtract 2 from the $y$-coordinates of the vertices of the preimage. Then graph the image.

$(x, y) \rightarrow (x + 2, y - 2)$

$A(1, -2) \rightarrow A'(3, -4)$

$B(3, -1) \rightarrow B'(5, -3)$

$C(0, 3) \rightarrow C'(2, 1)$

$D(-4, 1) \rightarrow D'(-2, -1)$

Graph $\triangle XYZ$ with vertices $X(2, 3)$, $Y(-3, 2)$, and $Z(-4, -3)$ and its image after the translation.

1. $(x, y) \rightarrow (x, y + 2)$
2. $(x, y) \rightarrow (x - 3, y)$
3. $(x, y) \rightarrow (x + 3, y - 1)$
4. $(x, y) \rightarrow (x + 4, y + 1)$

Graph $\triangle PQR$ with vertices $P(0, -4)$, $Q(1, 3)$, and $R(2, -5)$ and its image after the composition.

5. Translation: $(x, y) \rightarrow (x + 1, y + 2)$
6. Translation: $(x, y) \rightarrow (x, y + 3)$
7. Translation: $(x, y) \rightarrow (x - 4, y + 1)$
8. Translation: $(x, y) \rightarrow (x - 1, y + 1)$

4.2 Reflections (pp. 185–192)

Graph $\triangle ABC$ with vertices $A(1, -1)$, $B(3, 2)$, and $C(4, -4)$ and its image in the line $y = x$.

Graph $\triangle ABC$. Then use the coordinate rule for reflecting in the line $y = x$ to find the coordinates of the endpoints of the image.

$(a, b) \rightarrow (b, a)$

$A(1, -1) \rightarrow A'(-1, 1)$

$B(3, 2) \rightarrow B'(2, 3)$

$C(4, -4) \rightarrow C'(-4, 4)$

Graph the polygon and its image after a reflection in the given line.

7. $x = 4$
8. $y = 3$
9. How many lines of symmetry does the figure have?
4.3 Rotations (pp. 193–200)

Graph \( \triangle LMN \) with vertices \( L(1, -1), M(2, 3), \) and \( N(4, 0) \) and its image after a 270° rotation about the origin.

Use the coordinate rule for a 270° rotation to find the coordinates of the vertices of the image. Then graph \( \triangle LMN \) and its image.

\[
(a, b) \to (b, -a)
\]

\[L(1, -1) \to L'(-1, -1)\]
\[M(2, 3) \to M'(3, -2)\]
\[N(4, 0) \to N'(0, -4)\]

Graph the polygon with the given vertices and its image after a rotation of the given number of degrees about the origin.

10. \( A(-3, -1), B(2, 2), C(3, -3); 90° \)
11. \( W(-2, -1), X(-1, 3), Y(3, 3), Z(3, -3); 180° \)
12. Graph \( XY \) with endpoints \( X(5, -2) \) and \( Y(3, -3) \) and its image after a reflection in the \( x \)-axis and then a rotation of 270° about the origin.

Determine whether the figure has rotational symmetry. If so, describe any rotations that map the figure onto itself.

13. 
14.

4.4 Congruence and Transformations (pp. 203–210)

Describe a congruence transformation that maps quadrilateral \( ABCD \) to quadrilateral \( WXYZ \), as shown at the right.

\( AB \) falls from left to right, and \( WX \) rises from left to right. If you reflect quadrilateral \( ABCD \) in the \( x \)-axis as shown at the bottom right, then the image, quadrilateral \( A'B'C'D' \), will have the same orientation as quadrilateral \( WXYZ \). Then you can map quadrilateral \( A'B'C'D' \) to quadrilateral \( WXYZ \) using a translation of 5 units left.

\( \Rightarrow \) So, a congruence transformation that maps quadrilateral \( ABCD \) to quadrilateral \( WXYZ \) is a reflection in the \( x \)-axis followed by a translation of 5 units left.

Describe a congruence transformation that maps \( \triangle DEF \) to \( \triangle JKL \).

15. \( D(2, -1), E(4, 1), F(1, 2) \) and \( J(-2, -4), K(-4, -2), L(-1, -1) \)
16. \( D(-3, -4), E(-5, -1), F(-1, 1) \) and \( J(1, 4), K(-1, 1), L(3, -1) \)
17. Which transformation is the same as reflecting an object in two parallel lines? in two intersecting lines?
ANSWERS

18.

19.

20. 1.9 cm

21. Sample answer: reflection in the line $x = -1$ followed by a dilation with center $(-3, 0)$ and $k = 3$

22. Sample answer: dilation with center at the origin and $k = \frac{1}{2}$, followed by a reflection in the line $y = x$

23. Sample answer: 270° rotation about the origin followed by a dilation with center at the origin and $k = 2$

4.5 Dilations (pp. 211–218)

Graph trapezoid $ABCD$ with vertices $A(1, 1)$, $B(1, 3)$, $C(3, 2)$, and $D(3, 1)$ and its image after a dilation centered at $(0, 0)$ with a scale factor of 2.

Use the coordinate rule for a dilation centered at $(0, 0)$ with a scale factor of $\frac{1}{2}$.

$$
(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)
$$

Then graph trapezoid $ABCD$ and its image.

$$
A(1, 1) \rightarrow A'(2, 2)
$$

$$
B(1, 3) \rightarrow B'(2, 6)
$$

$$
C(3, 2) \rightarrow C'(6, 4)
$$

$$
D(3, 1) \rightarrow D'(6, 2)
$$

Graph the triangle and its image after a dilation with scale factor $k$.

18. $P(3, 3)$, $Q(5, 5)$, $R(9, 3)$; $C(1, 1)$, $k = \frac{1}{3}$

19. $X(-3, 2)$, $Y(2, 3)$, $Z(1, -1)$; $C(0, 0)$, $k = -3$

20. You are using a magnifying glass that shows the image of an object that is eight times the object’s actual size. The image length is 15.2 centimeters. Find the actual length of the object.

4.6 Similarity and Transformations (pp. 219–224)

Describe a similarity transformation that maps $\triangle FGH$ to $\triangle LMN$, as shown at the right.

$FG$ is horizontal, and $LM$ is vertical. If you rotate $\triangle FGH$ 90° about the origin as shown at the bottom right, then the image, $\triangle F'G'H'$, will have the same orientation as $\triangle LMN$. $\triangle LMN$ appears to be half as large as $\triangle F'G'H'$. Dilate $\triangle F'G'H'$ using center $(0, 0)$ and a scale factor of $\frac{1}{2}$.

$$
(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)
$$

$F'(-2, 2) \rightarrow F''(-1, 1)$

$G'(-2, 6) \rightarrow G''(-1, 3)$

$H'(-6, 4) \rightarrow H''(-3, 2)$

The vertices of $\triangle F'G'H'$ match the vertices of $\triangle LMN$.

So, a similarity transformation that maps $\triangle FGH$ to $\triangle LMN$ is a rotation of 90° about the origin followed by a dilation centered at $(0, 0)$ with a scale factor of $\frac{1}{2}$.

Describe a similarity transformation that maps $\triangle ABC$ to $\triangle RST$.

21. $A(1, 0)$, $B(-2, -1)$, $C(-1, -2)$ and $R(-3, 0)$, $S(6, -3)$, $T(3, -6)$

22. $A(6, 4)$, $B(-2, 0)$, $C(-4, 2)$ and $R(2, 3)$, $S(0, -1)$, $T(1, -2)$

23. $A(3, -2)$, $B(0, 4)$, $C(-1, -3)$ and $R(-4, -6)$, $S(8, 0)$, $T(-6, 2)$
Graph \( \triangle RST \) with vertices \( R(-4, 1), S(-2, 2), \) and \( T(3, -2) \) and its image after the translation.

1. \((x, y) \rightarrow (x - 4, y + 1)\)

2. \((x, y) \rightarrow (x + 2, y - 2)\)

Graph the polygon with the given vertices and its image after a rotation of the given number of degrees about the origin.

3. \(D(-1, -1), E(-3, 2), F(1, 4); \) \(270^\circ\)

4. \(J(-1, 1), K(3, 3), L(4, -3), M(0, -2); \) \(90^\circ\)

Determine whether the polygons with the given vertices are congruent or similar. Use transformations to explain your reasoning.

5. \(Q(2, 4), R(5, 4), S(6, 2), T(1, 2) \) and \( W(6, -12), X(15, -12), Y(18, -6), Z(-5, -6) \)

6. \(A(-6, 6), B(-6, 2), C(-2, -4) \) and \( D(9, 7), E(5, 7), F(-1, 3)\)

Determine whether the object has line symmetry and whether it has rotational symmetry. Identify all lines of symmetry and angles of rotation that map the figure onto itself.

7. \(\text{Basketball} \)

8. \(\text{Guitar} \)

9. \(\text{Card} \)

10. Draw a diagram using a coordinate plane, two parallel lines, and a parallelogram that demonstrates the Reflections in Parallel Lines Theorem (Theorem 4.2).

11. A rectangle with vertices \( W(-2, 4), X(2, 4), Y(2, 2), \) and \( Z(-2, 2) \) is reflected in the \( y\)-axis. Your friend says that the image, rectangle \( W'X'Y'Z' \), is exactly the same as the preimage. Is your friend correct? Explain your reasoning.

12. Write a composition of transformations that maps \( \triangle ABC \) onto \( \triangle CDB \) in the tessellation shown. Is the composition a congruence transformation? Explain your reasoning.

13. There is one slice of a large pizza and one slice of a small pizza in the box.
   a. Describe a similarity transformation that maps pizza slice \( ABC \) to pizza slice \( DEF \).
   b. What is one possible scale factor for a medium slice of pizza? Explain your reasoning. (Use a dilation on the large slice of pizza.)

14. The original photograph shown is 4 inches by 6 inches.
   a. What transformations can you use to produce the new photograph?
   b. You dilate the original photograph by a scale factor of \( \frac{1}{2} \). What are the dimensions of the new photograph?
   c. You have a frame that holds photos that are 8.5 inches by 11 inches. Can you dilate the original photograph to fit the frame? Explain your reasoning.

5. similar; Quadrilateral \( QRST \) can be mapped to quadrilateral \( WXYZ \) by a dilation with the center at the origin and \( k = 3 \), followed by a reflection in the \( x\)-axis. Because this composition has a rigid motion and a dilation, it is a similarity transformation.

6. congruent; \( \triangle ABC \) can be mapped to \( \triangle DEF \) by a \(270^\circ\) rotation about the origin followed by a translation 1 unit up and 3 units right. Because this is a composition of two rigid motions, the composition is rigid.

7. yes, yes; The lines of symmetry are vertically through the center of the ball and horizontally through the center of the ball; \(180^\circ\)

8. yes, no; The line of symmetry runs from the center of the base of the guitar, and through the sound hole to the center of the headstock of the guitar.

9. no, yes; \(180^\circ\)

10–14. See Additional Answers.
1. Which composition of transformations maps \( \triangle ABC \) to \( \triangle DEF \) (TEKS G.3.C)

   \( \text{A) Rotation: } 90° \text{ counterclockwise about the origin} \)
   \( \text{Translation: } (x, y) \rightarrow (x + 4, y - 3) \)

   \( \text{B) Translation: } (x, y) \rightarrow (x - 4, y - 3) \)
   \( \text{Rotation: } 90° \text{ counterclockwise about the origin} \)

   \( \text{C) Translation: } (x, y) \rightarrow (x + 4, y - 3) \)
   \( \text{Rotation: } 90° \text{ counterclockwise about the origin} \)

   \( \text{D) Rotation: } 90° \text{ counterclockwise about the origin} \)
   \( \text{Translation: } (x, y) \rightarrow (x - 4, y - 3) \)

2. The directed line segment \( ST \) has endpoints \( S(-3, -2) \) and \( T(4, 5) \). Point \( P \) lies on the directed line segment \( ST \) and the ratio of \( SP \) to \( PT \) is 3 to 4. What are the coordinates of point \( P \)? (TEKS G.2.A)

   \( \text{F} \) \((0, 1)\)
   \( \text{G} \) \(\left(\frac{21}{4}, \frac{31}{4}\right)\)
   \( \text{H} \) \((7, 8)\)
   \( \text{J} \) \(\left(\frac{9}{4}, \frac{10}{4}\right)\)

3. In the quilt pattern, which of the following transformations describes a rotation? (TEKS G.3.C)

   \( \text{A) Figure A to Figure B} \)
   \( \text{B) Figure A to Figure C} \)
   \( \text{C) Figure A to Figure D} \)
   \( \text{D) Figure B to Figure C} \)

4. A parallelogram with vertices \( J(1, 0) \), \( K(4, 1) \), \( L(5, -1) \), and \( M(2, -2) \) is rotated 90° about the origin. Which of the vertices is not correct? (TEKS G.3.A)

   \( \text{F} \) \(J'(0, 1)\)
   \( \text{G} \) \(K'(1, -4)\)
   \( \text{H} \) \(L'(1, 5)\)
   \( \text{J} \) \(M'(2, 2)\)

ANSWERS

1. B
2. F
3. B
4. G

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5. Which equation represents the line passing through the point (−6, 3) that is perpendicular to the line \( y = −\frac{1}{3}x - 5 \)? (TEKS G.2.C)
   
   \( \text{A} \) \( y = 3x + 21 \) \hspace{2cm} \( \text{B} \) \( y = −\frac{1}{3}x - 5 \)
   \( \text{C} \) \( y = 3x - 15 \) \hspace{2cm} \( \text{D} \) \( y = −\frac{1}{3}x + 1 \)

6. Which of the quadrilaterals are congruent to one another? (TEKS G.6.C)
   
   \( \text{F} \) \( ABCD \) and \( WXYZ \)
   \( \text{G} \) \( WXYZ \) and \( QRST \)
   \( \text{H} \) \( ABCD, WXYZ, \) and \( QRST \)
   \( \text{J} \) none of the above

7. GRIDDED ANSWER The vertices of \( \triangle ABC \) are \( A(4, 4), B(4, 1), \) and \( C(2, 1) \). The vertices of the image of the triangle after a dilation centered at \( (3, 0) \) are \( A'(6, 12), B'(6, 3), \) and \( C'(0, 3) \). What is the scale factor of the dilation? (TEKS G.3.C)

8. Your friend makes the statement shown. This statement is an example of which of the following? (TEKS G.4.A)
   
   “The product of two numbers that are both less than one is less than both of the numbers.”
   
   \( \text{A} \) a definition \hspace{2cm} \( \text{B} \) a postulate
   \( \text{C} \) a conjecture \hspace{2cm} \( \text{D} \) a theorem

9. The diagram shows a carving on a door frame. \( \angle HGD \) and \( \angle HGF \) are right angles, \( m\angle DGB = 21^\circ, m\angle HBG = 55^\circ, \angle DGB = \angle CGF, \) and \( \angle HBG = \angle HCG \). What is \( m\angle HGC \)? (TEKS G.5.C)
   
   \( \text{F} \) \( 21^\circ \) \hspace{2cm} \( \text{G} \) \( 69^\circ \)
   \( \text{H} \) \( 111^\circ \) \hspace{2cm} \( \text{J} \) \( 159^\circ \)