BIL Counter Evidence to Ed Reports Alignment
©2019 Big Ideas Math: Modeling Real Life, Grade 6

<table>
<tr><td colspan="2">GATEWAY TWO: Rigor and Mathematical Practices</td></tr>
<tr><td colspan="2">Rigor and Balance
Each grade's instructional materials reflect the balances in the Standards and help students meet the Standards' rigorous expectations, by helping students develop conceptual understanding, procedural skill and fluency, and application.</td></tr>
<tr><td colspan="2">Indicator 2a -- Attention to conceptual understanding: Materials develop conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings.</td></tr>
<tr><td>Ed Reports Review</td><td>BIL Counter Evidence</td></tr>
<tr><td>Indicator 2a --The instructional materials do not always provide students opportunities to independently demonstrate conceptual understanding throughout the grade-level. The shift from conceptual understanding, most prevalent in the Exploration Section, to procedural understanding is completed within the lesson. The Examples and Concepts, Skills, and Problem Solving sections have a focus that is primarily procedural with limited opportunities to demonstrate conceptual understanding.</td><td>Students have opportunity to independently demonstrate conceptual understanding in a variety of ways. We intentionally include conceptual problems throughout the Self Assessment for Concepts & Skills and Concepts, Skills, & Problem Solving practice sets. For example:

Self Assessment for Concepts & Skills
• 3.2 #4-6, page 117
• 3.4 #5, page 131
• 5.3 #10, page 217
• 5.4 #7, page 223
• 5.4 Laurie's Notes page T-223

Concepts, Skills, & Problem Solving
• 3.3 #38, page 128
• 5.4 #12-13, page 225
• 5.5 #15, 16, 33, page 231</td></tr>
<tr><td colspan="2">Indicator 2c -- Attention to Applications: Materials are designed so that teachers and students spend sufficient time working with engaging applications of the mathematics, without losing focus on the major work of each grade.</td></tr>
<tr><td>Ed Reports Review</td><td>BIL Counter Evidence</td></tr>
<tr><td>Indicator 2c -- The instructional materials present opportunities for students to engage in application of grade-level mathematics; however the problems are scaffolded through teacher-led questions and procedural explanation. The last example of each lesson is titled, "Modeling Real Life," which provides a real-life problem involving the key standards addressed for each lesson. This section provides a step-by-step solution for the problem; therefore, students do not fully engage in application. In addition, there are few non-routine problems presented.... Overall, there are limited opportunities for students to engage in non-routine problems throughout the grade level.</td><td>Students have abundant opportunity to engage in routine and non-routine application problems throughout the grade level. Examples of non-routine problems:

Self Assessment for Problem Solving
• 2.5 #19, page 77
• 3.6 #10, page 145
• 4.2 #18, page 172
• 5.1 #25, page 205
Concepts, Skills, & Problem Solving
• 3.1 #50, page 114
• 3.2 #32, page 120
• 6.1 #30, page 250
• 6.3 #44, page 264
Connecting Concepts
• Chapter 3 #2, page 149
• Chapter 6 #3, page 273
• Chapter 8 #2, page 399</td></tr>
</table>

Boldfaced italic type indicates pages where additional explanation is annotated on the supporting pages of this document.

GATEWAY TWO: Rigor and Mathematical Practices	
Indicator 2d -- Balance: The three aspects of rigor are not always treated together and are not always treated separately. There is a balance of the 3 aspects of rigor within the grade.	
Ed Reports Review	**BIL Counter Evidence**
Indicator 2d -- The instructional materials for Big Ideas Math: Modeling in Real Life, Grade 6 partially meet expectations that the three aspects of rigor are not always treated together and are not always treated separately.	The Big Ideas Math: Modeling Real Life program consistently across Grades K-8 strives for a balanced approach to rigor. Each section develops a concept from conceptual understanding (explorations) to procedural fluency (skill examples) to rigorous application (Modeling Real Life examples), engaging students in the mathematics and promoting active learning. Every set of practice problems reflects this balance, giving students the rigorous practice they need to achieve mastery. The Teaching Edition front matter was updated in a recent printing to provide detail on the program philosophy concerning rigor. • ***Front matter, page xxiii***
Indicator 2d -- The instructional materials present opportunities in most lessons for students to engage in each aspect of rigor, however, these are often treated together. There is also an over-emphasis on procedural skill and fluency.	The following are examples where conceptual understanding is treated by itself or is the focus. Chapter Explorations • Chapter 2 Exploration #1-8, page 44 • ***Chapter 3 Exploration #1-14, page 106*** • Chapter 6 Exploration #1-9, page 244 The following are examples where application is treated by itself or is the focus. • Connecting Concepts and Performance Task ♦ Chapter 1, page 33 ♦ Chapter 2, page 95 ♦ Chapter 5, page 233 ♦ ***Chapter 6, page 273*** ♦ Chapter 8, page 399 ♦ Chapter 9, page 445 • Chapters 9 and 10 ♦ All of the lessons in these chapters focus on real-life application.

GATEWAY TWO: Rigor and Mathematical Practices

Mathematical Practice - Content Connections

Materials meaningfully connect the Standards for Mathematical Content and the Standards for Mathematical Practice.

Indicator 2e -- The Standards for Mathematical Practice are identified and used to enrich mathematics content within and throughout each applicable grade.

Ed Reports Review	BIL Counter Evidence
Indicator 2e -- The MPs are identified within some sections in both the Teaching Edition (Exploration and Example sections) and Student Edition (Exploration 1 [within blue boxes], Concept, Skills, & Problem Solving section). In the Student Edition, MPs are labeled with "MP" and a shortened version of the MP, such as "Structure, Reasoning, Construct Arguments, Precision, etc." There is no document that correlates the abbreviated titles with the Standards for Mathematical Practice. For example, the label "MP Number Sense" could align to several MPs. Additionally, Big Ideas Math: Modeling in Real Life, Grade 6 added "MP Logic" as a Mathematical Practice. This added practice does not align with the CCSSM Standards for Mathematical Practice.	We will provide a correlation online to students and teachers at bigideasmath.com, aligning the MP labels and headings in the student edition with the Standards for Mathematical Practice. The correlation will be included in future printings, and is provided below. **MP1 Make sense of problems and persevere in solving them.** • Problem Solving **MP2 Reason abstractly and quantitatively.** • Reasoning • Number Sense **MP3 Construct viable arguments and critique the reasoning of others.** • Construct Arguments • Logic • You Be The Teacher • Different Words, Same Question • Which One Doesn't Belong? **MP4 Model with mathematics.** • Modeling Real Life • Problem Solving **MP5 Use appropriate tools strategically.** • Choose Tools • Using Tools **MP6 Attend to precision.** • Precision **MP7 Look for and make use of structure**. • Structure • Patterns **MP8 Look for and express regularity in repeated reasoning.** • Repeated Reasoning

GATEWAY TWO: Rigor and Mathematical Practices

Indicator 2f -- Materials carefully attend to the full meaning of each practice standard

Ed Reports Review	BIL Counter Evidence
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP1: Make Sense of Problems and Persevere in Solving Them. The instructional materials present few opportunities for students to make sense of problems and persevere in solving them.	While the examples are stepped out for students, they illustrate opportunities for students to make sense of problems and persevere in solving them when they independently solve the related problems. Students are encouraged to use the methods shown and the Problem-Solving Plan to think through and solve problems. For example: • 1.5 Example 4 & Self-Assessment for Prob. Solving #17-19, page 30 • ***3.3 Example 4 & Self-Assessment for Prob. Solving #9-10, page 125*** • 4.4 Example 6 & Self-Assessment for Prob. Solving #15-17, page 185 Connecting Concepts pages at the end of each chapter encourage students to make sense of problems and persevere in solving them. For example: • ***Chapter 2, page 95*** • Chapter 4, page 189 • Chapter 9, page 445 • Chapter 10, page 491 All non-routine problems listed under Indicator 2c also cover MP1. Teaching Edition notes labeled with MP1 give opportunities for the teacher to emphasize these habits to students and for students to use them going forward. For example: • ***1.4 page T-21*** • 2.5 page T-73 • 4.4 page T-184 The Teaching Edition front matter was updated in a recent printing to emphasize opportunities for in-class problem solving throughout the program. • ***Front matter, page xxiv***
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP4: Model with mathematics. The instructional materials present few opportunities for students to model with mathematics. Many MP4 notations are used in the Example sections throughout the grade level. In these examples, the work and a step-by-step description is provided for the student, eliminating students' use of models.	Modeling with mathematics is covered throughout our program. Every Modeling Real Life example is directly followed by corresponding problems for students to engage in MP4. In addition, every Concepts, Skills, & Problem-Solving set contains multiple opportunities for students to model with mathematics in the Modeling Real Life exercises. For example: Try Its and Self-Assessments: • 1.2 Self-Assessment #16-18, page 12 • ***3.2 Self-Assessment #9-11, page 118*** • 3.5 Try It #2, page 137 • 5.2 Self-Assessment #12-13, page 212 Concepts, Skills, & Prob-Solving: • 1.2 #41,42,44,45, page 14 • 3.2 #27-32, page 120 • 3.5 #28,29,34-39, page 140 • ***5.2 #30,39-41, page 214***

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GATEWAY TWO: Rigor and Mathematical Practices	
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP5: Use appropriate tools strategically. The instructional materials present few opportunities for students to choose their own tool, therefore, the full meaning of MP5 is not attended to. The instructional materials present limited opportunities for students to choose tools strategically, as the materials indicate what tools should be used.	Students to have opportunity to choose tools strategically. For example: • ***2.1 Explorations 1-2, page 45*** • 3.4 #24, page 134: Students choose which tool they prefer. • 3.6 Exploration 2, page 141: The Math Practice note asks students when certain tools are useful and not useful. In the Dynamic Student Edition, students have access to the following math tools at all times. • Algebra tiles • Balance scale • Desmos geometry tool • Desmos graphing calculator • Four function calculator • Fraction models • Number line • Place value • Probability tools • Scientific calculator • Simulation
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP6: Attend to Precision. The instructional materials do not support students to attend to precision. In most instances, teachers attend to precision for students.	The Student Edition often has opportunity for students to attend to precision. For example: • 3.5 Exploration 1 Math Practice note, page 135 • ***5.2 Self-Assessment for Concepts & Skills #11 page 211*** • 7.7 Exploration 1 Math Practice note, page 325 • 7.7 Self-Assessment for Concepts & Skills #5, page 327 • 8.7 Exploration 1 Math Practice note, page 383 While the MP6 notes in the Teaching Edition discuss the mathematics of the examples, the note exists for the teacher to encourage precision in the related student work that directly follows, in the Try Its, Self-Assessment for Concepts & Skills, and Concepts, Skills, & Problem Solving practice problems. For example: • ***1.4 Laurie's Notes page T-22*** • 4.4 Laurie's Notes page T-184 • 5.1 Laurie's Notes page T-204
Indicator 2f -- The instructional materials do not present opportunities for students to engage in MP7: Look for and make use of structure. The instructional materials often label content MP7 Structure, but the teaching notes and problems do not attend to the full meaning of the MP.	As stated in the EdReports Math Grades K-8 Evidence Guides, "Every instance of an MP being marked does not necessarily have to encompass the full meaning of an MP, but taken together there should be evidence that the materials carefully attend to the full meaning of each practice standard." Below are several examples across Grade 6 showing opportunities for students to practice the various aspects of MP7. • ***2.2 Exploration 2, page 53*** • 3.1 #49, page 114: Students apply ratio understanding to 3 numbers. • 3.4 #26, page 134 • 4.2 Exploration 1 Math Practice note, page 169 • 4.4 Self-Assessment for Concepts & Skills #14, page 184 • 6.4 Self-Assessment for Concepts & Skills #17, page 254 • 7.2 #29, page 296

GATEWAY TWO: Rigor and Mathematical Practices

Indicator 2g.i -- Materials prompt students to construct viable arguments and analyze the arguments of others concerning key grade-level mathematics detailed in the content standards.

Ed Reports Review	BIL Counter Evidence
Indicator 2g.i -- The Student Edition labels MP3 as "MP Construct Arguments," however, these activities do not always require students to construct arguments. "Construct Arguments" was labeled only twice for students and "Build Arguments" was labeled once for students.	We suggest that when explaining or comparing answers, students must use what they have learned in building a logical progression of statements that defends their answer. The ability to critique someone else's reasoning also helps students analyze their own work and formulate good explanations. For example: • 1.2 Self-Assessment for Concepts & Skills #13, page 11 • 1.3 #67, page 20 • 1.4 Exploration 2d, page 21 • 1.5 Exploration 2c, page 27 • ***1.5 #45-47, page 32*** • 2.3 #45, page 66 • 2.7 # 75, page 94 • 3.1 #46, page 114 • 4.2 Self-Assessment for Concepts & Skills #16, page 171 • 4.3 #44, page 180 • 5.1 Self-Assessment for Concepts & Skills #21, page 204 • 5.3 Exploration 1b, page 215 • 6.2 Self-Assessment for Concepts & Skills #15, page 254 • 7.1 Exploration 1, page 285 • 7.2 Exploration 1, page 291 • 7.3 Exploration 1, page 297

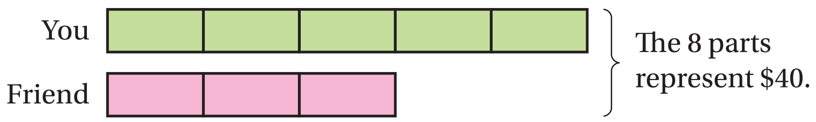
Indicator 2g.ii -- Materials assist teachers in engaging students in constructing viable arguments and analyzing the arguments of others concerning key grade-level mathematics detailed in the content standards.

Ed Reports Review	BIL Counter Evidence
Indicator 2g.ii -- There are some missed opportunities where the materials could assist teachers in engaging students teachers in both constructing viable arguments and analyzing the arguments of others.	The Teaching Edition contains many instances of guidance, along with probing questions the teacher can ask, to engage students in constructing arguments and analyzing the arguments of others. These are often indicated with either a MP3 inline head or a red "?" icon. For example: MP3 inline head • ***1.2 page T-10*** • 1.5 page T-29 • 2.3 page T-61 • 3.3 page T-121 • 5.3 pages T-215, T-217 Red "?" icon • 1.1 page T-5 • 1.4 page T-22 • 2.2 page T-55 • 2.7 page T-88 • 3.1 page T-107 • ***3.3 page T-125*** • 5.3 page T-218

EXAMPLE 3 Using a Tape Diagram to Solve a Ratio Problem

The ratio of your monthly allowance to your friend's monthly allowance is 5 : 3. The monthly allowances total $40. How much is each allowance?

Represent the ratio 5 : 3 using a tape diagram.



1 part represents $40 ÷ 8 = $5.

5 parts represent 5 × $5 = $25.

3 parts represent 3 × $5 = $15.

So, your allowance is $25, and your friend's allowance is $15.

Check Verify that the ratio of allowances is 5 : 3.

$$5 : 3 \xrightarrow{\times 5,\ \times 5} 25 : 15 \checkmark$$

Try It

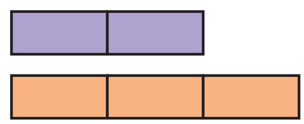
3. **WHAT IF?** Repeat Example 3 when the ratio of your monthly allowance to your friend's monthly allowance is 2 to 3.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

Indicator 2a - In #4-6, students demonstrate their conceptual understanding of ratios by using tape diagrams.

4. MP **STRUCTURE** What ratio is represented by the tape diagram? Can you use the tape diagram to model the ratio 6 : 9? Can you use the tape diagram to model the ratio 8 : 16? Explain your reasoning.



5. MP **REASONING** You are given a tape diagram and the total value of the parts. How can you find the value of 1 part?

6. **DRAWING A TAPE DIAGRAM** Describe two ways that you can represent the ratio 12 : 4 using a tape diagram.

USING A TAPE DIAGRAM **You are given the number of tickets in a bag and the ratio of winning tickets to losing tickets. How many of each kind of ticket are in the bag?**

7. 35 tickets; 1 to 4

8. 80 tickets; 2 : 8

EXAMPLE 2 Using a Graph to Solve a Ratio Problem

You buy dark chocolate cashews for $12.50 per pound.

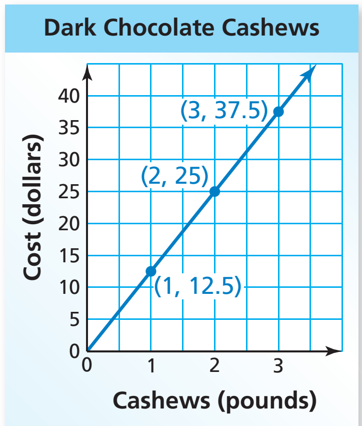
Relationships involving cost are often graphed with cost on the vertical axis.

a. Represent the ratio relationship using a graph.

Create a ratio table.

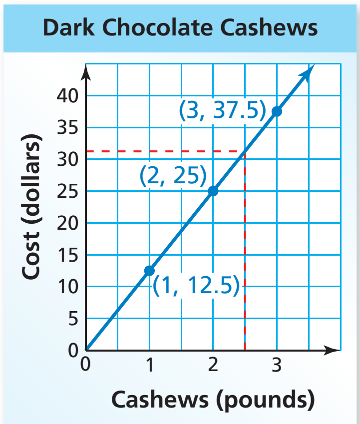
Cashews (pounds)	1	2	3
Cost (dollars)	12.5	25	37.5

The ordered pairs (cashews, cost) are (1, 12.5), (2, 25), and (3, 37.5). Plot the ordered pairs and draw a line, starting at (0, 0), through the points.



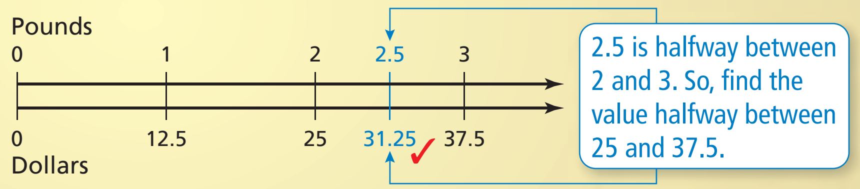
b. How much does 2.5 pounds of dark chocolate cashews cost?

Using the graph, you can see that the cost of 2.5 pounds is halfway between $25 and $37.50.



So, 2.5 pounds of dark chocolate cashews cost $31.25.

Another Method Use a double number line to find the cost.



Try It

3. WHAT IF? Repeat Example 2 when the cost of the dark chocolate cashews is $15 per pound.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

4. GRAPHING A RATIO RELATIONSHIP Represent the ratio relationship using a graph.

Rain (inches)	Snow (inches)
3	5
6	10
9	15

5. CRITICAL THINKING Use what you know about equivalent ratios to explain why the graph of a ratio relationship passes through (0, 0).

6. WHICH ONE DOESN'T BELONG? Which ordered pair does *not* belong with the other three? Explain your reasoning.

(4, 1) (8, 2) (12, 3) (24, 4)

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 Using Properties to Write Equivalent Expressions

a. Simplify the expression $9 \cdot 0 \cdot p$.

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ Associative Property of Multiplication

$= 0 \cdot p$ Multiplication Property of Zero

$= 0$ Multiplication Property of Zero

b. Simplify the expression $4.5 \cdot r \cdot 1$.

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 4.5 \cdot r$ Multiplication Property of One

$= 4.5r$ Rewrite.

Try It **Simplify the expression. Explain each step.**

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING PROPERTIES **Simplify the expression. Explain each step.**

7. $(7 + c) + 4$ **8.** $4(b \cdot 6)$ **9.** $0 \cdot b \cdot 9$

10. **WRITING** Explain what it means for expressions to be equivalent. Then give an example of equivalent expressions.

11. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.

In an algebraic expression, **like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

like terms | like terms

$$\underbrace{5x + 2x}_{\text{like terms}} + \underbrace{2 + 19}_{\text{like terms}}$$

You can use the Distributive Property to *combine* like terms.

EXAMPLE 2 Combining Like Terms

Simplify each expression.

a. $3x + 9 + 2x - 5$

$3x + 9 + 2x - 5 = 3x + 2x + 9 - 5$	Commutative Property of Addition
$= (3 + 2)x + 9 - 5$	Distributive Property
$= 5x + 4$	Simplify.

b. $y + y + y$

$y + y + y = 1y + 1y + 1y$	Multiplication Property of One
$= (1 + 1 + 1)y$	Distributive Property
$= 3y$	Add coefficients.

c. $7z + 2(z - 5y)$

$7z + 2(z - 5y) = 7z + 2(z) - 2(5y)$	Distributive Property
$= 7z + 2z - 10y$	Multiply.
$= (7 + 2)z - 10y$	Distributive Property
$= 9z - 10y$	Add coefficients.

When you combine like terms, you are using the Distributive Property. You are applying the rules
$ab + ac = a(b + c)$
and
$ab - ac = a(b - c)$.

Try It **Simplify the expression.**

5. $8 + 3z - z$

6. $3(b + 5) + b + 2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

7. WRITING One meaning of the word *distribute* is *to give something to each member of a group*. How can this help you remember the Distributive Property?

SIMPLIFYING EXPRESSIONS **Use the Distributive Property to simplify the expression.**

8. $3(x + 10)$

9. $15(4n - 2)$

10. $2w + 4 + 13w + 1$

Extra Example 2

Simplify each expression.

a. $7w + 11 + 3w - 4$ $10w + 7$

b. $b + b + b + b$ $4b$

c. $2c + 3(f + 5c)$ $17c + 3f$

Try It

5. $8 + 2z$

6. $4b + 17$

ELL Support

Provide support for Self-Assessment for Concepts & Skills Exercise 7 by reading the ELL Support on page T-221 to students. Allow students to work in pairs for Exercises 8–10. Have pairs display their answers on whiteboards for your review.

Self-Assessment
for Concepts & Skills

7. *Sample answer:* You must distribute or give the number outside the parentheses to the terms inside the parentheses.

8. $3x + 30$

9. $60n - 30$

10. $15w + 5$

Laurie's Notes

Discuss

? Write $5x + 2x + 2 + 19$ on the board. Ask, "How many terms are in the expression?" 4 "Why are 2 and 19 **like terms**?" They are both constants. "Why are $5x$ and $2x$ like terms?" They both have the same variable raised to the same exponent.

EXAMPLE 2

- Refer students to the push-pin note. The Distributive Property works in reverse. Remind them that this is similar to what they did with the n boxes in the exploration.
- Work through parts (a) and (b) as shown. These problems should make sense to students.
- In part (a), you may want to show combining like terms that contain variables using the visual shown.

$$3x + 2x = \underbrace{\overbrace{x + x + x}^{3x} + \overbrace{x + x}^{2x}}_{5x} = 5x$$

? Part (c) is more involved. Write part (c) and ask, "How many operations do you see?" 5 operations "What are they?" multiply, add, multiply, subtract, multiply

- **Look for and Make Use of Structure:** Help students recognize the structure of this expression. I like to read it as, "7 times the variable z plus 2 times the quantity z minus $5y$." Before working with the $7z$, the Distributive Property must be used.
- Use arrows to help students see 2 is distributed to both z and $5y$.

Try It

- **Neighbor Check:** Have students work independently, and then have their neighbors check their work. Have students discuss any discrepancies.
- **Thumbs Up:** Ask students to assess their understanding of using the Distributive Property to combine like terms.

Self-Assessment for Concepts & Skills

- Exercise 7 asks students to explain their interpretations of the Distributive Property before applying the skill in Exercises 8–10. This will help you analyze your students' level of understanding.
- Identify the reasons for incorrect answers for Exercises 8–10. Are the errors computational? Can students complete Exercises 8 and 9 successfully but not 10, or vice versa? Make sure students are aware of the reasons for any mistakes.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

31. **MODELING REAL LIFE** You make candles by adding 2 fluid ounces of scented oil for every 22 fluid ounces of wax. Your friend makes candles by adding 3 fluid ounces of the same scented oil for every 37 fluid ounces of wax. Whose candles are more fragrant? Explain your reasoning.

32. **MODELING REAL LIFE** A mint milk shake contains 1.25 fluid ounces of milk for every 4 ounces of ice cream. A strawberry milk shake contains 1.75 fluid ounces of milk for every 5 ounces of ice cream. Which milk shake is thicker? Explain.

CRITICAL THINKING **Two whole numbers *A* and *B* satisfy the following conditions. Find *A* and *B*.**

33. $A + B = 30$
$A : B$ is equivalent to $2 : 3$.

34. $A + B = 44$
$A : B$ is equivalent to $4 : 7$.

35. $A - B = 18$
$A : B$ is equivalent to $11 : 5$.

36. $A - B = 25$
$A : B$ is equivalent to $13 : 8$.

Nutrition Facts
8 servings per container
Serving size 1 ounce (28g)
Amount per serving
Calories 161
% Daily Value*
Total Fat 13g **20%**
Saturated Fat 3g **13%**
Includes 0g Added Sugars **0%**
Protein 4g
Vitamin D 0mcg 0%
Calcium 14.8mg 1%
Iron 2.67mg 15%
Potassium 264mg 8%
* The % Daily Value (DV) tells you how much a nutrient in a serving of food contributes to a daily diet. 2,000 calories a day is used for general nutrition advice.

Indicator 2a - In #38, students extend their understanding of ratios to a ratio involving 3 numbers.

37. **MODELING REAL LIFE** A nutrition label shows that there are 161 calories in 28 grams of dry roasted cashews. You eat 9 cashews totaling 12 grams.

a. Do you think it is possible to find the number of calories you consume? Explain your reasoning.

b. How many cashews are in one serving?

38. **MP REASONING** The ratio of three numbers is $4 : 5 : 3$. The sum of the numbers is 54. What are the three numbers?

39. **CRITICAL THINKING** Seven out of every 8 students surveyed own a bike. The difference between the number of students who own a bike and those who do not is 72. How many students were surveyed?

40. **MP LOGIC** You and a classmate have a bug collection for science class. You find 5 out of every 9 bugs in the collection. You find 4 more bugs than your classmate. How many bugs are in the collection?

41. **MP PROBLEM SOLVING** You earn \$72 for every 8 hours you spend shoveling snow. You earn \$60 for every 5 hours you spend babysitting. For every 3 hours you spend babysitting, you spend 2 hours shoveling snow. You babysit for 15 hours in January. How much money do you earn in January?

42. **DIG DEEPER!** You and a friend each have a collection of tokens. Initially, for every 8 tokens you had, your friend had 3. After you give half of your tokens to your friend, your friend now has 18 more tokens than you. Initially, how many more tokens did you have than your friend?

5.4 Practice

Go to **BigIdeasMath.com** to get HELP with solving the exercises.

Review & Refresh

Simplify the expression. Explain each step.

1. $(s + 4) + 8$ **2.** $(12 + x) + 2$ **3.** $3(4n)$

You are given the difference of the numbers of boys and girls in a class and the ratio of boys to girls. How many boys and how many girls are in the class?

4. 3 more boys; 5 for every 4 **5.** 8 more girls; 3 for every 5

6. 4 more girls; 9 for every 13 **7.** 6 more boys; 7 for every 4

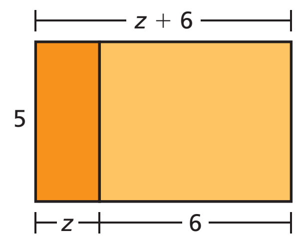
Divide.

8. $301 \div 7$ **9.** $1722 \div 14$ **10.** $629 \div 12$ **11.** $8068 \div 31$

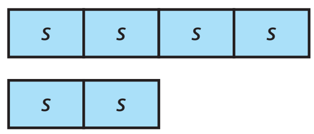
Concepts, Skills, & Problem Solving

USING MODELS **Use the model to simplify the expression. Explain your reasoning.** (See Exploration 1, p. 221.)

12. $5(z + 6) =$ ☐



13. $4s + 2s =$ ☐



SIMPLIFYING EXPRESSIONS **Use the Distributive Property to simplify the expression.**

14. $3(x + 4)$ **15.** $10(b - 6)$ **16.** $6(s - 9)$ **17.** $7(8 + y)$

18. $8(12 + a)$ **19.** $9(2n + 1)$ **20.** $12(6 - k)$ **21.** $18(5 - 3w)$

22. $9(3 + c + 4)$ **23.** $\frac{1}{4}(8 + x + 4)$ **24.** $8(5g + 5 - 2)$ **25.** $6(10 + z + 3)$

26. $4(x + y)$ **27.** $25(x - y)$ **28.** $7(p + q + 9)$ **29.** $\frac{1}{2}(2n + 4 + 6m)$

MATCHING **Match the expression with an equivalent expression.**

30. $6(n + 4)$ **31.** $2(3n + 9)$ **32.** $6(n + 2)$ **33.** $3(2n + 3)$

A. $3(2n + 6)$ **B.** $6n + 9$ **C.** $3(2n + 8)$ **D.** $6n + 12$

34. MP **STRUCTURE** Each day, you run on a treadmill for r minutes and lift weights for 15 minutes. Which expressions can you use to find how many minutes of exercise you do in 5 days? Explain your reasoning.

$5(r + 15)$ $5r + 5 \cdot 15$ $5r + 15$ $r(5 + 15)$

5.5 Practice

Go to *BigIdeasMath.com* to get HELP with solving the exercises.

Review & Refresh

Use the Distributive Property to simplify the expression.

1. $2(n + 8)$ **2.** $3(4 + m)$ **3.** $7(b - 3)$ **4.** $10(4 - w)$

Write the phrase as an expression.

5. 5 plus a number p

6. 18 less than a number r

7. 11 times a number d

8. a number c divided by 25

Decide whether the rates are equivalent.

9. 84 feet in 12 seconds
217 feet in 31 seconds

10. 12 cups of soda for every 54 cups of juice
8 cups of soda for every 36 cups of juice

Match the decimal with its equivalent percent.

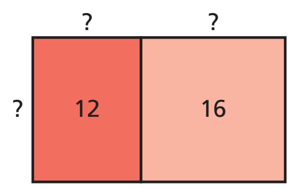
11. 0.36 **12.** 3.6 **13.** 0.0036 **14.** 0.036

A. 0.36% **B.** 360% **C.** 36% **D.** 3.6%

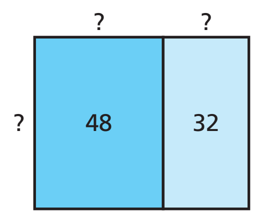
Concepts, Skills, & Problem Solving

FINDING DIMENSIONS **The model shows the area (in square units) of each part of a rectangle. Use the model to find missing values that complete the expression. Explain your reasoning.** (See Exploration 1, p. 227.)

15. $12 + 16 = \square(\square + \square)$



16. $48 + 32 = \square(\square + \square)$



FACTORING NUMERICAL EXPRESSIONS **Factor the expression using the GCF.**

17. $7 + 14$ **18.** $12 + 42$ **19.** $22 + 11$ **20.** $70 + 95$

21. $60 - 36$ **22.** $100 - 80$ **23.** $84 + 28$ **24.** $48 + 80$

25. $19 + 95$ **26.** $44 - 11$ **27.** $18 - 12$ **28.** $48 + 16$

29. $98 - 70$ **30.** $58 + 28$ **31.** $72 - 39$ **32.** $69 + 84$

33. MP **REASONING** The whole numbers a and b are divisible by c, where b is greater than a. Is $a + b$ divisible by c? Is $b - a$ divisible by c? Explain your reasoning.

34. **MULTIPLE CHOICE** Which expression is *not* equivalent to $81x + 54$?

A. $27(3x + 2)$ **B.** $3(27x + 18)$ **C.** $9(9x + 6)$ **D.** $6(13x + 9)$

EXAMPLE 4 Modeling Real Life


Zinc: $2.41 per ounce

A science teacher buys 3.25 ounces of zinc for an experiment. The teacher pays with a $10 bill. How much change does the teacher receive?

Find the cost of 3.25 ounces of zinc at $2.41 per ounce. Then subtract that amount from $10.

Step 1: Multiply 2.41 by 3.25 to find the cost of the zinc.

$$\begin{array}{r l}
2.41 & \leftarrow \text{ 2 decimal places} \\
\times\ 3.25 & \leftarrow \text{ + 2 decimal places} \\
\hline
1205 & \\
482 & \\
723 & \\
\hline
7.8325 & \leftarrow \text{ 4 decimal places}
\end{array}$$

The cost of 3.25 ounces of zinc is $7.83.

Step 2: Subtract the cost of the zinc from the amount of money the teacher uses to buy the zinc.

$10.00 - 7.83 = 2.17$

So, the teacher receives $2.17 in change.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. You earn $9.15 per hour painting a fence. It takes 6.75 hours to paint the fence. Did you earn enough money to buy the jersey shown? If so, how much money do you have left? If not, how much money do you need to earn?

18. A sand volleyball court is a rectangle that has a length of 52.5 feet and a width that is half of the length. In case of rain, the court is covered with a tarp. How many square feet of tarp are needed to cover the court?

19. **DIG DEEPER!** You buy 4 cases of bottled water and 5 bottles of fruit punch for a birthday party. Each case of bottled water costs $2.75, and each bottle of fruit punch costs $1.35. You hand the cashier a $20 bill. How much change will you receive?

EXAMPLE 4 Modeling Real Life

A runner's goal is to complete a mile in 4 minutes or less. The runner's speed is 20 feet per second. Does the runner meet the goal? If not, how much faster (in feet per second) must the runner be to meet the goal?

To meet the goal, the runner must complete 1 mile in 4 minutes or less. The minimum speed required is 1 mile per 4 minutes.

To compare this speed to the runner's speed of 20 feet per second, convert the minimum speed of 1 mile per 4 minutes to feet per second.

1 min = 60 sec   1 mi = 5280 ft

$$\frac{1\text{ mi}}{4\text{ min}} = \frac{1\cancel{\text{ mi}}}{4\cancel{\text{ min}}} \times \frac{1\cancel{\text{ min}}}{60\text{ sec}} \times \frac{5280\text{ ft}}{1\cancel{\text{ mi}}} = \frac{22\text{ ft}}{1\text{ sec}}$$

So, the runner did not meet the goal because a speed of 20 feet per second is below the minimum speed of 22 feet per second. The runner must be $22 - 20 = 2$ feet per second faster to meet the goal.

Check Verify that the distances traveled in 4 minutes at the runner's speed and in 4 minutes at the additional speed have a sum of 1 mile.

At runner's speed

$$4\cancel{\text{ min}} \times \frac{60\cancel{\text{ sec}}}{1\cancel{\text{ min}}} \times \frac{20\text{ ft}}{1\cancel{\text{ sec}}} = 4800\text{ ft}$$

At additional speed

$$4\cancel{\text{ min}} \times \frac{60\cancel{\text{ sec}}}{1\cancel{\text{ min}}} \times \frac{2\text{ ft}}{1\cancel{\text{ sec}}} = 480\text{ ft}$$

The sum of the distances is $4800 + 480 = 5280$ feet, or 1 mile. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

8. Will all of the water from a full two-liter bottle fit into a two-quart pitcher? Explain.

9. **DIG DEEPER!** The speed of light is about 300,000 kilometers per second. The Sun is about 93 million miles from Earth. How many minutes does it take for sunlight to reach Earth?

10. A race car driver's goal is to complete a 1000-kilometer auto race in 4 hours or less. The driver's average speed is 4200 meters per minute. Does the driver meet the goal? If not, how much faster (in meters per minute) must the driver be to meet the goal?

EXAMPLE 4 Modeling Real Life

The figure shows the portions of ultraviolet (UV) rays reflected by four different surfaces. How many times more UV rays are reflected by water than by sea foam?

The diagram shows that sea foam reflects 25% of UV rays and water reflects $\frac{21}{25}$ of UV rays. First, write 25% and $\frac{21}{25}$ as decimals.

Sea foam: $25\% = 25.\cancel{\%} = 0.25$ **Water:** $\frac{21}{25} = \frac{84}{100} = 0.84$

Next, divide 0.84 by 0.25.

$$0.25\overline{)0.84} \longrightarrow \begin{array}{r} 3.36 \\ 25\overline{)84.00} \\ -75 \\ \hline 9\,0 \\ -7\,5 \\ \hline 1\,50 \\ -1\,50 \\ \hline 0 \end{array}$$

So, water reflects 3.36 times more UV rays than sea foam.

Another Method First, write 25% as a fraction:

$25\% = \frac{25}{100} = \frac{1}{4}.$

Then divide $\frac{21}{25}$ by $\frac{1}{4}$.

$$\frac{21}{25} \div \frac{1}{4} = \frac{21}{25} \cdot 4$$
$$= \frac{84}{25}$$
$$= 3\frac{9}{25}, \text{ or } 3.36 \checkmark$$

Indicator 2c - #18 is non-routine because students solve a problem to then answer a bigger question. Students first have to determine how many times more salt is in the Dead Sea than the Indian Ocean. Then they have to use a given bucket with an amount of salt from the Indian Ocean to find the amount of salt in a bucket of the same size from the Dead Sea.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. Write the amount of occupied space on the computer as a percent.

Volume	Capacity	Free Space
(C:)	150 GB	132 GB

18. *Salinity* is a measure of the salt content of a body of water. One researcher measures the salinity of the Indian Ocean as 3.2%. Another researcher measures the salinity of the Dead Sea as 34%. A bucket of water from the Indian Ocean contains 56 grams of salt. How much salt is contained in the same amount of water from the Dead Sea? Justify your answer.

EXAMPLE 6 Modeling Real Life

You are saving to buy a meteorite fragment for \$125. You begin with \$45 and you save \$3 each week. The expression $45 + 3w$ gives the amount of money you save after w weeks. Can you buy the meteorite after 20 weeks?

Understand the problem.

You are given an expression that represents your savings after w weeks. You are asked whether you have enough money to buy a \$125 meteorite after 20 weeks.

Make a plan.

To find the amount of money you save after 20 weeks, evaluate the expression when $w = 20$. Then compare the value of the expression to the price of the meteorite.

Solve and check.

$45 + 3w = 45 + 3(20)$ Substitute 20 for w.

$= 45 + 60$ Multiply 3 and 20.

$= 105$ Add 45 and 60.

You cannot buy the \$125 meteorite after 20 weeks because you only have \$105.

Another Method You start with \$45, so you need to save another $125 - 45 = \$80$. At \$3 per week, it will take you $\frac{80}{3} \approx 27$ weeks of saving.

$$45 + 3(27) = 45 + 81 = \$126 ✓$$

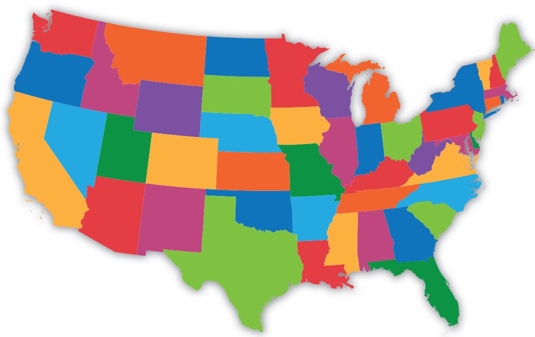
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

24. The expression $12.25m + 29.99$ gives the cost (in dollars) of a gym membership for m months. You have \$180 to spend on a membership. Can you buy a one-year membership?

25. DIG DEEPER! The expression $p - 15$ gives the amount (in dollars) you pay after using the coupon when the original amount of a purchase is p dollars. The expression $30 + 6n$ gives the amount of money (in dollars) you save after n weeks. A jacket costs \$78. Can you buy the jacket after 6 weeks? Explain.

43. MP **PROBLEM SOLVING** In the contiguous United States, the ratio of states that border an ocean to states that do not border an ocean is 7 : 9. How many of the states border an ocean?



44. MP **REASONING** The value of a ratio is $\frac{4}{3}$. The second quantity in the ratio is how many times the first quantity in the ratio? Explain your reasoning.

45. **MODELING REAL LIFE** A train moving at a constant speed travels 3 miles every 5 minutes. A car moving at a constant speed travels 12 miles every 20 minutes. Are the vehicles traveling at the same speed? If not, which is faster?

46. **CRITICAL THINKING** To win a relay race, you must swim 200 yards before your opponent swims 190 yards. You swim at a pace of 50 yards every 40 seconds. Your opponent swims at a pace of 10 yards every 8.5 seconds. Who wins the race? Justify your answer.

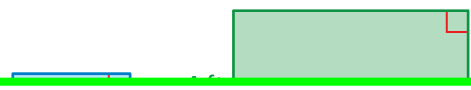
47. **DIG DEEPER!** There are 3 boys for every 2 girls in a dance competition. Does it make sense for there to be a total of 9 people in the competition? Explain.

48. **GEOMETRY** Use the blue and green rectangles.

a. Find the ratio of the length of the blue rectangle t

Repeat this

b. Compare y



Indicator 2c - #50 is non-routine because students have to find which option for tokens is the best deal. They then have to use what they found to make suggestions to the restaurant on how it could modify the price of tokens.

49. MP **STRUCTURE** The ratio of the side lengths of a triangle is 2 : 3 : 4. The shortest side is 15 inches. What is the perimeter of the triangle? Explain.

TOKENS	
1 Token	$0.50
10 Tokens	$5.00
25 Tokens	$10.00
50 Tokens	$25.00
90 Tokens	$40.00

50. MP **PROBLEM SOLVING** A restaurant sells tokens that customers use to play games while waiting for their orders.

a. Which option is the best deal? Justify your answer.

b. What suggestions, if any, would you give to the restaurant about how it could modify the prices of tokens?

51. **DIG DEEPER!** There are 12 boys and 10 girls in your gym class. If 6 boys joined the class, how many girls would need to join for the ratio of boys to girls to remain the same? Justify your answer.

USING A TAPE DIAGRAM **A bowl contains blueberries and strawberries. You are given the total number of berries in the bowl and the ratio of blueberries to strawberries. How many of each berry are in the bowl?**

21. 16 berries; 3 : 1

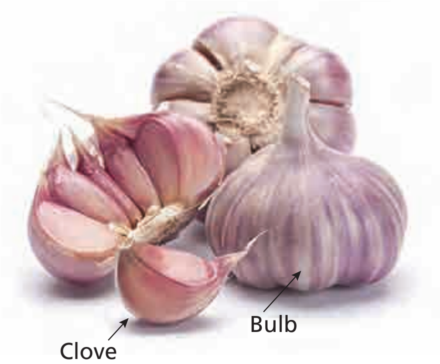
22. 10 berries; 2 for every 3

23. 12 berries; 1 to 2

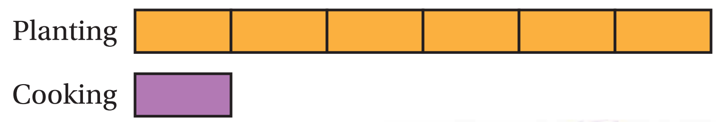
24. 20 berries; 4 : 1

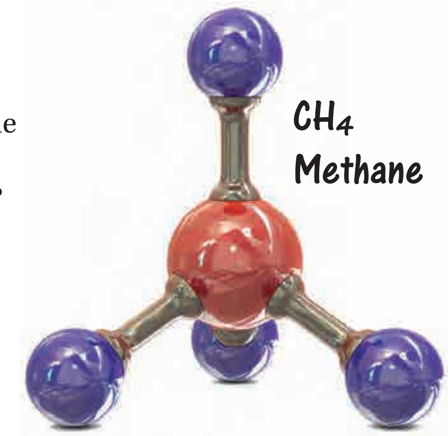
25. 48 berries; 9 to 3

26. 46 berries; 11 for every 12



27. MP **PROBLEM SOLVING** You separate bulbs of garlic into two groups: one for planting and one for cooking. The tape diagram represents the ratio of bulbs for planting to bulbs for cooking. You use 6 bulbs for cooking. Each bulb has 8 cloves. How many cloves of garlic will you plant?



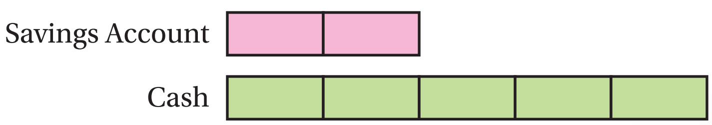


28. **MODELING REAL LIFE** Methane gas contains carbon atoms and hydrogen atoms in the ratio of 1 : 4. A sample of methane gas contains 92 hydrogen atoms. How many carbon atoms are in the sample? How many total atoms are in the sample?

29. **MODELING REAL LIFE** There are 8 more girls than boys in a school play. The ratio of boys to girls is 5 : 7. How many boys and how many girls are in the play?

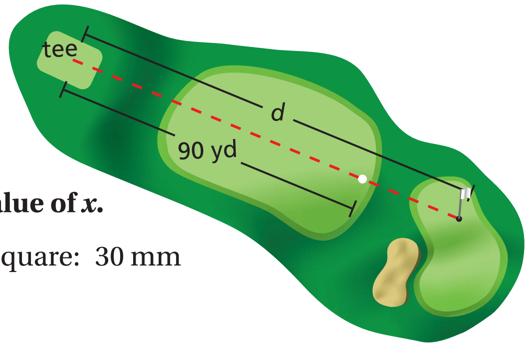
30. **DIG DEEPER!** A baseball team sells tickets for two games. The ratio of sold tickets to unsold tickets for the first game was 7 : 3. For the second game, the ratio was 13 : 2. There were 240 unsold tickets for the second game. How many tickets were sold for the first game?

31. MP **PROBLEM SOLVING** You have \$150 in a savings account and you have some cash. The tape diagram represents the ratio of the amounts of money. You want to have twice the amount of money in your savings account as you have in cash. How much of your cash should you deposit into your savings account?



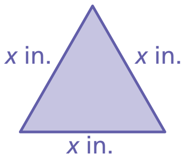
32. **DIG DEEPER!** A fish tank contains tetras, guppies, and minnows. The ratio of tetras to guppies is 4 : 2. The ratio of minnows to guppies is 1 : 3. There are 60 fish in the tank. How many more tetras are there than minnows? Justify your answer.

25. **MODELING REAL LIFE** You hit a golf ball 90 yards. It travels three-fourths of the distance to the hole. Write an equation you can use to find the distance d (in yards) from the tee to the hole.

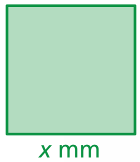


GEOMETRY **Write an equation you can use to find the value of x.**

26. Perimeter of triangle: 16 in.



27. Perimeter of square: 30 mm



28. **MODELING REAL LIFE** You sell instruments at a Caribbean music festival. You earn \$326 by selling 12 sets of maracas, 6 sets of claves, and x djembe drums. Find the number of djembe drums you sold.

29. **MP PROBLEM SOLVING** Neil Armstrong set foot on the Moon 109.4 hours after *Apollo 11* departed from the Kennedy Space Center. *Apollo 11* landed on the Moon about 6.6 hours before Armstrong's first step. How many hours did it take for *Apollo 11* to reach the Moon?

30. **MP LOGIC** You buy a basket of 24 strawberries. You eat them as you walk to the beach. It takes the same amount of time to walk each block. When you are halfway there, half of the berries are gone. After walking 3 more blocks, you still have 5 blocks to go. You reach the beach 28 minutes after you began. One-sixth of your strawberries are left.

a. Is there enough information to find the time it takes to walk each block? Explain.

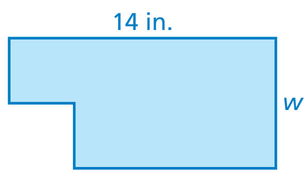
b. Is there enough information to find how many strawberries you ate while walking the last block? Explain.

31. **DIG DEEPER!** Find a sales receipt from a store that shows the total price of the items and the total amount paid including sales tax.

a. Write an equation you can use to find the sales tax rate r.

b. Can you use r to find the *percent* for the sales tax? Explain.

32. **GEOMETRY** A square is cut from a rectangle. The side length of the square is half of the unknown width w. The area of the shaded region is 84 square inches. Write an equation you can use to find the width (in inches).



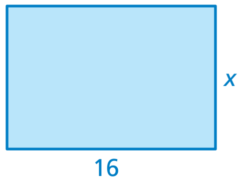
38. **MODELING REAL LIFE** Forty-five basketball players participate in a three-on-three tournament. Write and solve an equation to find the number of three-person teams in the tournament.

39. **MODELING REAL LIFE** A theater has 1200 seats. Each row has 20 seats. Write and solve an equation to find the number of rows in the theater.

GEOMETRY **Solve for *x*. Check your answer.**

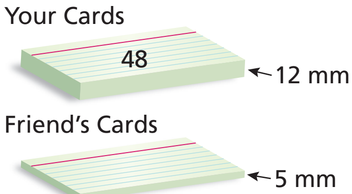
40. Area = 45 square units

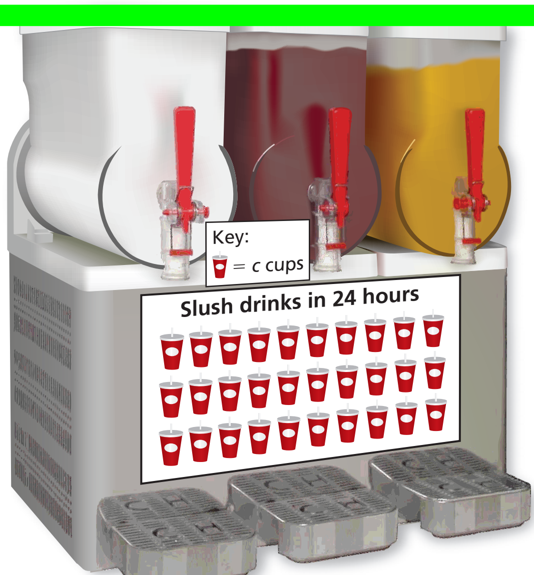
41. Area = 176 square units



42. **MP LOGIC** On a test, you earn 92% of the possible points by correctly answering 6 five-point questions and 8 two-point questions. How many points p is the test worth?

43. **MODELING REAL LIFE** You use index cards to play a homemade game. The object is to be the first to get rid of all your cards. How many cards are in your friend's stack?



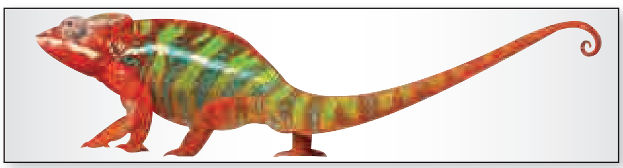


44. **DIG DEEPER!** A slush drink machine fills 1440 cups in 24 hours.

a. Find the number c of cups each symbol represents.

b. To lower costs, you replace the cups with paper cones that hold 20% less. Find the number n of paper cones that the machine can fill in 24 hours.

45. **MP NUMBER SENSE** The area of the picture is 100 square inches. The length is 4 times the width. Find the length and width of the picture.



3 Connecting Concepts

Using the Problem-Solving Plan

1. You mix water, glue, and borax in the ratio of 3 : 1 : 2 to make slime. How many gallons of each ingredient should you use to make 0.75 gallon of slime?

Understand the problem.

You know the ratio of the ingredients in the slime and that you are making 0.75 gallon of slime. You are asked to find the number of gallons of each ingredient needed to make 0.75 gallon of slime.

Make a plan.

Represent the ratio 3 : 1 : 2 using a tape diagram. Because there are 6 parts that represent 0.75 gallon, divide 0.75 by 6 to find the value of one part of the tape diagram. Then use the value of one part to find the number of gallons of each ingredient you should use.

Solve and check.

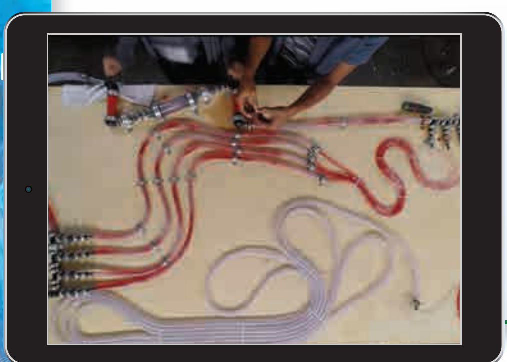
Use the plan to solve the problem. Then check your solution.

2. You buy yogurt cups and frozen fruit bars for a party. Yogurt cups are sold in packages of six. The ratio of the number of yogurt cups in a package to the number of frozen fruit bars in a package is 3 : 2. What are the least numbers of packages you should buy in order to have the same numbers of yogurt cups and frozen fruit bars?

3. The greatest common factor of two whole numbers is 9. The ratio of the greater number to the lesser number is 6 : 5. What are the two numbers? Justify your answer.

Performance Task

Oops! Unit Conversion Mistakes



At the beginning of this chapter, you watched a STEAM Video called "Human Circulatory System." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

6 Connecting Concepts

Using the Problem-Solving Plan

1. A tornado forms 12.25 miles from a weather station. It travels away from the station at an average speed of 440 yards per minute. How far from the station is the tornado after 30 minutes?

Understand the problem.

You know the initial distance between the tornado and the station, and the average speed the tornado is traveling away from the station. You are asked to determine how far the tornado is from the station after 30 minutes.

Make a plan.

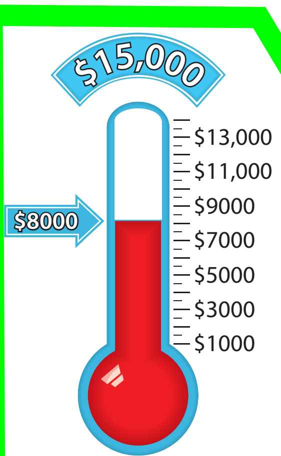
First, convert the average speed to miles per minute. Then write an equation that represents the distance d (in miles) between the tornado and the station after t minutes. Use the equation to find the value of d when $t = 30$.

Solve and check.

Use the plan to solve the problem. Then check your solution.

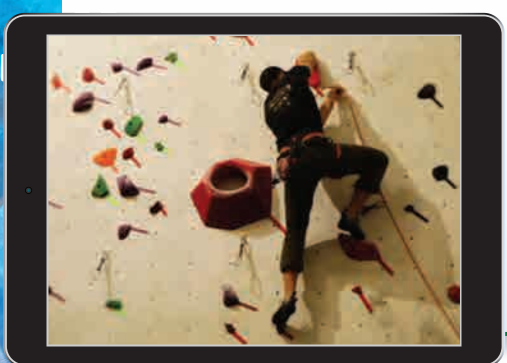
2. You buy 96 cans of soup to donate to a food bank. The store manager discounts the cost of each case for a total discount of \$40. Use an equation in two variables to find the discount for each case of soup. What is the total cost when each can of soup originally costs \$1.20?

1 case = 12 cans



3. The diagram shows the initial amount raised by an organization for cancer research. A business agrees to donate \$2 for every \$5 donated by the community during an additional fundraising event. Write an equation that represents the total amount raised (in dollars). How much money does the community need to donate for the organization to reach its fundraising goal?

Performance Task



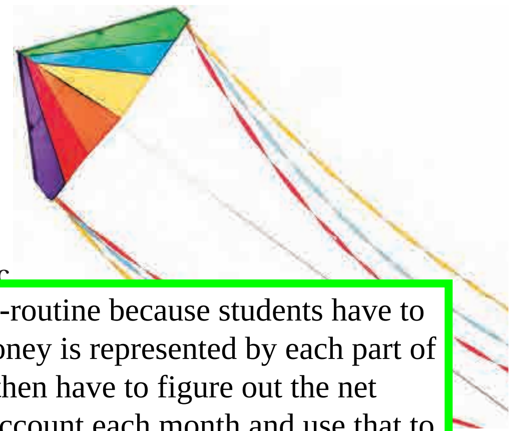
Planning the Climb

At the beginning of this chapter, you watched a STEAM video called "Rock Climbing." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

8 Connecting Concepts

Using the Problem-Solving Plan

1. You use a coordinate plane to design a kite for a competition. The vertices of the design are $A(0, 0)$, $B(13.5, 9)$, $C(27, 0)$, and $D(13.5, -36)$. The coordinates are measured in inches. Find the least number of square yards of fabric you need to make the kite.



Understand the problem.

You know the vertices of your kite design in a coordinate plane, where the coordinates are measured in inches. You are asked to find the least number of square yards of fabric needed to make the kite.

Indicator 2c - #2 is non-routine because students have to figure out how much money is represented by each part of the tape diagram. They then have to figure out the net amount that leaves the account each month and use that to determine when the account has a balance of $0.

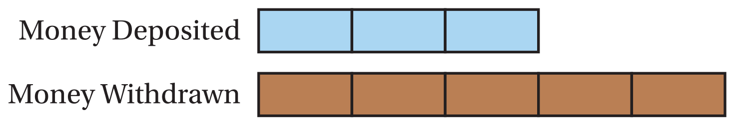
Make a plan.

First, draw a diagram of the des decompose the figure into two in square inches. Finally, conve

Solve and check.

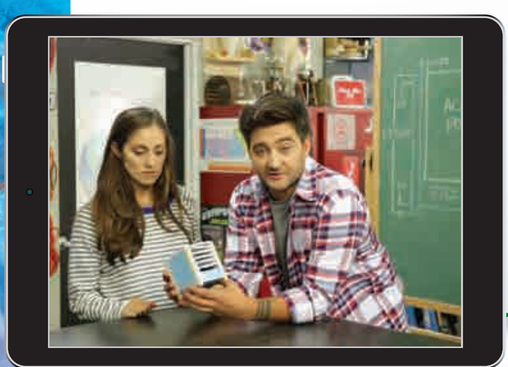
Use the plan to solve the problem. Then check your solution.

2. You have $240 in a savings account. You deposit $60 per month. The tape diagram represents the ratio of money deposited to money withdrawn each month. Find the monthly change in your account balance. How long will it take for the account to have a balance of $0? Justify your answer.



3. A cord made of synthetic fiber can support 630 pounds, which is at least 450% of the weight that can be supported by a cord made of steel. Graph the possible weights that can be supported by the steel cord.

Performance Task

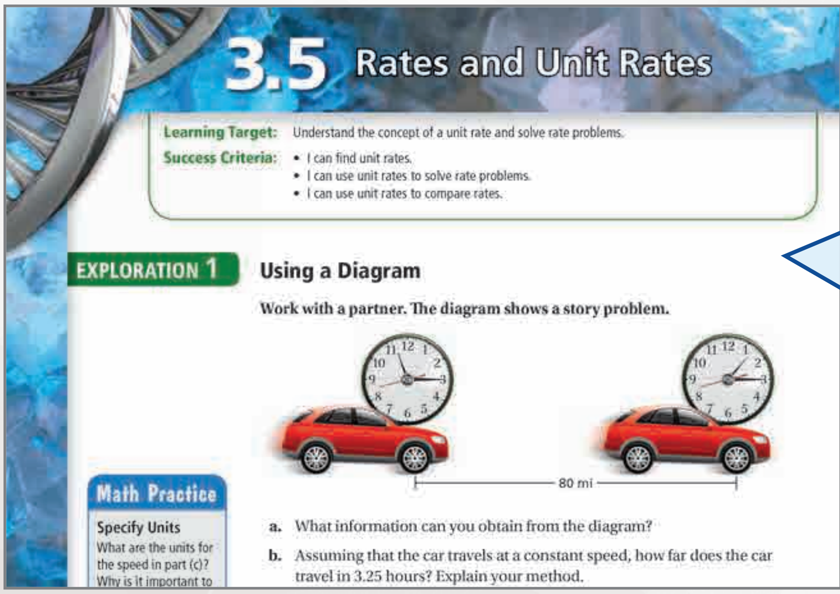


Launching a CubeSat

At the beginning of this chapter, you watched a STEAM Video called “Designing a CubeSat.” You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Professional Development

Rigorous by Design



3.5 Rates and Unit Rates

Learning Target: Understand the concept of a unit rate and solve rate problems.

Success Criteria:
- I can find unit rates.
- I can use unit rates to solve rate problems.
- I can use unit rates to compare rates.

EXPLORATION 1 Using a Diagram

Work with a partner. The diagram shows a story problem.

a. What information can you obtain from the diagram?

b. Assuming that the car travels at a constant speed, how far does the car travel in 3.25 hours? Explain your method.

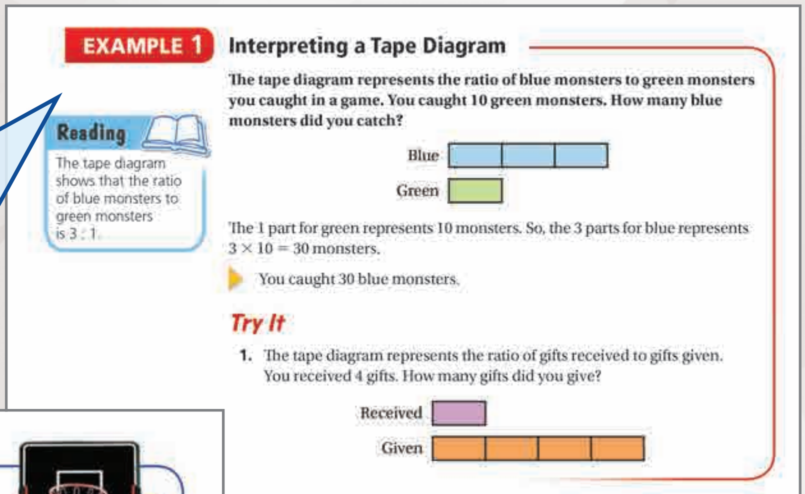
Math Practice
Specify Units
What are the units for the speed in part (c)? Why is it important to

Conceptual Understanding

Explorations help students reach a deeper level of conceptual understanding.

Procedural Fluency

Lessons follow a gradual release model and give teachers opportunities for flexible instruction, providing opportunities for all levels of learners to attain procedural fluency. Self-Assessments provide students the opportunity to asses their understanding of the success criteria, taking ownership of their learning.



EXAMPLE 1 Interpreting a Tape Diagram

The tape diagram represents the ratio of blue monsters to green monsters you caught in a game. You caught 10 green monsters. How many blue monsters did you catch?

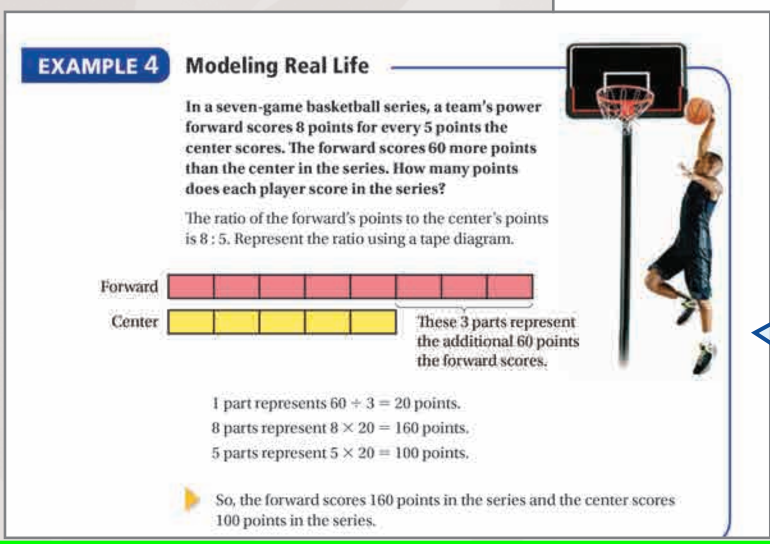
The 1 part for green represents 10 monsters. So, the 3 parts for blue represents $3 \times 10 = 30$ monsters.

You caught 30 blue monsters.

Reading
The tape diagram shows that the ratio of blue monsters to green monsters is 3 : 1.

Try It

1. The tape diagram represents the ratio of gifts received to gifts given. You received 4 gifts. How many gifts did you give?



EXAMPLE 4 Modeling Real Life

In a seven-game basketball series, a team's power forward scores 8 points for every 5 points the center scores. The forward scores 60 more points than the center in the series. How many points does each player score in the series?

The ratio of the forward's points to the center's points is 8 : 5. Represent the ratio using a tape diagram.

1 part represents $60 \div 3 = 20$ points.
8 parts represent $8 \times 20 = 160$ points.
5 parts represent $5 \times 20 = 100$ points.

So, the forward scores 160 points in the series and the center scores 100 points in the series.

Application

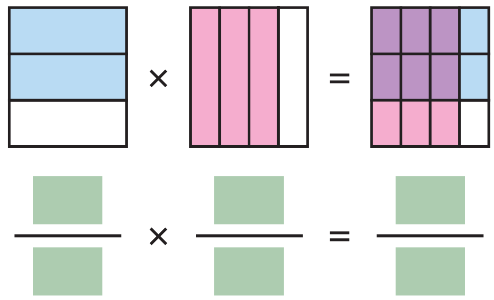
Modeling Real Life examples bring problem solving into the classroom, promoting application of concepts and skills and reaching higher levels of DOK.

Getting Ready for Chapter 2

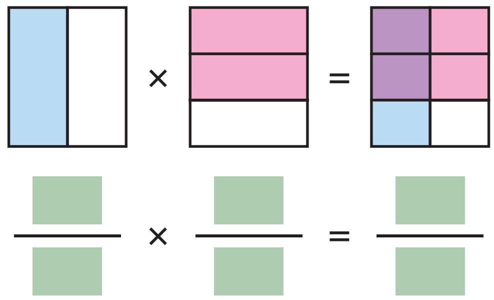
Chapter Exploration

Work with a partner. The area model represents the multiplication of two fractions. Copy and complete the statement.

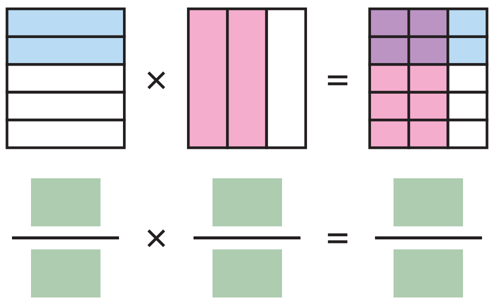
1.



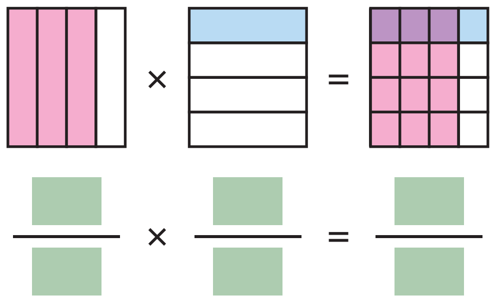
2.



3.



4.



Work with a partner. Use an area model to find the product.

5. $\frac{1}{2} \times \frac{1}{3}$ **6.** $\frac{4}{5} \times \frac{1}{4}$ **7.** $\frac{1}{6} \times \frac{3}{4}$ **8.** $\frac{3}{5} \times \frac{1}{4}$

9. MODELING REAL LIFE You have a recipe that serves 6 people. The recipe uses three-fourths of a cup of milk.

a. How can you use the recipe to serve *more* people? How much milk would you need? Give 2 examples.

b. How can you use the recipe to serve *fewer* people? How much milk would you need? Give 2 examples.

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

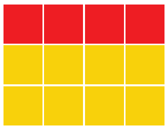
reciprocals multiplicative inverses

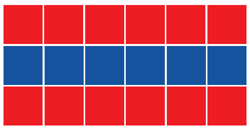
Indicator 2d - In this Chapter Exploration, one aspect of rigor (conceptual understanding) is emphasized. The focus is on using color tiles to introduce the conceptual understanding for ratios.

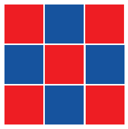
Getting Ready for Chapter 3

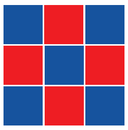
Chapter Exploration

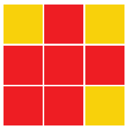
Work with a partner. What portion of the rectangle is red? How did you write your answer?

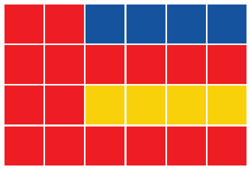
1. 

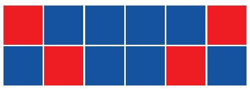
2. 

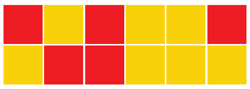
3. 

4. 

5. 

6. 

7. 

8. 

9. 

10. Work with a partner. In Exercises 1–9, which of the rectangles have the same portion of red tiles? Explain your reasoning.

Work with a partner. Use square color tiles to build two different-sized rectangles that represent the description.

11. Five-sixths of the tiles are blue.

12. Three-fourths of the tiles are yellow.

13. Four-fifths of the tiles are green.

14. Five-sevenths of the tiles are red.

15. **MODELING REAL LIFE** Work with a partner. The soccer committee has 8 girls and 6 boys. The tennis committee has 9 girls and 8 boys. A friend tells you that the tennis committee has a greater portion of girls than the soccer committee. Is your friend correct? Explain. If not, how many boys could you add to the soccer committee so that your friend is correct?

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

ratio	rate	equivalent rates
equivalent ratios	unit rate	

Getting Ready for Chapter 6

Chapter Exploration

Work with a partner. Every equation that has an unknown variable can be written as a question. Write a question that represents the equation. Then answer the question.

	Equation	Question	Answer to Question
1.	$x + 3 = 7$		
2.	$5 - x = 2$		
3.	$3x = 12$		
4.	$x \div 5 = 3$		
5.	$20 \div x = 4$		

Work with a partner. Write an equation that represents the question. Then answer the question.

	Question	Equation	Answer to Question
6.	What number can be added to 7 to get 12?		
7.	What number can be subtracted from 11 to get 3?		
8.	What number can be multiplied by 10 to get 30?		
9.	What number can be divided by 7 to get 3?		
10.	**MODELING REAL LIFE** Your friend says that he will be 21 years old in 7 years. How old is he now?		

Vocabulary

The following vocabulary terms are defined in this chapter. Think about what each term might mean and record your thoughts.

equation
inverse operations
equation in two variables
independent variable
dependent variable

1 Connecting Concepts

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Using the Problem-Solving Plan

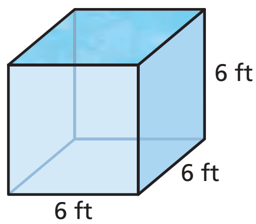
1. A sports team gives away shirts at the stadium. There are 60 large shirts, 1.6 times as many small shirts as large shirts, and 1.5 times as many medium shirts as small shirts. The team wants to divide the shirts into identical groups to be distributed throughout the stadium. What is the greatest number of groups that can be formed using every shirt?

Understand the problem. You know the number of large shirts and two relationships among the numbers of small, medium, and large shirts. You are asked to find the greatest number of identical groups that can be formed using every shirt.

Make a plan. Break the problem into parts. First use multiplication to find the number of each size shirt. Then find the GCF of these numbers.

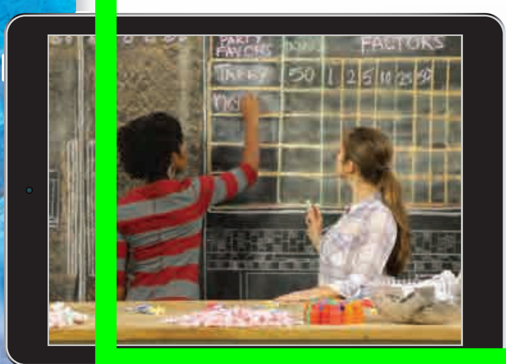
Solve and check. Use the plan to solve the problem. Then check your solution.

2. An escape artist fills the tank shown with water. Find the number of cubic feet of water needed to fill the tank. Then find the number of cubic yards of water that are needed to fill the tank. Justify your answer.



Performance Task

Setting the Table



At the beginning of this chapter, you watched a STEAM video called "Filling Piñatas." You are now ready to complete the performance task for this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

2 Connecting Concepts

Using the Problem-Solving Plan

1. You change the water jug on the watercooler. How many glasses can be completely filled before you need to change the water jug again?

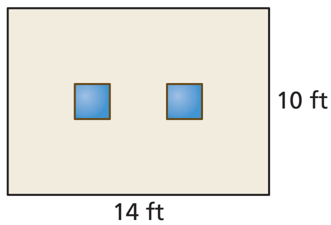
Understand the problem. You know the capacities of the water jug and the glass. You are asked to determine how many glasses the water jug can fill.

Make a plan. First, use what you know about converting measures to find the number of fluid ounces in 5 gallons. Then divide this amount by the capacity of the glass to find the number of glasses that can be filled.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. Two ferries just departed from their docks at the same time. Ferry A departs from its dock every 1.2 hours. Ferry B departs from its dock every 1.8 hours. How long will it be until both ferries depart from their docks at the same time again?

3. You want to paint the ceiling of your bedroom. The ceiling has two square skylights as shown. Each skylight has a side length of $1\frac{7}{8}$ feet. How many square feet will you paint? Justify your answer.



Performance Task

Space Explorers

At the beginning of this chapter, you watched a STEAM video called "Space is Big." You are now ready to complete the performance task for this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

5 Connecting Concepts

Using the Problem-Solving Plan

1. A store sells 18 pairs of the wireless earbuds shown. Customers saved a total of $882 on the earbuds. Find the original price of the earbuds.

Understand the problem. You know the percent discount on a pair of wireless earbuds, the number of pairs of earbuds sold, and the total amount of money that customers saved. You are asked to find the original price of the earbuds.

Make a plan. First, write an expression that represents the total amount of money that customers pay for the earbuds. Then factor the expression to find the discount (in dollars) on each pair of earbuds. Finally, solve a percent problem to find the original price.

Solve and check. Use the plan to solve the problem. Then check your solution.

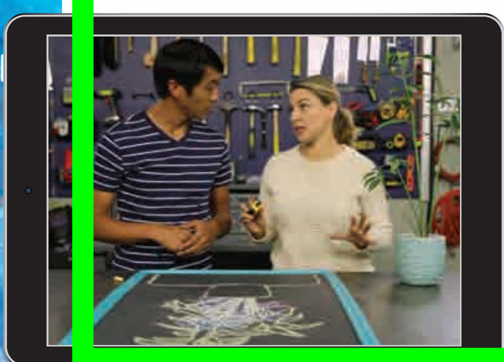
2. All of the weight plates in a gym are labeled in kilograms. You want to convert the weights to pounds. Write an expression to find the number of pounds in z kilograms. Then find the weight in pounds of a plate that weighs 20.4 kilograms.

3. You buy apple chips and banana chips in the ratio of 2 : 7.

a. How many ounces of banana chips do you buy when you buy n ounces of apple chips? Explain.

b. You buy 12 ounces of apple chips. How many ounces of banana chips do you buy?

Performance Task



Describing Change

At the beginning of this chapter, you watched a STEAM video called "Shadow Drawings." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Indicator 2d - In the exercises and the Performance Task, one aspect of rigor (application) is emphasized. Students use their learning from the chapter and previous chapters to complete the exercises.

6 Connecting Concepts

Using the Problem-Solving Plan

1. A tornado forms 12.25 miles from a weather station. It travels away from the station at an average speed of 440 yards per minute. How far from the station is the tornado after 30 minutes?

Understand the problem.

You know the initial distance between the tornado and the station, and the average speed the tornado is traveling away from the station. You are asked to determine how far the tornado is from the station after 30 minutes.

Make a plan.

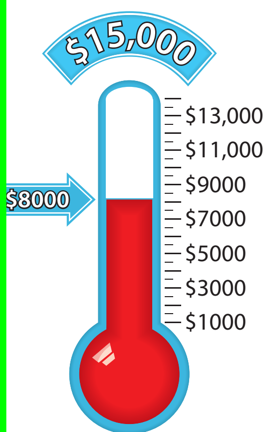
First, convert the average speed to miles per minute. Then write an equation that represents the distance d (in miles) between the tornado and the station after t minutes. Use the equation to find the value of d when $t = 30$.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. You buy 96 cans of soup to donate to a food bank. The store manager discounts the cost of each case for a total discount of $40. Use an equation in two variables to find the discount for each case of soup. What is the total cost when each can of soup originally costs $1.20?

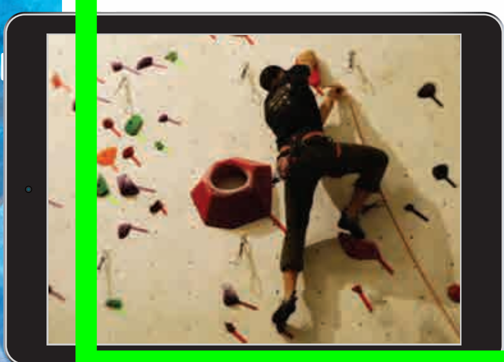
1 case = 12 cans



3. The diagram shows the initial amount raised by an organization for cancer research. A business agrees to donate $2 for every $5 donated by the community during an additional fundraising event. Write an equation that represents the total amount raised (in dollars). How much money does the community need to donate for the organization to reach its fundraising goal?

Performance Task

Planning the Climb

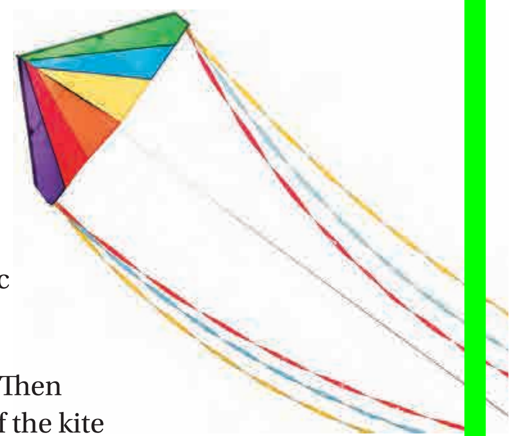


At the beginning of this chapter, you watched a STEAM video called "Rock Climbing." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

8 Connecting Concepts

Using the Problem-Solving Plan

1. You use a coordinate plane to design a kite for a competition. The vertices of the design are $A(0, 0)$, $B(13.5, 9)$, $C(27, 0)$, and $D(13.5, -36)$. The coordinates are measured in inches. Find the least number of square yards of fabric you need to make the kite.

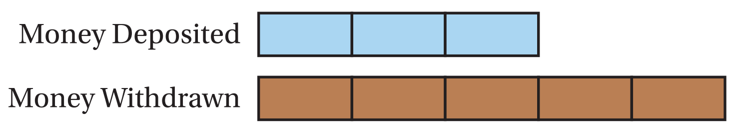


Understand the problem. You know the vertices of your kite design in a coordinate plane, where the coordinates are measured in inches. You are asked to find the least number of square yards of fabric needed to make the kite.

Make a plan. First, draw a diagram of the design in a coordinate plane. Then decompose the figure into two triangles to find the area of the kite in square inches. Finally, convert the area from square inches to square yards.

Solve and check. Use the plan to solve the problem. Then check your solution.

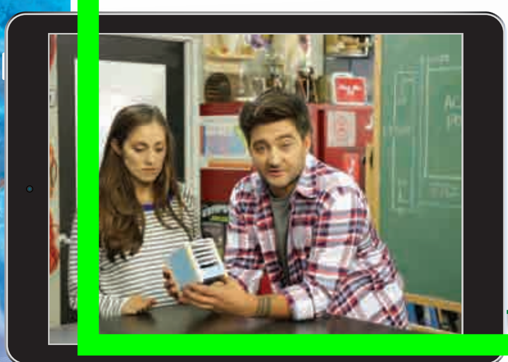
2. You have \$240 in a savings account. You deposit \$60 per month. The tape diagram represents the ratio of money deposited to money withdrawn each month. Find the monthly change in your account balance. How long will it take for the account to have a balance of \$0? Justify your answer.



3. A cord made of synthetic fiber can support 630 pounds, which is at least 450% of the weight that can be supported by a cord made of steel. Graph the possible weights that can be supported by the steel cord.

Performance Task

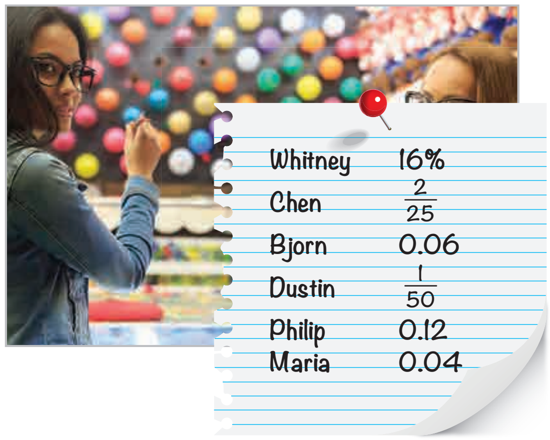
Launching a CubeSat



At the beginning of this chapter, you watched a STEAM Video called "Designing a CubeSat." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

9 Connecting Concepts

Using the Problem-Solving Plan



1. Six friends play a carnival game in which a person throws darts at balloons. Each person throws the same number of darts and then records the portion of the balloons that pop. Find and interpret the mean, median, and MAD of the data.

Understand the problem. You know that each person throws the same number of darts. You are given the portion of balloons popped by each person as a fraction, a decimal, or a percent.

Make a plan. First, write each fraction and each decimal as a percent. Next, order the percents from least to greatest. Then find and interpret the mean, median, and MAD of the data.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. The cost c (in dollars) to rent skis at a resort for n days is represented by the equation $c = 22n$. The durations of several ski rentals are shown in the table. Find the range and interquartile range of the costs of the ski rentals. Then determine whether any of the costs are outliers.

Duration of Rentals (days)							
1	5	1	3	1	2	5	4
3	12	1	12	5	7	4	1

Performance Task

Which Measure of Center Is Best: Mean, Median, or Mode?

At the beginning of this chapter, you watched a STEAM Video called "Daylight in the Big City." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

EXAMPLE 4 Modeling Real Life

One firefly flashes every 8 seconds. Another firefly flashes every 10 seconds. Both fireflies just flashed. After how many seconds will both fireflies flash at the same time again?

Understand the problem.

You are given the numbers of seconds between flashes for two different fireflies. You are asked when the fireflies will flash at the same time again.

Make a plan.

The LCM of the numbers of seconds between flashes represents the number of seconds it will take for both fireflies to flash at the same time again. So, find the LCM of 8 and 10 by listing the multiples of each number.

Solve and check.

Multiples of 8: 8, 16, 24, 32, 40, . . .

Multiples of 10: 10, 20, 30, 40, 50, . . .

The LCM of 8 and 10 is 40.

So, both fireflies will flash at the same time again after 40 seconds.

Another Method Find the LCM using prime factorizations.

$8 = 2 \cdot 2 \cdot 2 \qquad 10 = 2 \cdot 5$

So, the LCM is $2 \cdot 2 \cdot 2 \cdot 5 = 40$. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. A geyser erupts every fourth day. Another geyser erupts every sixth day. Today both geysers erupted. In how many days will both geysers erupt on the same day again?

18. A water park has two large buckets that slowly fill with water. One bucket dumps water every 12 minutes. The other bucket dumps water every 10 minutes. Five minutes ago, both buckets dumped water. When will both buckets dump water at the same time again?

19. DIG DEEPER! You purchase disposable plates, cups, and forks for a cookout. Plates are sold in packages of 24, cups in packages of 32, and forks in packages of 48. What are the least numbers of packages you should buy in order to have the same number of plates, cups, and forks?

EXAMPLE 4 Modeling Real Life

You and your teacher make colored frosting. You add 3 drops of red food coloring for every 1 drop of blue food coloring. Your teacher adds 5 drops of red for every 3 drops of blue. Whose frosting is redder?

Understand the problem.

You are g
coloring t
frosting. Y
frosting is

Make a plan.

Use ratio
ratios in v
number c
drops is t
to determ

Solve and check.

Create ra
Include a

Dro of R
3
6
9
12
15

Indicator 2f - In Example 4, the Problem-Solving Plan is shown to help students make sense of problems and persevere in solving them. Students then use the Problem-Solving Plan to help them solve #9-10.

MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.... Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Look Back
The tables show that when both frostings have a total of 16 drops, your frosting has 2 more drops of red and 2 fewer drops of blue. So, your frosting is redder. ✓

When bo[...] drops of red than your teacher's frosting.

So, your frosting is redder than your teacher's frosting.

Self-Assessment for Problem Solving

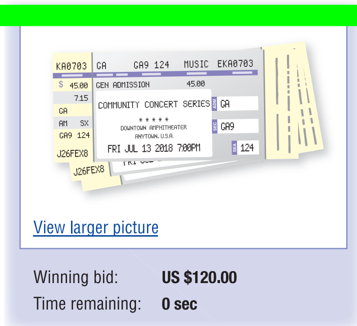
Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. You mix 7 tablespoons of vinegar for every 4 tablespoons of baking soda to produce a chemical reaction. You use 15 tablespoons of baking soda. How much vinegar do you use?

10. You make a carbonated beverage by adding 7 ounces of soda water for every 3 ounces of regular water. Your friend uses 11 ounces of soda water for every 4 ounces of regular water. Whose beverage is more carbonated?

EXAMPLE 6 Modeling Real Life

You win an online auction for concert tickets. Your winning bid is 60% of your maximum bid. How much more were you willing to pay for the tickets than you actually paid?



A. \$72 **B** \$80

C \$120 **D.** \$200

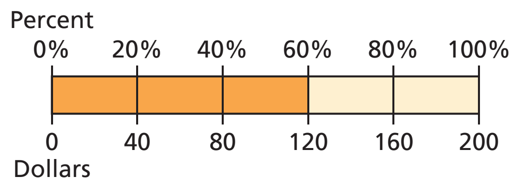
Understand the problem.

You are given the winning bid and the percent of your maximum bid represented by the winning bid. You are asked to find how much more you were willing to pay for the tickets than you actually paid.

Make a plan.

Your maximum bid is the whole and your winning bid is the part. Create a model using the fact that 60% of the whole is \$120 to find the maximum bid. Then subtract the winning bid from the maximum bid to determine how much more you were willing to pay.

Solve and check.



Your maximum bid is \$200, and your winning bid is \$120. So, you were willing to pay $\$200 - \$120 = \$80$ more for the tickets.

The correct answer is **B**.

Look Back
Verify that the additional \$80 you were willing to pay is $100\% - 60\% = 40\%$ of the maximum bid.

The model shows that 40% of the maximum bid is \$80. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

15. You raise \$420 during a fundraising event. The amount of money that you raise is 120% of your goal. How much more did you raise than your goal?

16. A shirt is on sale for 60% of the original price. The original price is \$28 more than the sale price. What was the original price?

17. You have a meal at a restaurant. The sales tax is 8%. You leave a tip for the waitress that is 20% of the pretax price. You spend a total of \$23.04. What is the pretax price of the meal?

2 Connecting Concepts

Using the Problem-Solving Plan

1. You change the water jug on the watercooler. How many glasses can be completely filled before you need to change the water jug again?

Understand the problem.

You know the capacities of the water jug and the glass. You are asked to determine how many glasses the water jug can fill.

Make a plan.

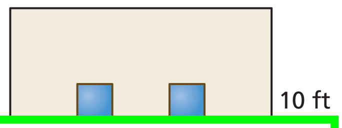
First, use what you know about converting measures to find the number of fluid ounces in 5 gallons. Then divide this amount by the capacity of the glass to find the number of glasses that can be filled.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. Two ferries just departed from their docks at the same time. Ferry A departs from its dock every 1.2 hours. Ferry B departs from its dock every 1.8 hours. How long will it be until both ferries depart from their docks at the same time again?

3. You want to paint the ceiling of your bedroom. The ceiling has two square skylights as shown. Each skylight has a side length of $1\frac{7}{}$ feet. How many square feet will



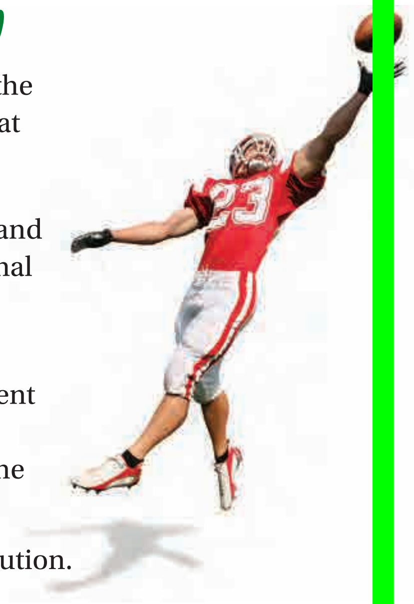
Indicator 2f - In each exercise, students have to use their knowledge of the current chapter and previous concepts to solve them. This helps students make sense of problems and persevere in solving them. For instance, #2 requires the skills of multiplying decimals and least common multiple to solve.

MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.... Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

4 Connecting Concepts

Using the Problem-Solving Plan

1. During a football game, a total of 63 points are scored by the two teams. Team A scores 80% of the number of points that Team B scores. What is the final score of the game?



Understand the problem.

You know that Team A's score is 80% of Team B's score, and the total points scored is 63. You are asked to find the final score of the football game.

Make a plan.

Because 80% means 80 per 100, write the relationship between the scores of the teams as a ratio. Then represent the situation using a tape diagram. Use the total points to find the value of each part of the tape diagram and the final score of the game.

Solve and check.

Use the plan to solve the problem. Then check your solution.

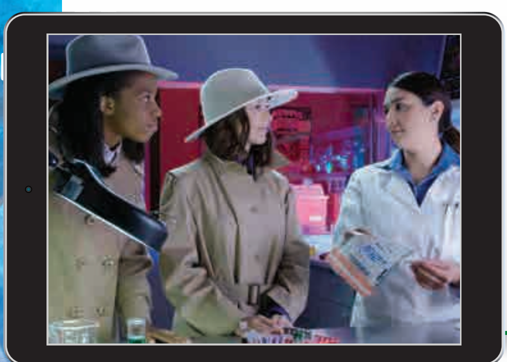
2. A pen at a pet store contains male and female guinea pigs. The ratio of female guinea pigs to male guinea pigs is 7 to 3. Find the percent of guinea pigs in the pen that are male. Justify your answer.

3. You multiply two numbers. The first number, 21, is 6.25% of the product. What is the second number? Justify your answer.

4. You have a bag containing dollar coins, dimes, and pennies. The bag contains 40 coins. The number of dollar coins is 20% of the total number of coins. The number of pennies is $\frac{7}{9}$ of the number of dimes. How much money is in the bag?

Performance Task

Genetic Ancestry

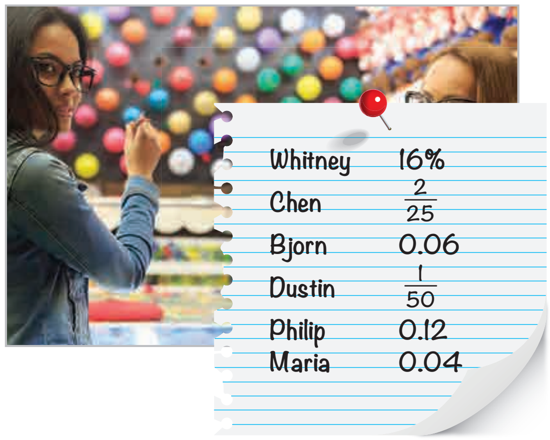


At the beginning of this chapter, you watched a STEAM video called "Chargaff's Rules." You are now ready to complete the performance task for this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

9 Connecting Concepts

Using the Problem-Solving Plan

1. Six friends play a carnival game in which a person throws darts at balloons. Each person throws the same number of darts and then records the portion of the balloons that pop. Find and interpret the mean, median, and MAD of the data.



Understand the problem.

You know that each person throws the same number of darts. You are given the portion of balloons popped by each person as a fraction, a decimal, or a percent.

Make a plan.

First, write each fraction and each decimal as a percent. Next, order the percents from least to greatest. Then find and interpret the mean, median, and MAD of the data.

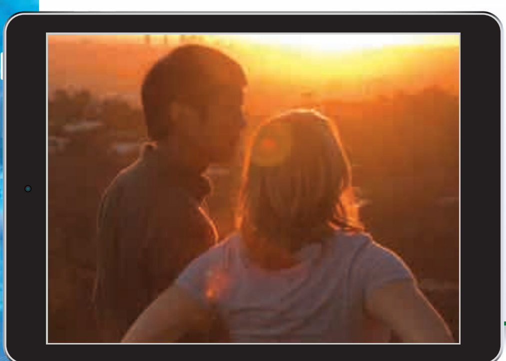
Solve and check.

Use the plan to solve the problem. Then check your solution.

2. The cost c (in dollars) to rent skis at a resort for n days is represented by the equation $c = 22n$. The durations of several ski rentals are shown in the table. Find the range and interquartile range of the costs of the ski rentals. Then determine whether any of the costs are outliers.

Duration of Rentals (days)							
1	5	1	3	1	2	5	4
3	12	1	12	5	7	4	1

Performance Task



Which Measure of Center Is Best: Mean, Median, or Mode?

At the beginning of this chapter, you watched a STEAM Video called "Daylight in the Big City." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

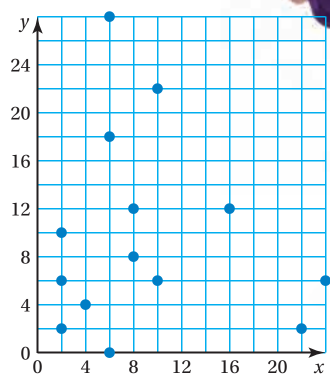
10 Connecting Concepts

Using the Problem-Solving Plan

1. The locations of pitches in an at-bat are shown in the coordinate plane, where the coordinates are measured in inches. Describe the location of a typical pitch in the at-bat.

Understand the problem.

You know the locations of the pitches. You are asked to find the location of a typical pitch in the at-bat.



Make a plan.

First, use the coordinates of the pitches to create two data sets, one for the x-coordinates of the pitches and one for the y-coordinates of the pitches. Next, make a box-and-whisker plot for each data set. Then use the most appropriate measure of center for each data set to find the location of a typical pitch.

Solve and check.

Use the plan to solve the problem. Then check your solution.

2. A set of 20 data values is described below. Sketch a histogram that could represent the data set. Explain.

- least value: 10
- third quartile: 34
- first quartile: 25
- greatest value: 48
- mean: 29
- MAD: 7

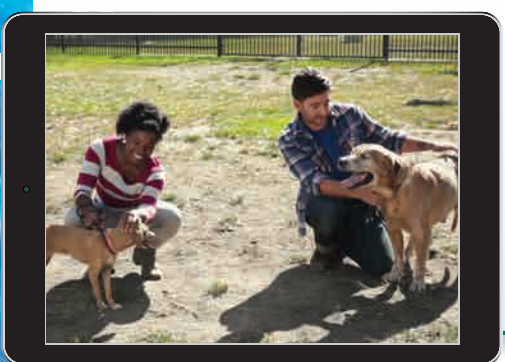
3. The chart shows the dimensions (in inches) of several flat-rate shipping boxes. Each box is in the shape of a rectangular prism. Describe the distribution of the volumes of the boxes. Then find the most appropriate measures to describe the center and the variation of the volumes.

SHIPPING RATES

Dimensions	Price
5 × 5 × 4	$6.80
8 × 5 × 3	$8.30
9 × 8 × 5	$9.75
10 × 6 × 6	$10.75
10 × 10 × 4	$10.75
8 × 7 × 5	$10.75
12 × 10 × 3	$11.25
15 × 10 × 3	$12.25
12 × 12 × 5	$17.40

Performance Task

Classifying Dog Breeds by Size



At the beginning of this chapter, you watched a STEAM Video called "Choosing a Dog." You are now ready to complete the performance task related to this video, available at ***BigIdeasMath.com***. Be sure to use the problem-solving plan as you work through the performance task.

Learning Target

Find the greatest common factor of two numbers.

Success Criteria

- Explain the meaning of factors of a number.
- Use lists of factors to identify the greatest common factor of numbers.
- Use prime factors to identify the greatest common factor of numbers.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL Support

Explain the comparative and superlative word endings *–er* and *–est* by writing *small, smaller,* and *smallest* on the board. Ask a volunteer to draw a small circle. Then ask another student to draw a smaller circle. Ask, "Which one is smallest?" Explain that the word *great* means large, and that the greatest common factor is the largest of the factors that are shared by two or more numbers.

Exploration 1

a–d. See Additional Answers.

e. It is the greatest of the common factors; The greatest common factors are circled in diagrams for parts (a) through (d).

Exploration 2

a. 18 and 27, 180 and 55;
$2 \cdot 3 \cdot 3 = 18$, $3 \cdot 3 \cdot 3 = 27$,
$2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 = 180$, $5 \cdot 11 = 55$

b–d. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.NS.B.4

Preparing to Teach

- The ability to find all factors, including prime factors, is essential to finding the **greatest common factor** (GCF). **Common factors** and the GCF are useful when working with fractions, applications of division, and eventually algebraic
- Students s sets of nur
- **MP5 Use A** or method able to ass factors is factorizati

Motivate

- Introduce a cat or a Teaching
- Use the su cat and a the Venn of each re

Explorati

- Expect tha factors of
- **MP5 Use A** students i
- ❓ Have stud number in
- **FYI:** Two n
- **Connectio** factors for can be sev

Indicator 2f - This note appears in the Teaching Edition to point out that students have to make sense of the Venn diagram to be able to answer the question in Exploration 2(a). Students then have to apply this knowledge by making their own Venn diagrams in part (b). (See the next page which shows the corresponding Student Edition content.)

MP1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.... Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Exploration 2

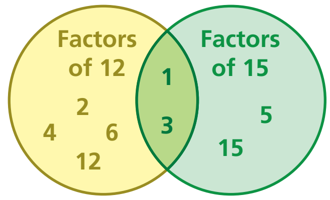
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students must make sense of the structure of the Venn diagram, and how it visually represents the factors of two numbers. Each number in the overlap is a factor of both original numbers.
- **Note:** Part (b) is similar to part (a) in the first exploration; however, part (b) only asks for prime factors.
- Ask volunteers to share their reasoning about each part.
- In part (d), ask guiding questions so students recognize two methods for finding the GCF: using lists of factors and using prime factorizations.
- **Common Error:** Students may choose only the largest prime number in the overlap of the two circles. Guide those students through another example to clarify any misunderstanding.

1.4 Greatest Common Factor

Learning Target: Find the greatest common factor of two numbers.

Success Criteria:
- I can explain the meaning of factors of a number.
- I can use lists of factors to identify the greatest common factor of numbers.
- I can use prime factors to identify the greatest common factor of numbers.

A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the factors of 12 and 15. Numbers that are factors of both 12 and 15 are represented by the overlap of the two circles.



EXPLORATION 1 Identifying Common Factors

Work with a partner. In parts (a)–(d), create a Venn diagram that represents the factors of each number and identify any *common factors*.

a. 36 and 48

b. 16 and 56

c. 30 and 75

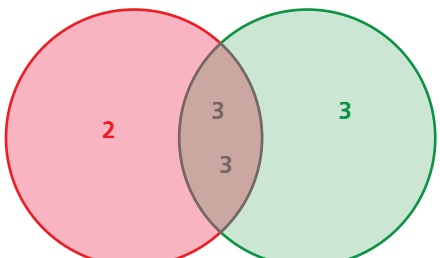
e. Look at the Venn diagrams in parts (… the *greatest common factor* of each p … in each diagram.

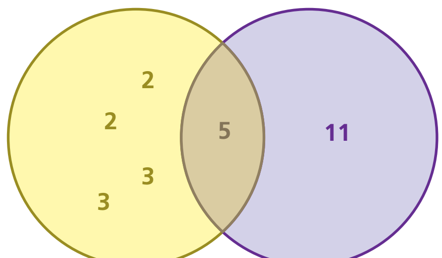
Indicator 2f - In Exploration 2a, students must make sense of the Venn diagram to be able to answer the question. Students then have to apply this knowledge by making their own Venn diagrams in part (b).

EXPLORATION 2 Using Prime Factors

Work with a partner.

a. Each Venn diagram represents the prime factorizations of two numbers. Identify each pair of numbers. Explain your reasoning.





b. Create a Venn diagram that represents the prime factorizations of 36 and 48.

c. Repeat part (b) for the remaining number pairs in Exploration 1.

d. MP **STRUCTURE** Make a conjecture about the relationship between the greatest common factors you found in Exploration 1 and the numbers in the overlaps of the Venn diagrams you just created.

Math Practice

Interpret a Solution

What does the diagram representing the prime factorizations mean?

Learning Target

Multiply decimals and solve problems involving multiplication of decimals.

Success Criteria

- Multiply decimals by whole numbers.
- Multiply decimals by decimals.
- Evaluate expressions involving multiplication of decimals.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Point out that many different words can indicate multiplication, such as *times*, *multiply*, and *product*. Have students keep a record of the different phrases they read in word problems that indicate multiplication, so they will become familiar with them.

Exploration 1

a. i. 0.8×0.5; 0.4; *Sample answer:* 40 squares are shaded twice, and $\frac{40}{100} = 0.4$.

ii. 0.4×0.9; 0.36; *Sample answer:* 36 squares are shaded twice, and $\frac{36}{100} = 0.36$.

iii. 0.5×1.5; 0.75; *Sample answer:* 75 squares are shaded twice, and $\frac{75}{100} = 0.75$.

iv 0.7×1.7; 1.19; *Sample answer:* 119 squares are shaded twice, and $\frac{119}{100} = 1.19$.

bc . See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

COMMON CORE **STATE STANDARDS**
6.NS.B.3

Preparing to Teach

- Although students are required to multiply decimals using the standard algorithm, they can develop that procedural knowledge by building upon the area models used in prior grades.
- Beginning with decimals represented on a 10-by-10 grid, and then a 10-by-20 grid, gives all students an entry point to understanding decimal multiplication.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** The area model for multiplication is a visual tool used to help students make sense of decimal multiplication. The factors are represented by the dimensions of the overlapping shaded rectangle and the area is the product. Making connections within mathematics is a habit you want all students to develop.

Motivate

- Show the class a quarter. Ask students to tell you the value of *x* quarters, where *x* is a multiple or power of 10. Record the value of the quarters.

❓"What is the value of 10 quarters?" $10 \times \$0.25 = \$2.50$

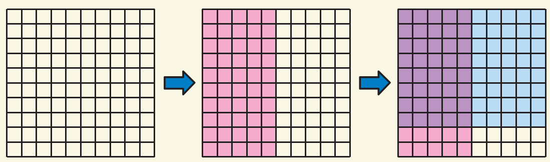
❓"What is the value of 20 quarters?" $20 \times \$0.25 = \$5.00$

❓"What is the value of 100 quarters?" $100 \times \$0.25 = \$25.00$

- In today's lesson, students will extend their understanding of decimal multiplication beyond the context of money.

Exploration 1

❓Discuss model (i) and say, "The large square is the unit square, representing the whole number 1. What does each small square represent?" 0.01 "When pink and blue are mixed together, what color is made?" purple It might be helpful to sketch the stages shown.



❓"What number is represented by the pink shading? the blue shading?" 0.5; 0.8 "How is the product represented in the area model?" the purple shading

- Help students to understand that the area being considered is the rectangle with dimensions 0.5 by 0.8. Because the large square represents 1, the area must be less than 1.
- If available, offer each pair of students colored pencils and base-ten grid paper.

❓Ask, "Why are the last two models larger than the first two models?" Because one factor is greater than 1.

❓After completing part (a) for model (iv), ask, "Do you see a relationship to the problem 17×7?" The digits are the same, but the decimal point is in a different location.

- Have pairs discuss part (b). As you circulate, ask students to explain part (b) before starting part (c). Students should describe the process of multiplying decimals as the same as multiplying whole numbers, except that the decimal point will be placed differently.

2.5 Multiplying Decimals

Learning Target: Multiply decimals and solve problems involving multiplication of decimals.

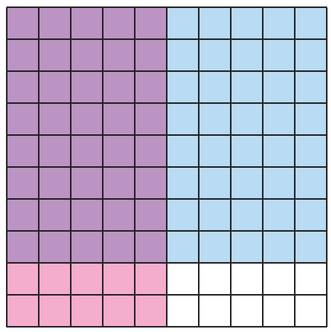
Success Criteria:
- I can multiply decimals by whole numbers.
- I can multiply decimals by decimals.
- I can evaluate expressions involving multiplication of decimals.

EXPLORATION 1 Multiplying Decimals

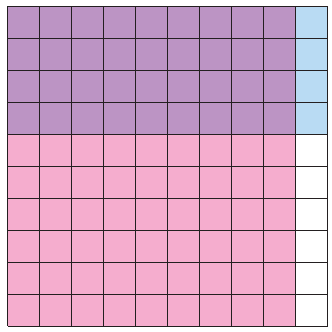
Work with a partner.

a. Write the multiplication expression represented by each area model. Then find the product. Explain how you found your answer.

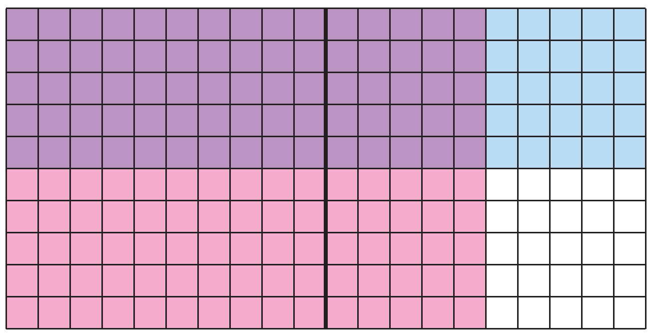
i.



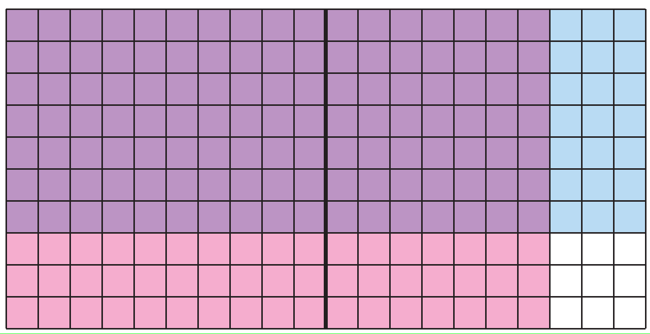
ii.



iii.



iv.



b. How can you find the products in part (a) without using a model? How do you know where to place the decimal points in the answers?

c. Find the product of 0.55 and 0.45. Explain how you found your answer.

Math Practice

View as Components

How can you use an area model to find the product?

Laurie's Notes

EXAMPLE 5

- ? "What information is given?" The length of the room is 15 feet and the width is 80% of the length.
- Write "80% of 15 = ?" and ask students what method(s) can be used to answer the question.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students should know a variety of ways to find 80% of 15. It is important that students find an approach to solving a problem that makes sense.
- Students need a basic understanding that if the width is 80% of the length, the width is shorter than the length. This may not be obvious to all students.
- The percent bar model visually shows the 5 equal parts of 15. Students can use this model to find 20%, 40%, 60%, or 80% of 15.
- **MP6 Attend to Precision:** Be sure that students label the answer with the correct units. Note that the question asks for the area.

Try It

- **Turn and Talk:** Tell students to answer the question using a percent bar model and a decimal.
- You may need to remind students to find the area, not just the width. Read the question out loud to pairs who stop too soon.

Self-Assessment for Concepts & Skills

- Some of these exercises differ from the examples. Probe students' understanding of the concept of percents and the ways in which problems can be presented.
- In Exercise 10, do students comprehend the different ways in which a percent problem can be written? There are many ways of saying the same thing.
- Consider students' explanations in Exercise 13. Well-developed number sense allows students to quickly decide if the answer is greater than or less than the original number. Allow students who have mastered this understanding to explain to others who are struggling.
- ? Distribute *Response Cards* with A, B, and C to students. "Which percent representation do you find most helpful? Hold up A for *equation*, B for *ratio table*, or C for *bar model*." Student responses will provide insight into your students' stage of thinking.

ELL Support

Have students complete Exercises 11 and 12 with a partner. Have each pair display their answers on a whiteboard for your review. Then have two pairs form a group to discuss and complete Exercises 13 and 14. Have two groups compare their answers. If there is disagreement, provide support.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Extra Example 5

The length of a rectangular garden is 225% of its width. The width of the garden is 4 meters. What is the area of the garden? 36 square meters

Try It

9. 7920 ft^2

Formative Assessment Tip

Response Cards
This technique is used as a quick check to see whether students' knowledge of a skill, technique, or procedure is correct. Students are given cards at the beginning of class that are held up in front of them in response to a question. The cards can be prepared in advance with particular responses, such as A, B, C, and D; or 1, 2, and 3; or True and False. The cards can also be left blank for students to write their responses on. If you plan to use the cards multiple times, consider laminating them. *Response Cards* give all students the opportunity to participate in the lesson, because you are soliciting information from everyone and not just those who raise their hands. Because the cards are held facing the teacher, it is a private way to gather quick information about students' understanding.

Self-Assessment for Concepts & Skills

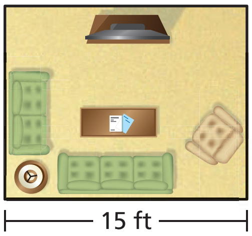
10. Twenty percent of what number is 30?; 150; 6

11. 9 **12.** 60

13. less than 52; 52 is more than 100% of the number.

14. Move the decimal point of the number one place to the left; *Sample answer:* Multiplying by 10% is the same as dividing by 10.

EXAMPLE 5 Solving a Percent Problem

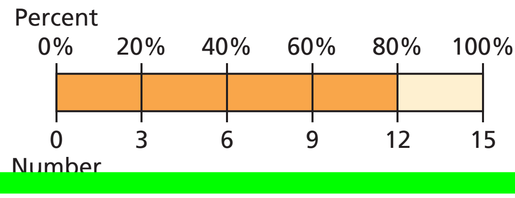


The width of a rectangular room is 80% of its length. What is the area of the room?

Find 80% of 15 feet to determine the width of the room.

$$80\% \text{ of } 15 = 0.8 \times 15$$
$$= 12$$

The width is 12 feet.



Use the formula for the area of a rectangle.

$$\text{Area} = \ell w$$
$$= 15(12)$$
$$= 180$$

So, the area of the room is 180 square feet.

Try It

9. The width of a rectangular stage is 55% of its length. The stage is 120 feet long. What is the area of the stage?

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

- What is twenty percent of 30?
- What is one-fifth of 30?
- Twenty percent of what number is 30?
- What is two-tenths of 30?

11. **FINDING THE PERCENT OF A NUMBER** Find 12% of 75.

12. **FINDING THE WHOLE** 35% of what number is 21?

13. MP **NUMBER SENSE** If 52 is 130% of a number, is the number greater than or less than 52? Explain.

14. MP **STRUCTURE** How can you find 10% of any number without multiplying or dividing? Explain your reasoning.

In-Class Problem Solving

5 Connecting Concepts

Using the Problem-Solving Plan

1. A store sells 18 pairs of the wireless earbuds shown. Customers saved a total of $882 on the earbuds. Find the original price of the earbuds.

Understand the problem. You know the percent discount on a pair of wireless earbuds, the number of pairs of earbuds sold, and the total amount of money that customers saved. You are asked to find the original price of the earbuds.

Make a plan. First, write an expression that represents the total amount of money that customers pay for the earbuds. Then factor the expression to find the discount (in dollars) on each pair of earbuds. Finally, solve a percent problem to find the original price.

Solve and check. Use the plan to solve the problem. Then check your solution.

2. All of the weight plates in a gym are l
kilograms. You want to convert the w
Write an expression to find the num
z kilograms. Then find the weight in
that weighs 20.4 kilograms.

3. You buy apple chips and banana chips in the ratio of 2 : 7.
   a. How many ounces of banana chips do you buy when you buy n ounces of apple chips? Explain.
   b. You buy 12 ounces of apple chips. How many ounces of banana chips do you buy?

Performance Task

Describing Change

At the beginning of this chapter, you watched a STEAM video called "Shadow Drawings." You are now ready to complete the performance task related to this video, available at *BigIdeasMath.com*. Be sure to use the problem-solving plan as you work through the performance task.

Connecting Concepts pages combine previously learned skills with concepts from the current chapter, so students practice problem solving for high-stakes assessments. Students use the Problem-Solving Plan along with a variety of problem-solving strategies.

Problem-Solving Strategies

Using an appropriate strategy will help you make sense of problems as you study the mathematics in this course. You can use the following strategies to solve problems that you encounter.

- Use a verbal model.
- Draw a diagram.
- Write an equation.
- Solve a simpler problem.
- Sketch a graph or number line.
- Make a table.
- Make a list.
- Break the problem into parts.

Self-Assessment for Problem Solving gives teachers the opportunity for continual formative assessment and allows students to communicate mathematically in every lesson.

Self-Assessment for Problem Solving

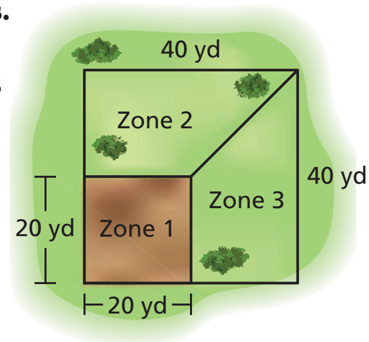
Solve each exercise. Then rate your understanding of the success criteria in your journal.

13. A youth club receives a discount on each pizza purchased for a party. The original price of each pizza is x dollars. The club leader purchases 8 pizzas for a total of $(8x - 32)$ dollars. Factor the expression. What can you conclude about the discount?

14. Three crates of food are packed on a shuttle departing for the Moon. Each crate weighs x pounds. On the Moon, the combined weight of the crates is $(3x - 81)$ pounds. What can you conclude about the weight of each crate on the Moon?

EXAMPLE 4 Modeling Real Life

The diagram shows landing zones for skydivers. Zone 1 is for experts. The remaining space is divided in half and designated as Zones 2 and 3 for tandem divers. What is the area of Zone 2?



Understand the problem.

You are given the dimensions of landing zones and that the areas of Zones 2 and 3 are equal. You are asked to find the area of Zone 2.

Make a plan.

Use a verbal model to write an expression. Subtract the area of Zone 1 from the total area to find the combined area of Zones 2 and 3. Then multiply the combined area by one-half.

Solve and check.

Verbal Model One-half (Total area − Area of Zone 1)

Expression $\frac{1}{2}$ (40^2 − 20^2)

$$\frac{1}{2}(40^2 - 20^2) = \frac{1}{2}(1600 - 400) \quad \text{Evaluate powers in parentheses.}$$

$$= \frac{1}{2}(1200) \quad \text{Perform operation in parentheses.}$$

$$= 600 \quad \text{Multiply } \frac{1}{2} \text{ and 1200.}$$

The area of Zone 2 is 600 square yards.

Check Verify that the areas of the three zones have a sum equal to the total area.

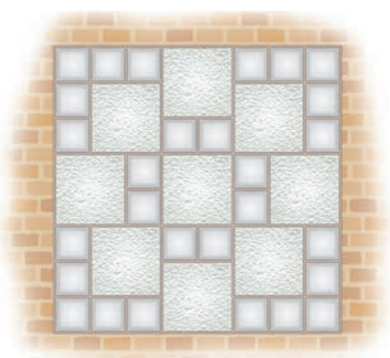
$400 + 600 + 600 \stackrel{?}{=} 1600$

$1600 = 1600$ ✓

Self-Assessment for Problem Solving

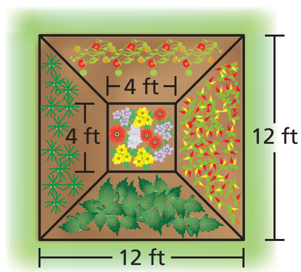
Solve each exercise. Then rate your understanding of the success criteria in your journal.

16. A square plot of land has side lengths of 40 meters. An archaeologist divides the land into 64 equal parts. What is the area of each part?



17. A glass block window is made of two different-sized glass squares. The window has side lengths of 40 inches. The large glass squares have side lengths of 10 inches. Find the total area of the small glass squares.

18. **DIG DEEPER!** A square vegetable garden has side lengths of 12 feet. You plant flowers in the center portion as shown. You divide the remaining space into 4 equal sections and plant tomatoes, onions, zucchini, and peppers. What is the area of the onion section?



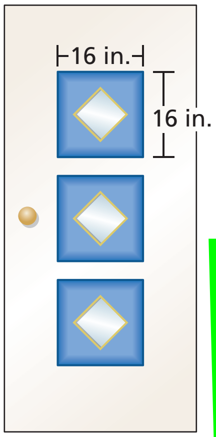
USING ORDER OF OPERATIONS **Evaluate the expression.**

28. $12 - 2(7 - 4)$ **29.** $4(3 + 5) - 3(6 - 2)$ **30.** $6 + \frac{1}{4}(12 - 8)$

31. $9^2 - 8(6 + 2)$ **32.** $4(3 - 1)^3 + 7(6) - 5^2$ **33.** $8\left[\left(1\frac{1}{6} + \frac{5}{6}\right) \div 4\right]$

34. $7^2 - 2\left(\frac{11}{8} - \frac{3}{8}\right)$ **35.** $8(7.3 + 3.7 - 8) \div 2$ **36.** $2^4(5.2 - 3.2) \div 4$

37. $\frac{6^2(3 + 5)}{4}$ **38.** $\frac{12^2 - 4(6) + 1}{11^2}$ **39.** $\frac{26 \div 2 + 5}{3^2 - 3}$

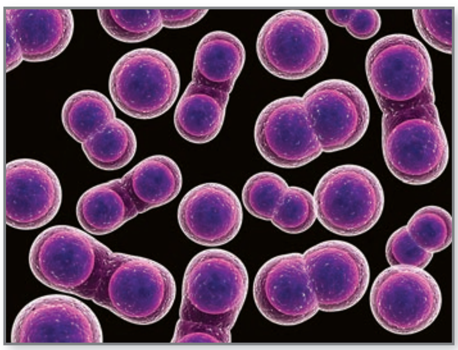


40. MP **PROBLEM SOLVING** Before a show, there are 8 people in a theater. Five groups of 4 people enter, and then three groups of 2 people leave. Evaluate the expression $8 + 5(4) - 3(2)$ to find how many people are in the theater.

41. **MODELING REAL LIFE** The front door of a house is painted white and blue. Each window is a square with a side length of 7 inches. What is the area of the door that is painted blue?

42. MP **PROBLEM SOLVING** You buy 6 notebooks, 10 folders, 1 pack of pencils, and 1 lunch box for school. After using a $10 gift card, how much do you owe? Explain how you solved the problem.

43. **OPEN-ENDED** Use all four operations and at least one exponent to write an expression that has a value of 100.



44. MP **REPEATED REASONING** A Petri dish contains 35 cells. Every day, each cell in the Petri dish divides into 2 cells in a process called *mitosis*. How many cells are there after 14 days? Justify your answer.

45. MP **REASONING** Two groups collect litter along the side of a road. It takes each group 5 minutes to clean up a 200-yard section. How long does it take both groups working together to clean up 2 *miles*? Explain how you solved the problem.

46. MP **NUMBER SENSE** Copy each statement. Insert $+$, $-$, $\times$, or $\div$ symbols to make each statement true.

a. $27 \square 3 \square 5 \square 2 = 19$ **b.** $9^2 \square 11 \square 8 \square 4 \square 1 = 60$

c. $5 \square 6 \square 15 \square 9 = 24$ **d.** $14 \square 2 \square 7 \square 3 \square 9 = 10$

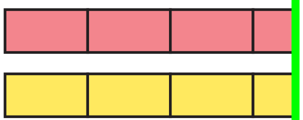
EXAMPLE 4 Modeling Real Life

In a seven-game basketball series, a team's power forward scores 8 points for every 5 points the center scores. The forward scores 60 more points than the center in th[...] does each player sc[...]

The ratio of the forwa[...] is 8 : 5. Represent the[...]

Forward

Center



1 part represents[...]

8 parts represent[...]

5 parts represent[...]

So, the forward s[...] 100 points in the[...]

Indicator 2f - In #9-11, students model with mathematics by being presented real-life situations. Students can use tape diagrams if they want to solve the problems.

MP4 Model with mathematics - Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community.... Mathematically proficient students who can apply what they know... are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

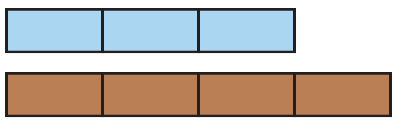
Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. The tape diagram represents the ratio of the numbers of planets in two different solar systems. There are 8 planets in Solar System B. How many planets are in Solar System A?

Solar System A

Solar System B



10. You and your friend play an arcade game. You score 5 points for every 9 points that your friend scores. You score 320 points less than your friend. How many points do you each score?

11. **DIG DEEPER!** Your team wins 18 medals at a track meet. The medals are gold, silver, and bronze in a ratio of 2 : 2 : 5. How many of each medal were won by your team?

USING A TAPE DIAGRAM **A bowl contains blueberries and strawberries. You are given the total number of berries in the bowl and the ratio of blueberries to strawberries. How many of each berry are in the bowl?**

21. 16 berries; 3 : 1

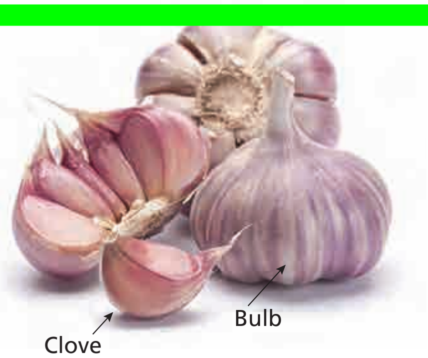
22. 10 berries; 2 for every 3

23. 12 berries; 1 to 2

24. 20 berries; 4 : 1

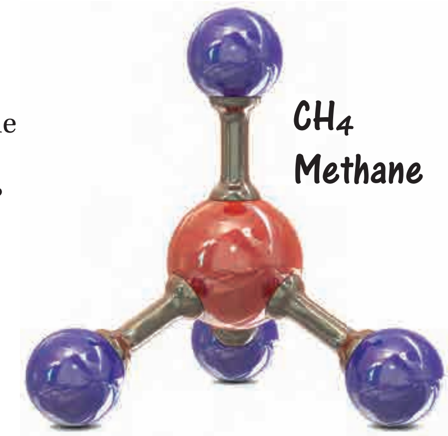
25. 48 berries; 9 to 3

26. 46 berries; 11 for every 12



27. **MP PROBLEM SOLVING** You separate bulbs of garlic into two groups: one for planting and one for cooking. The tape diagram represents the ratio of bulbs for planting to bulbs for cooking. You use 6 bulbs for cooking. Each bulb has 8 cloves. How many cloves of garlic will you plant?

Planting	▭▭▭▭▭▭ (6 units)
Cooking	▭ (1 unit)



28. **MODELING REAL LIFE** Methane gas contains carbon atoms and hydrogen atoms in the ratio of 1 : 4. A sample of methane gas contains 92 hydrogen atoms. How many carbon atoms are in the sample? How many total atoms are in the sample?

29. **MODELING REAL LIFE** There are 8 more girls than boys in a school play. The ratio of boys to girls is 5 : 7. How many boys and how many girls are in the play?

30. **DIG DEEPER!** A baseball team sells tickets for two games. The ratio of sold tickets to unsold tickets for the first game was 7 : 3. For the second game, the ratio was 13 : 2. There were 240 unsold tickets for the second game. How many tickets were sold for the first game?

31. **MP PROBLEM SOLVING** You have $150 in a savings account and you have some cash. The tape diagram represents the ratio of the amounts of money. You want to have twice the amount of money in your savings account as you have in cash. How much of your cash should you deposit into your savings account?

Savings Account	▭▭ (2 units)
Cash	▭▭▭▭▭ (5 units)

32. **DIG DEEPER!** A fish tank contains tetras, guppies, and minnows. The ratio of tetras to guppies is 4 : 2. The ratio of minnows to guppies is 1 : 3. There are 60 fish in the tank. How many more tetras are there than minnows? Justify your answer.

EXAMPLE 2 Using a Unit Rate to Solve a Rate Problem

A piece of space junk travels 5 miles every 6 seconds.

a. How far does the space junk travel in 30 seconds?

The ratio of miles to seconds is 5 : 6. Divide by 6 to find the unit rate in miles per second. Then multiply each quantity by 30 to find the distance traveled in 30 seconds.

	$\div 6$	$\times 30$	
Distance (miles)	5	$\frac{5}{6}$	25
Time (seconds)	6	1	30
	$\div 6$	$\times 30$	

The space junk travels 25 miles in 30 seconds.

b. How many seconds does it take the space junk to travel 2 miles?

The ratio of seconds to miles is 6 : 5. Divide by 5 to find the unit rate in seconds per mile. Then multiply each quantity by 2 to find the time to travel 2 miles.

	$\div 5$	$\times 2$	
Time (seconds)	6	$\frac{6}{5}$	$\frac{12}{5}$
Distance (miles)	5	1	2
	$\div 5$	$\times 2$	

It takes $\frac{12}{5} = 2\frac{2}{5}$ seconds for the space junk to travel 2 miles.

In Example 2, notice that you can use one step in each ratio table. Multiply by $\frac{1}{6} \times 30 = 5$ in part (a) and $\frac{1}{5} \times 2 = \frac{2}{5}$ in part (b).

Try It

2. WHAT IF? Repeat Example 2 when the space junk travels 3 miles every 5 seconds.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING UNIT RATES **Write a unit rate for the situation.**

3. 5 revolutions in 50 seconds

4. 1400 words for every 4 pages

5. WHICH ONE DOESN'T BELONG? Which rate does *not* belong with the other three? Explain your reasoning.

8 pounds for every 2 feet	12 pounds per 3 feet
20 pounds per 4 feet	24 pounds for every 6 feet

28. **MODELING REAL LIFE** Lightning strikes Earth 1000 times in 10 seconds.

a. How many times does lightning strike in 12 seconds?

b. How many seconds does it take for lightning to strike 7250 times?

29. **MODELING REAL LIFE** You earn $35 for washing 7 cars.

a. How much do you earn for washing 4 cars?

b. You earn $45. How many cars did you wash?

COMPARING RATES **Decide whether the rates are equivalent.**

30. 24 laps in 6 minutes
72 laps in 18 minutes

31. 126 points for every 3 games
210 points for every 5 games

32. 15 breaths for every 36 seconds
90 breaths for every 3 minutes

33. $16 for 4 pounds
$1 for 4 ounces

34. **MODELING REAL LIFE** An office printer prints 25 photos in 12.5 minutes. A home printer prints 15 photos in 6 minutes. Which printer is faster? How many more photos can you print in 12 minutes using the faster printer?

35. **MODELING REAL LIFE** You jog 2 kilometers in 12 minutes. Your friend jogs 3 kilometers in 16.5 minutes. Who jogs faster? How much sooner will the faster jogger finish a five-kilometer race?

36. **MP PROBLEM SOLVING** A softball team has a budget of $200 for visors. The athletic director pays $90 for 12 sun visors. Is there enough money in the budget to purchase 15 more sun visors? Explain your reasoning.

37. **DIG DEEPER!** The table shows the amounts of food collected by two homerooms. Homeroom A collects 21 additional items of food. How many more items does Homeroom B need to collect to have more items per student?

	Homeroom A	Homeroom B
Students	24	16
Canned Food	30	22
Dry Food	42	24

38. **MP REASONING** A runner completed a 26.2-mile marathon in 210 minutes.

a. Estimate the unit rate, in miles per minute.

b. Estimate the unit rate, in minutes per mile.

c. Another runner says, "I averaged 10-minute miles in the marathon." Is this runner talking about the unit rate described in part (a) or in part (b)? Explain your reasoning.

39. **DIG DEEPER!** You can complete one-half of a job in an hour. Your friend can complete one-third of the same job in an hour. How long will it take to complete the job if you work together?

EXAMPLE 4 Modeling Real Life

You plant a cypress tree that is 10 inches tall. Each year, its height increases by 15 inches. Write an expression that represents the height (in inches) after *t* years. What is the height after 9 years?

Make a table showing the height of the tree each year for the first several years. Use the results to write an expression and evaluate the expression when $t = 9$.

The height is *increasing*, so *add* 15 each year, as shown in the table.

Year, t	Height (inches)
0	10
1	$10 + 15(1) = 25$
2	$10 + 15(2) = 40$
3	$10 + 15(3) = 55$
4	$10 + 15(4) = 70$

When t is 0, the height is 10 inches.

You can see that an expression is $10 + 15t$.

Sometimes, as in Example 3, a variable represents a single value. Other times, as in Example 4, a variable can represent more than one value.

Evaluate $10 + 15t$ when $t = 9$.

$$10 + 15t = 10 + 15(9) = 145$$

So, the height (in inches) after t years is $10 + 15t$. After 9 years, the height of the tree is 145 inches.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. A company rents paddleboards by charging a rental fee plus an hourly rate. Write an expression that represents the cost (in dollars) of renting a paddleboard for h hours. How much does an eight-hour rental cost?

13. **DIG DEEPER!** A county fair charges an entry fee of \$7 and \$0.75 for each ride token. You have \$15. Write an expression that represents the amount (in dollars) you have left after entering the fair and purchasing n tokens. How many tokens can you purchase? How much money do you have left after purchasing 6 tokens?

30. MP **PROBLEM SOLVING** To rent a moving truck for the day, it costs \$33 plus \$1 for each mile driven.

a. Write an expression that represents the cost (in dollars) to rent the truck.

b. You drive the truck 300 miles. How much do you pay?

WRITING PHRASES **Give two ways to write the expression as a phrase.**

31. $n + 6$ **32.** $4w$ **33.** $15 - b$ 3

Indicator 2f - In #30 and #39-41, students model with mathematics by being presented real-life situations. Students write and evaluate expressions for these situations.

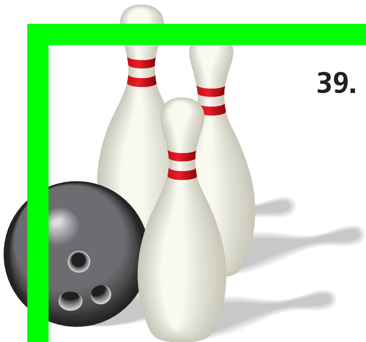
EVALUATING EXPRESSIONS **Write the phrase as an expression. T** **expression when $x = 5$ and $y = 20$.**

35. 3 less than the quotient of a number y and 4

36. the sum of a numb all divided by 3

37. 6 more than the product of 8 and a number x

38. the quotient of 40 and the difference of a number y and 16



39. **MODELING REAL LIFE** It costs \$3 to bowl a game and \$2 for shoe rental.

a. Write an expression that represents the total cost (in dollars) of g games.

b. Use your expression to find the total cost of 8 games.

40. **MODELING REAL LIFE** Florida has 8 less than 5 times the number of counties in Arizona. Georgia has 25 more than twice the number of counties in Florida.

a. Write an expression that represents the number of counties in Florida.

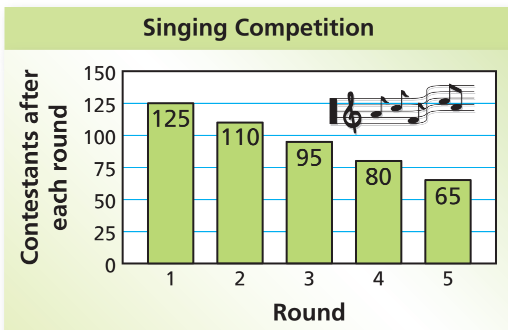
b. Write an expression that represents the number of counties in Georgia.

c. Arizona has 15 counties. How many do Florida and Georgia have?

41. MP **PATTERNS** There are 140 people in a singing competition. The graph shows the results for the first five rounds.

a. Write an expression that represents the number of people after each round.

b. Assuming this pattern continues, how many people compete in the ninth round? Explain your reasoning.



42. MP **NUMBER SENSE** The difference between two numbers is 8. The lesser number is a. Write an expression that represents the greater number.

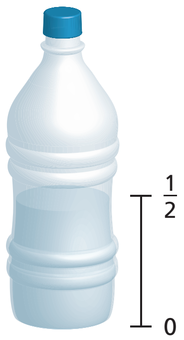
43. MP **NUMBER SENSE** One number is four times another. The greater number is x. Write an expression that represents the lesser number.

2.1 Multiplying Fractions

Learning Target: Find products involving fractions and mixed numbers.

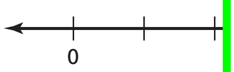
Success Criteria:
- I can draw a model to explain fraction multiplication.
- I can multiply fractions.
- I can find products involving mixed numbers.
- I can interpret products involving fractions and mixed numbers to solve real-life problems.

EXPLORATION 1 Using Models to Solve a Problem



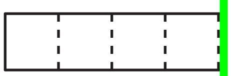
Work with a partner. A bottle of water is $\frac{1}{2}$ full. You drink $\frac{2}{3}$ of the water. Use one of the models to find the portion of the bottle of water that you drink. Explain your steps.

- number line



- area model

- tape diagram



Indicator 2f - In Exploration 1, students are presented 3 models in Exploration 1 and choose 1 of them to solve both Exploration 1 and Exploration 2.

MP5 Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations.... Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

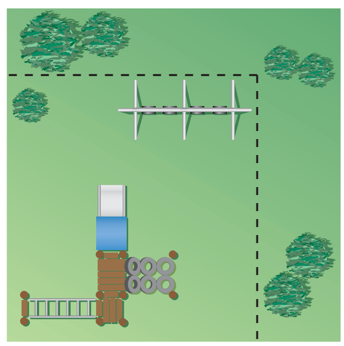
EXPLORATION 2 Solving a Prob

Math Practice

Find General Methods

How can you use your answer to find a method for multiplying fractions?

Work with a partner. A park has a playground that is $\frac{3}{4}$ of its width and $\frac{4}{5}$ of its length.



a. Use a model to find the portion of the park that is covered by the playground. Explain your steps.

b. How can you find the solution of part (a) without using a model?

19. **MODELING REAL LIFE** A radio station collects donations for a new broadcast tower. The cost to construct the tower is \$25.50 per inch.

a. Represent the ratio relationship using a graph.

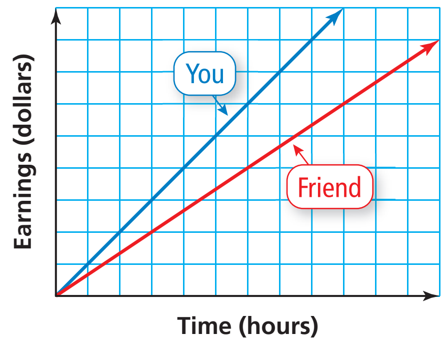
b. How much does it cost to fund 4.5 inches of the construction?

20. **MODELING REAL LIFE** Your school organizes a clothing drive as a fundraiser for a class trip. The school earns \$100 for every 400 pounds of donated clothing.

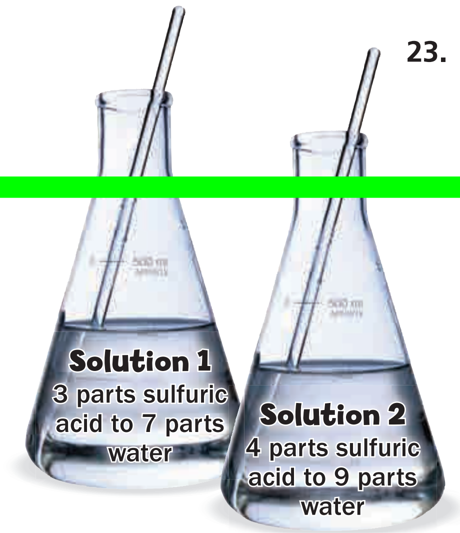
a. Represent the ratio relationship using a graph.

b. How much money does your school earn for donating 2200 pounds of clothing?

21. MP **NUMBER SENSE** Just by looking at the graph, determine who earns a greater hourly wage. Explain.



22. **MODELING REAL LIFE** An airplane traveling from Chicago to Los Angeles travels 15 miles every 2 minutes. On the return trip, the plane travels 25 miles every 3 minutes. Graph each ratio relationship in the same coordinate plane. Does the plane fly faster when traveling to Los Angeles or to Chicago?



23. **MODELING REAL LIFE** Your freezer produces 8 ice cubes every 2 hours. Your friend's freezer produces 24 ice cubes every 5 hours. Graph each ratio relationship in the same coordinate plane. Whose freezer produces ice faster?

24. MP **CHOOSE TOOLS** A chemist prepares two acid solutions.

a. Use a ratio table to determine which solution is more acidic.

b. Use a graph to determine which solution is more acidic.

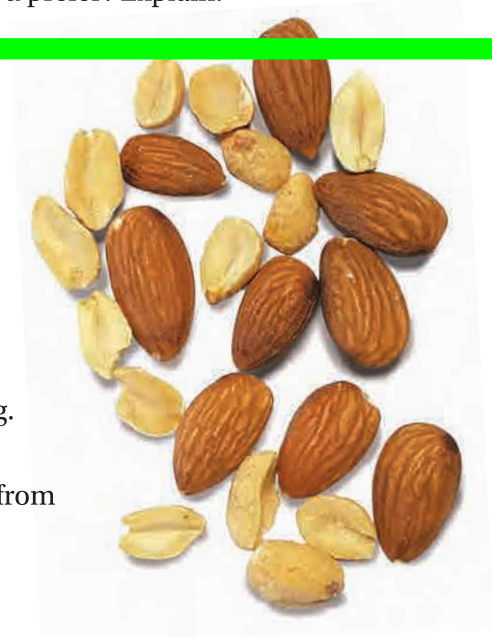
c. Which method do you prefer? Explain.

25. **DIG DEEPER!** A company offers a nut mixture with 7 peanuts for every 3 almonds. The company changes the mixture to have 9 peanuts for every 5 almonds, but the number of nuts per container does not change.

a. How many nuts are in the smallest possible container?

b. Graph each ratio relationship. What can you conclude?

c. Almonds cost more than peanuts. Should the company change the price of the mixture? Explain your reasoning.



26. MP **STRUCTURE** The point (p, q) is on the graph of values from a ratio table. What are two additional points on the graph?

3.6 Converting Measures

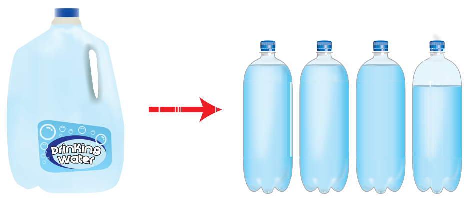
Learning Target: Use ratio reasoning to convert units of measure.

Success Criteria:
- I can write conversion facts as unit rates.
- I can convert units of measure using ratio tables.
- I can convert units of measure using conversion factors.
- I can convert rates using conversion factors.

EXPLORATION 1 Estimating Unit Conversions

Work with a partner. You are given 4 one-liter containers and a one-gallon container.

a. A full one-gallon container can be used to fill the one-liter containers, as shown below. Write a unit rate that estimates the number of liters per gallon.

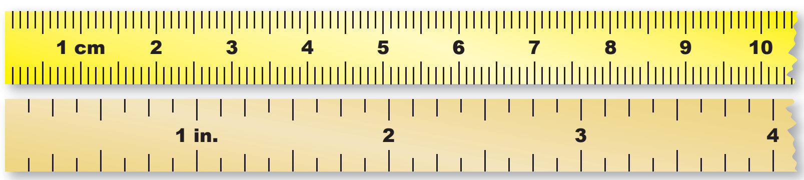


b. A full one-liter container can be used to partially fill the one-gallon container, as shown below. Write a unit rate that estimates the number of gallons per liter.

c. Estimate the number of liters in 5.5 gallons and the number of gallons in 12 liters. What method(s) did you use? What other methods could you have used?

EXPLORATION 2 Converting Units in a Rate

Work with a partner. The rate that a caterpillar moves is given in inches per minute. Using the rulers below, how can you convert the rate to centimeters per second? Justify your answer.



Math Practice

Recognize Usefulness of Tools

When would rulers be a useful tool for converting between centimeters and inches? When would they not be useful?

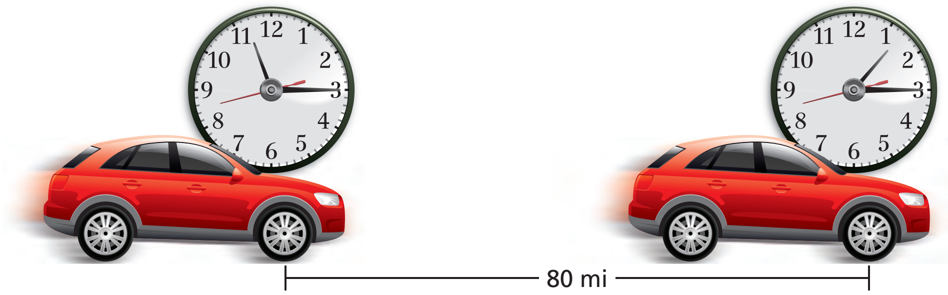
3.5 Rates and Unit Rates

Learning Target: Understand the concept of a unit rate and solve rate problems.

Success Criteria:
- I can find unit rates.
- I can use unit rates to solve rate problems.
- I can use unit rates to compare rates.

EXPLORATION 1 Using a Diagram

Work with a partner. The diagram shows a story problem.



a. What information can you obtain from the diagram?

b. Assuming that the car travels at a constant speed, how far does the car travel in 3.25 hours? Explain your method.

c. Draw a speedometer that shows the speed of the car. How can you use the speedometer to answer part (b)?

Math Practice

Specify Units

What are the units for the speed in part (c)? Why is it important to keep track of units in ratio problems?

EXPLORATION 2 Using Equivalent Ratios

Work with a partner. Count the number of times you can clap your hands in 12 seconds. Have your partner record your results. Then switch roles with your partner and repeat the process.

a. Using your results and your partner's results, write ratios that represent the numbers of claps for every 12 seconds.

b. Explain how you can use the ratios in part (a) to find the numbers of times you and your partner can clap your hands in 2 minutes, in 2.5 minutes, and in 3 minutes.

EXAMPLE 3 Writing an Algebraic Expression

The length of Interstate 90 from the West Coast to the East Coast is 153.5 miles more than 2 times the length of Interstate 15 from southern California to northern Montana. Let *m* be the length of Interstate 15. Which expression can you use to represent the length of Interstate 90?

A. $2m + 153.5$ **B.** $2m - 153.5$

C. $153.5 - 2m$ **D.** $153.5m + 2$

Variables can be lowercase or uppercase. Make sure you consistently use the same case for a variable when solving a problem.

The word *times* means *multiplication*. So, multiply 2 and *m*.

The phrase *more than* means *addition*. So, add $2m$ and 153.5.

$2m + 153.5$

The correct answer is **A**.

Try It

7. Your friend has 5 more than twice as many game tokens as you. Let *t* be the number of game tokens you have. Write an expression for the ...r of game tokens your friend has.

Indicator 2f - In #11, students have to use precision to determine whether the phrases have the same meaning.

MP6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other.

...ssessment for Concepts & Skills

...ercise. Then rate your understanding of the success criteria ...al.

...PRESSIONS **Write the phrase as an expression.**

...of 7 and 11 **9.** 5 subtracted from 9

...ENT WORDS, SAME QUESTION Which is different? ...oth" expressions.

...more than x

x increased by 12

...ake away 12

the sum of x and 12

11. **MP PRECISION** Your friend says that the phrases below have the same meaning. Is your friend correct? Explain your reasoning.

the difference of a number x and 12

the difference of 12 and a number x

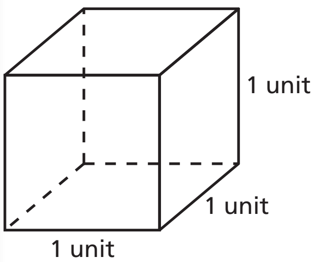
7.7 Volumes of Rectangular Prisms

Learning Target: Find volumes and missing dimensions of rectangular prisms.

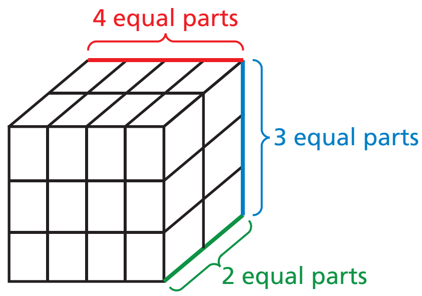
Success Criteria:
- I can use a formula to find the volume of a rectangular prism.
- I can use a formula to find the volume of a cube.
- I can use the volume of a rectangular prism and two of its dimensions to find the other dimension.
- I can apply volumes of rectangular prisms to solve real-life problems.

Recall that the **volume** of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in *cubic units*.

EXPLORATION 1 Using a Unit Cube

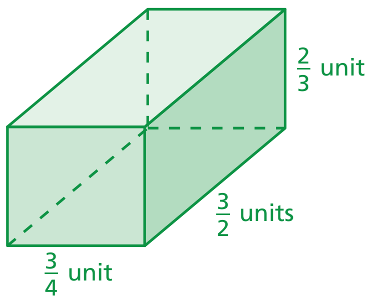


Work with a partner. A *unit cube* is a cube with an edge length of 1 unit. The parallel edges of the unit cube have been divided into 2, 3, and 4 equal parts to create smaller rectangular prisms that are identical.

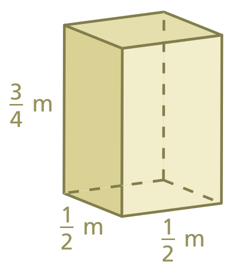


a. The volumes of the identical prisms are equal. What else can you determine about the volumes of the prisms? Explain.

b. Use the identical prisms in part (a) to find the volume of the prism below. Explain your reasoning.



c. How can you use a unit cube to find the volume of the prism below? Explain.



Math Practice

Communicate Precisely

In part (c), explain why you decided to divide the unit cube in the way you did.

d. Do the formulas $V = Bh$ and $V = \ell wh$ work for rectangular prisms with fractional edge lengths? Give examples to support your answer.

EXAMPLE 2 Finding a Missing Dimension of a Rectangular Prism

Volume = 1792 in.^{3}

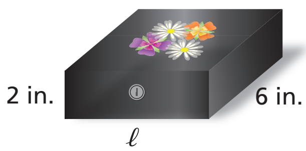
Find the height of the computer tower.

$V = \ell wh$	Write formula for volume.
$1792 = 16(7)h$	Substitute values.
$1792 = 112h$	Simplify.
$\frac{1792}{112} = \frac{112h}{112}$	Division Property of Equality
$16 = h$	Simplify.

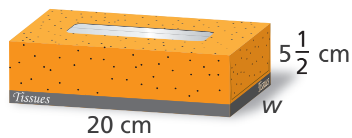
So, the height of the computer tower is 16 inches.

Try It **Find the missing dimension of the prism.**

3. Volume = 72 in.^{3}



4. Volume = 1375 cm^3



Self-Assessment for Concepts & Skills

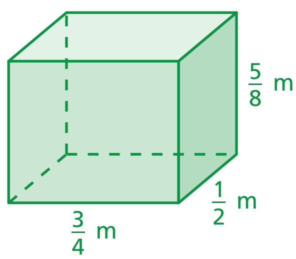
Solve each exercise. Then rate your understanding of the success criteria in your journal.

5. **CRITICAL THINKING** Explain how volume and surface area are different.

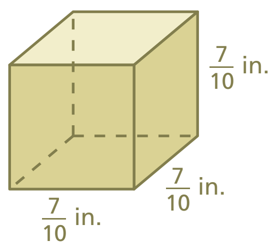
6. **FINDING A MISSING DIMENSION** The base of a rectangular prism has an area of 24 square millimeters. The volume of the prism is 144 cubic millimeters. Make a sketch of the prism. Then find the height of the prism.

FINDING VOLUME **Find the volume of the prism.**

7.



8.



8.7 Writing and Graphing Inequalities

Learning Target: Write inequalities and represent solutions of inequalities on number lines.

Success Criteria:
- I can write word sentences as inequalities.
- I can determine whether a value is a solution of an inequality.
- I can graph the solutions of inequalities.

EXPLORATION 1 Understanding Inequality Statements

Work with a partner. Create a number line on the floor with both positive and negative integers.

a. For each statement, stand at a number on your number line that makes the statement true. On what other numbers can you stand?

- Class starts more than 3 minutes late.

- You need at least 3 peaches for a recipe.

- The temperature is at most 3 degrees Celsius.

- After playing a video game for 5 minutes, you have fewer than 3 points.

Math Practice

State the Meaning of Symbols

You know the inequality symbols $<$ and $>$. What do the symbols $\leq$ and $\geq$ mean?

b. How can you represent the solutions of each statement in part (a) on a number line?

Laurie's Notes

Scaffolding Instruction

- Students have used a powerful visual representation of **common factors** to find the **greatest common factor** (GCF). Now they will move on to more efficient methods: lists of factors and factor trees.
- **Emerging:** Students who want to continue using Venn diagrams, or who struggled with the two different approaches in the explorations, will benefit from guided instruction in Examples 1 and 2. The Try It exercises will allow you to assess their progress.
- **Proficient:** Students who were confident in both explorations, and were able to make the connection between the greatest common factor and common prime factors, can self-assess using the Try It exercises.

EXAMPLE 1

- Using lists of factors to find the GCF is similar to the method used in Exploration 1.
- "What are the factors of 24?" 1, 2, 3, 4, 6, 8, 12, 24
- "What are the factors of 40?" 1, 2, 4, 5, 8, 10, 20, 40
- "Which factors appear in both lists?" 1, 2, 4, 8
- **FYI:** Students should not just say, "The greatest common factor is 8." The complete answer is, "The greatest common factor of 24 and 40 is 8."

Indicator 2f - This note exists for teachers to remind students that they have to be precise with their answers. Students should provide answers that clearly answer the question.

MP6 Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other.

Try It

- **Neighbor Check:** Students should complete
then share their methods and answers with
reasoning in Exercises 2 and 3.
- "How do you find the GCF of three numbers?
all three numbers and choose the greatest.

EXAMPLE 2

- Using prime factorizations to find the GCF is
Exploration 2.
- "Which prime factors do 12 and 56 have in c
- The greatest common factor of two (or more
prime factors that they have in common. The
12 and 56 is 2 • 2, or 4.
- The solution can be checked by the first me
- "Does it matter which method you use? Expl
you the correct answer.
- "How do you decide which method to use?"
few factors, use the lists of factors method.
large numbers of factors, use the prime fact

Try It

- Have students use whiteboards to complete the exercises. As they share their factor trees, encourage students to study other boards and compare their factor trees to other factor trees. This is a good opportunity to show that even though factor trees may differ, they result in the same prime factorization.

Extra Example 1

Find the GCF of 36 and 54 using lists of factors. 18

ELL Support

Have students work in pairs to complete Try It Exercises 1–3. Each partner should factor one of the given numbers by listing factors, as demonstrated in Example 1. Then partners should compare their lists and circle the common factors.

Beginner: State the greatest common factor.

Intermediate: Use a complete sentence to verbally identify the greatest common factor.

Advanced: Identify the greatest common factor, and explain the
...nd it.

using

Try It

4. 5

5. 2

6. 15

1.4 Lesson

Key Vocabulary
Venn diagram, *p. 21*
common factors, *p. 22*
greatest common factor, *p. 22*

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

EXAMPLE 1 Finding the GCF Using Lists of Factors

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Circle the common factors.

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

So, the GCF of 24 and 40 is 8.

Try It **Find the GCF of the numbers using lists of factors.**

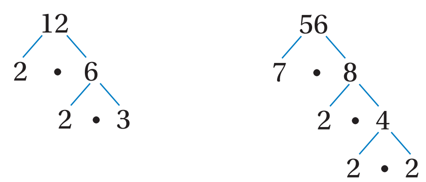
1. 8, 36 **2.** 18, 72 **3.** 14, 28, 49

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Make a factor tree for each number.



Write the prime factorization of each number.

$12 = 2 \cdot 2 \cdot 3$

$56 = 2 \cdot 2 \cdot 2 \cdot 7$

Circle the common prime factors.

$2 \cdot 2 = 4$

Find the product of the common prime factors.

So, the GCF of 12 and 56 is 4.

Examples 1 and 2 show two different methods for finding the GCF. After solving with one method, you can use the other method to check your answer.

Try It **Find the GCF of the numbers using prime factorizations.**

4. 20, 45 **5.** 32, 90 **6.** 45, 75, 120

Laurie's Notes

EXAMPLE 5

? "What information is given?" The length of the room is 15 feet and the width is 80% of the length.

- Write "80% of 15 = ?" and ask students what method(s) can be used to answer the question.
- **MP1 Make Sense of Problems and Persevere in Solving Them:** Students should know a variety of ways to find 80% of 15. It is important that students find an approach to solving a problem that makes sense.
- Students need a basic understanding that if the width is 80% of the length, the width is shorter than the length. This may not be obvious to all students.
- The percent bar model visually shows the 5 equal parts of 15. Students can
- **MP6 Attend to Precision:** Be sure that students label the answer with the correct units. Note that the question asks for the area.

Try It

- **Turn and Talk:** Tell students to answer the question using a percent bar model and a decimal.
- You may need to remind students to find the area, not just the width. Read the question out loud to pairs who stop too soon.

Self-Assessment for Concepts & Skills

- Some of these exercises differ from the examples. Probe students' understanding of the concept of percents and the ways in which problems can be presented.
- In Exercise 10, do students comprehend the different ways in which a percent problem can be written? There are many ways of saying the same thing.
- Consider students' explanations in Exercise 13. Well-developed number sense allows students to quickly decide if the answer is greater than or less than the original number. Allow students who have mastered this understanding to explain to others who are struggling.

? Distribute *Response Cards* with A, B, and C to students. "Which percent representation do you find most helpful? Hold up A for *equation*, B for *ratio table*, or C for *bar model*." Student responses will provide insight into your students' stage of thinking.

ELL Support

Have students complete Exercises 11 and 12 with a partner. Have each pair display their answers on a whiteboard for your review. Then have two pairs form a group to discuss and complete Exercises 13 and 14. Have two groups compare their answers. If there is disagreement, provide support.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Extra Example 5

The length of a rectangular garden is 225% of its width. The width of the garden is 4 meters. What is the area of the garden? 36 square meters

Try It

9. 7920 ft^2

Formative Assessment Tip

Response Cards

This technique is used as a quick check to see whether students' knowledge of a skill, technique, or procedure is correct. Students are given cards at the beginning of class that are held up in front of them in response to a question. The cards can be prepared in advance with particular responses, such as A, B, C, and D; or 1, 2, and 3; or True and False. The cards can also be left blank for students to write their responses on. If you plan to use the cards multiple times, consider laminating them. *Response Cards* give all students the opportunity to participate in the lesson, because you are soliciting information from everyone and not just those who raise their hands. Because the cards are held facing the teacher, it is a private way to gather quick information about students' understanding.

Self-Assessment for Concepts & Skills

10. Twenty percent of what number is 30?; 150; 6

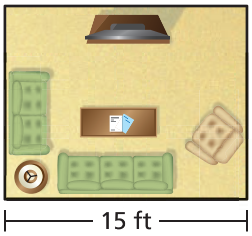
11. 9 **12.** 60

13. less than 52; 52 is more than 100% of the number.

14. Move the decimal point of the number one place to the left; *Sample answer:* Multiplying by 10% is the same as dividing by 10.

EXAMPLE 5 Solving a Percent Problem

The width of a rectangular room is 80% of its length. What is the area of the room?

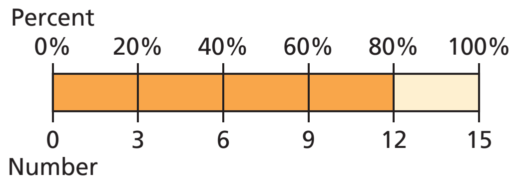


Find 80% of 15 feet to determine the width of the room.

80% of $15 = 0.8 \times 15$

$= 12$

The width is 12 feet.



Use the formula for the area of a rectangle.

$$\begin{aligned} \text{Area} &= \ell w \\ &= 15(12) \\ &= 180 \end{aligned}$$

So, the area of the room is 180 square feet.

Try It

9. The width of a rectangular stage is 55% of its length. The stage is 120 feet long. What is the area of the stage?

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

- What is twenty percent of 30?
- What is one-fifth of 30?
- Twenty percent of what number is 30?
- What is two-tenths of 30?

11. **FINDING THE PERCENT OF A NUMBER** Find 12% of 75.

12. **FINDING THE WHOLE** 35% of what number is 21?

13. MP **NUMBER SENSE** If 52 is 130% of a number, is the number greater than or less than 52? Explain.

14. MP **STRUCTURE** How can you find 10% of any number without multiplying or dividing? Explain your reasoning.

Laurie's Notes

EXAMPLE 5

- Students are asked to evaluate expressions involving more than one operation. They need to remember the order of operations.
- ? Write the expressions and ask, "How many operations will be performed in part (a)? in part (b)?" 2; 2
- ? "In part (a), which operation should you perform first?" multiplication "In part (b), which operation should you perform first?" evaluate the exponent
- These questions usually elicit a comment about the order of operations, at which time you can probe for understanding.
- Make students aware that they are working on the third success criterion, so that they can assess their understanding in the Self-Assessment for Concepts & Skills.

Try It

- Have students complete the exercises independently and then display their answers on whiteboards. Check students' work and look for errors involving the order of operations.

Self-Assessment for Concepts & Skills

- **MP6 Attend to Precision:** Have students work in groups to complete Exercise 17. This exercise should lead to a rich discussion that includes vocabulary, such as *terms*, *constants*, *coefficients*, *variables*, *expressions*, and *operations*. Students should share their reasoning using precise mathematical language.
- Have students complete Exercises 18–23 independently.
- In Exercises 22 and 23, check their level of understanding of the second and third success criteria.

ELL Support

Allow students to practice language by working in pairs. Have two pairs compare their answers. If there is a disagreement, pairs should work together to reach a consensus. Check comprehension of Exercises 17–19 by having each group write their final answers on a whiteboard. Discuss Exercises 21–23 with students to check for understanding.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Extra Example 5

a. Evaluate $\frac{t}{5} + 6$ when $t = 45$. 15

b. Evaluate $v^2 - 4$ when $v = 9$. 77

Try It

13. 31

14. 26

15. 29

16. 37.5

Self-Assessment for Concepts & Skills

17. $3(4) + 5$; the other three are algebraic expressions.

18. Terms: $9h$, 1
Coefficient: 9
Constant: 1

19. 1 **20.** 44

21. decrease; When you subtract greater and greater values from 20, you will have less and less left.

22. *Sample answer:* $5x + 4$; Use the order of operations.

23. no;
$8.2 \div m \cdot m \cdot m \cdot m$
$= \left(\frac{8.2}{m}\right) \cdot m^3$
and $8.2 \div m^4 = \frac{8.2}{m^4}$

EXAMPLE 5 Evaluating Expressions with Two Operations

a. Evaluate $3x - 14$ when $x = 5$.

$$\begin{aligned} 3x - 14 &= 3(5) - 14 && \text{Substitute 5 for } x. \\ &= 15 - 14 && \text{Using order of operations, multiply 3 and 5.} \\ &= 1 && \text{Subtract 14 from 15.} \end{aligned}$$

b. Evaluate $n^2 + 8.5$ when $n = 2$.

$$\begin{aligned} n^2 + 8.5 &= 2^2 + 8.5 && \text{Substitute 2 for } n. \\ &= 4 + 8.5 && \text{Using order of operations, evaluate } 2^2. \\ &= 12.5 && \text{Add 4 and 8.5.} \end{aligned}$$

Try It **Evaluate the expression when $y = 6$.**

13. $5y + 1$

14. $30 - 24 \div y$

15. $y^2 - 7$

16. $1.5 + y^2$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. WHICH ONE DOESN'T BELONG? Which expression does *not* belong with the other three? Explain your reasoning.

$2x + 1$ $\qquad 5w \cdot c$ $\qquad 3(4) + 5$ $\qquad 2y \cdot z$

18. ALGEBRAIC EXPRESSIONS Identify the terms, coefficients, and constants in the expression $9h + 1$.

EVALUATING EXPRESSIONS **Evaluate the expression when $m = 8$.**

19. $m - 7$

20. $5m + 4$

21. MP **NUMBER SENSE** Does the value of the expression $20 - x$ *increase*, *decrease*, or *stay the same* as x increases? Explain.

22. OPEN-ENDED Write an algebraic expression using more than one operation. When you evaluate the expression, how do you know which operation to perform first?

23. MP **STRUCTURE** Is the expression $8.2 \div m \cdot m \cdot m \cdot m$ the same as the expression $8.2 \div m^4$? Explain your reasoning.

2.2 Dividing Fractions

Learning Target: Compute quotients of fractions and solve problems involving division by fractions.

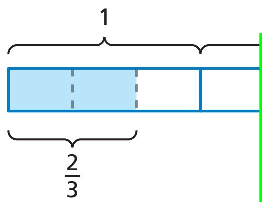
Success Criteria:
- I can draw a model to explain division of fractions.
- I can find reciprocals of numbers.
- I can divide fractions by fractions.
- I can divide fractions and whole numbers.

EXPLORATION 1 Dividing by Fractions

It may help to create a context for each question in Exploration 1. In part (a), suppose you want to cut 4 yards of string into pieces that are $\frac{2}{3}$ of a yard. How many pieces can you cut?

Work with a partner. Answer each question using a model.

a. How many two-thirds are in four?



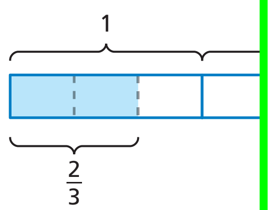
Indicator 2f - In Exploration 2, students look at what they did in Exploration 1 to figure out a pattern for writing a division expression and a multiplication expression.

MP7 Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property.... They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects....

b. How many three-fou

c. How many two-fifths

d. How many two-third



e. How many one-third

EXPLORATION 2 Finding a Pattern

Math Practice

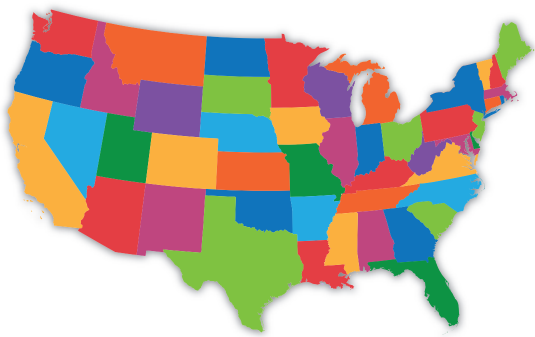
Look for Structure

Can the pattern you found be applied to division by a whole number? Why or why not?

Work with a partner. The table shows the division expressions from Exploration 1. Complete each multiplication expression so that it has the same value as the division expression above it. What can you conclude about dividing by fractions?

Division Expression	$4 \div \frac{2}{3}$	$3 \div \frac{3}{4}$	$\frac{4}{5} \div \frac{2}{5}$	$3 \div \frac{2}{3}$	$\frac{5}{6} \div \frac{1}{3}$
Multiplication Expression	$4 \times ?$	$3 \times ?$	$\frac{4}{5} \times ?$	$3 \times ?$	$\frac{5}{6} \times ?$

43. MP **PROBLEM SOLVING** In the contiguous United States, the ratio of states that border an ocean to states that do not border an ocean is 7 : 9. How many of the states border an ocean?



44. MP **REASONING** The value of a ratio is $\frac{4}{3}$. The second quantity in the ratio is how many times the first quantity in the ratio? Explain your reasoning.

45. **MODELING REAL LIFE** A train moving at a constant speed travels 3 miles every 5 minutes. A car moving at a constant speed travels 12 miles every 20 minutes. Are the vehicles traveling at the same speed? If not, which is faster?

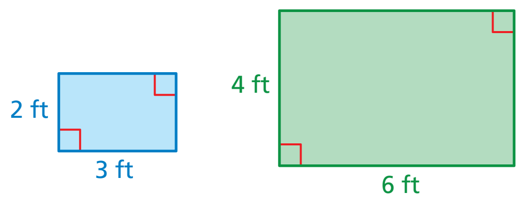
46. **CRITICAL THINKING** To win a relay race, you must swim 200 yards before your opponent swims 190 yards. You swim at a pace of 50 yards every 40 seconds. Your opponent swims at a pace of 10 yards every 8.5 seconds. Who wins the race? Justify your answer.

47. **DIG DEEPER!** There are 3 boys for every 2 girls in a dance competition. Does it make sense for there to be a total of 9 people in the competition? Explain.

48. **GEOMETRY** Use the blue and green rectangles.

a. Find the ratio of the length of the blue rectangle to the length of the green rectangle. Repeat this for width, perimeter, and area.

b. Compare your ratios in part (a).



49. MP **STRUCTURE** The ratio of the side lengths of a triangle is 2 : 3 : 4. The shortest side is 15 inches. What is the perimeter of the triangle? Explain.

TOKENS	
1 Token	$0.50
10 Tokens	$5.00
25 Tokens	$10.00
50 Tokens	$25.00
90 Tokens	$40.00

50. MP **PROBLEM SOLVING** A restaurant sells tokens that customers use to play games while waiting for their orders.

a. Which option is the best deal? Justify your answer.

b. What suggestions, if any, would you give to the restaurant about how it could modify the prices of tokens?

51. **DIG DEEPER!** There are 12 boys and 10 girls in your gym class. If 6 boys joined the class, how many girls would need to join for the ratio of boys to girls to remain the same? Justify your answer.

19. **MODELING REAL LIFE** A radio station collects donations for a new broadcast tower. The cost to construct the tower is \$25.50 per inch.

a. Represent the ratio relationship using a graph.

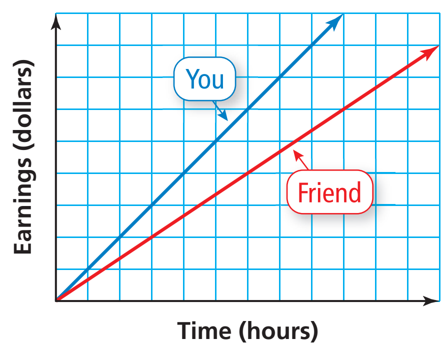
b. How much does it cost to fund 4.5 inches of the construction?

20. **MODELING REAL LIFE** Your school organizes a clothing drive as a fundraiser for a class trip. The school earns \$100 for every 400 pounds of donated clothing.

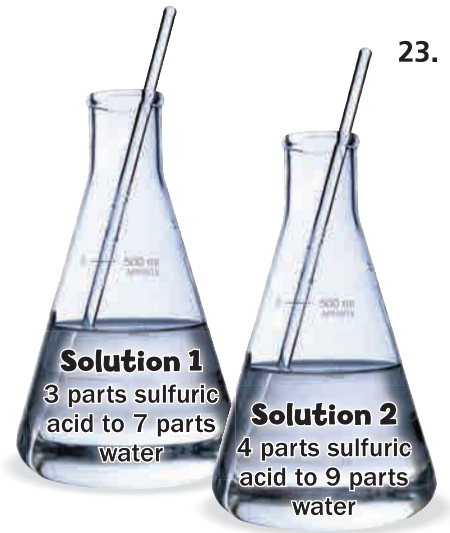
a. Represent the ratio relationship using a graph.

b. How much money does your school earn for donating 2200 pounds of clothing?

21. MP **NUMBER SENSE** Just by looking at the graph, determine who earns a greater hourly wage. Explain.



22. **MODELING REAL LIFE** An airplane traveling from Chicago to Los Angeles travels 15 miles every 2 minutes. On the return trip, the plane travels 25 miles every 3 minutes. Graph each ratio relationship in the same coordinate plane. Does the plane fly faster when traveling to Los Angeles or to Chicago?



23. **MODELING REAL LIFE** Your freezer produces 8 ice cubes every 2 hours. Your friend's freezer produces 24 ice cubes every 5 hours. Graph each ratio relationship in the same coordinate plane. Whose freezer produces ice faster?

24. MP **CHOOSE TOOLS** A chemist prepares two acid solutions.

a. Use a ratio table to determine which solution is more acidic.

b. Use a graph to determine which solution is more acidic.

c. Which method do you prefer? Explain.

25. **DIG DEEPER!** A company offers a nut mixture with 7 peanuts for every 3 almonds. The company changes the mixture to have 9 peanuts for every 5 almonds, but the number of nuts per container does not change.

a. How many nuts are in the smallest possible container?

b. Graph each ratio relationship. What can you conclude?

c. Almonds cost more than peanuts. Should the company change the price of the mixture? Explain your reasoning.

26. MP **STRUCTURE** The point (p, q) is on the graph of values from a ratio table. What are two additional points on the graph?

4.2 Percents and Decimals

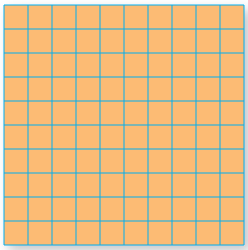
Learning Target: Write percents as decimals and decimals as percents.

Success Criteria:
- I can draw models to represent decimals.
- I can explain why the decimal point moves when multiplying and dividing by 100.
- I can write percents as decimals.
- I can write decimals as percents.

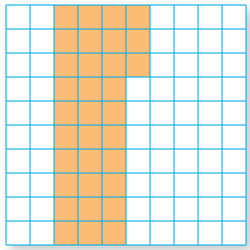
EXPLORATION 1 Interpreting Models

Work with a partner. Write a percent and a decimal shown by each model. How are percents and decimals related?

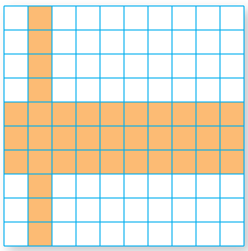
a.



b.



c.



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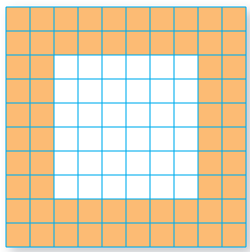
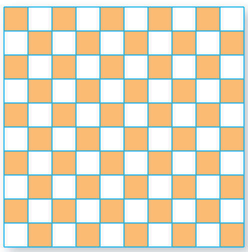
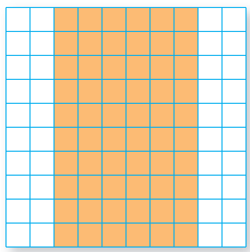


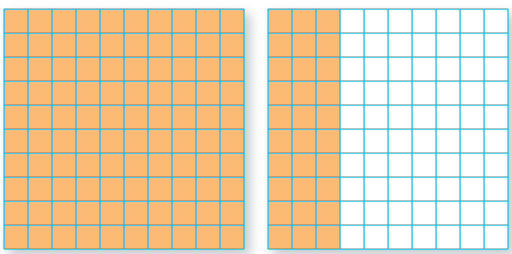
e.



f.



g.



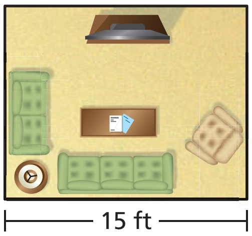
Math Practice

Look for Patterns

How does the decimal point move when you rewrite a percent as a decimal and a decimal as a percent? Explain why this occurs.

EXAMPLE 5 Solving a Percent Problem

The width of a rectangular room is 80% of its length. What is the area of the room?

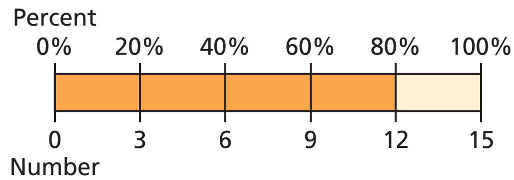


Find 80% of 15 feet to determine the width of the room.

80% of $15 = 0.8 \times 15$

$= 12$

The width is 12 feet.



Use the formula for the area of a rectangle.

$$\text{Area} = \ell w$$
$$= 15(12)$$
$$= 180$$

So, the area of the room is 180 square feet.

Try It

9. The width of a rectangular stage is 55% of its length. The stage is 120 feet long. What is the area of the stage?

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

10. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

What is twenty percent of 30?	What is one-fifth of 30?
Twenty percent of what number is 30?	What is two-tenths of 30?

11. **FINDING THE PERCENT OF A NUMBER** Find 12% of 75.

12. **FINDING THE WHOLE** 35% of what number is 21?

13. MP **NUMBER SENSE** If 52 is 130% of a number, is the number greater than or less than 52? Explain.

14. MP **STRUCTURE** How can you find 10% of any number without multiplying or dividing? Explain your reasoning.

EXAMPLE 3 Solving Equations Using Subtraction

a. Solve $x + 2 = 9$.

$x + 2 = 9$	Write the equation.
$\underline{-2} \quad \underline{-2}$	Subtraction Property of Equality
$x = 7$	Simplify.

Undo the addition.

The solution is $x = 7$.

Check

$x + 2 = 9$

$7 + 2 \stackrel{?}{=} 9$

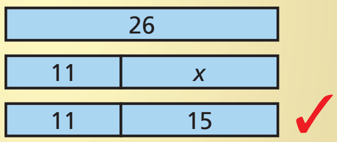
$9 = 9$ ✓

b. Solve $26 = 11 + x$.

$26 = 11 + x$	Write the equation.
$\underline{-11} \quad \underline{-11}$	Subtraction Property of Equality
$15 = x$	Simplify.

The solution is $x = 15$.

Another Method



Try It Solve the equation. Check your solution.

8. $s + 8 = 17$

9. $9 = y + 6$

10. $13 + m = 20$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

CHECKING SOLUTIONS **Tell whether the given value is a solution of the equation.**

11. $n + 8 = 42;\ n = 36$

12. $g - 9 = 24;\ g = 35$

SOLVING EQUATIONS **Solve the equation. Check your solution.**

13. $x - 8 = 12$

14. $b + 14 = 33$

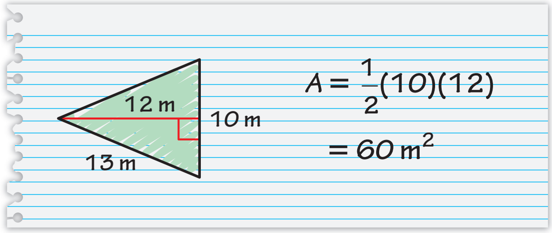
15. **WRITING** When solving $x + 5 = 16$, why do you subtract 5 from the left side of the equation? Why do you subtract 5 from the right side of the equation?

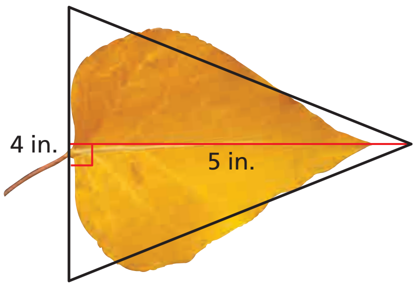
16. MP **REASONING** Do the equations have the same solution? Explain your reasoning.

$$x - 8 = 6$$
$$x - 6 = 8$$

17. MP **STRUCTURE** Just by looking at the equation $x + 6 + 2x = 2x + 6 + 4$, find the value of x. Explain your reasoning.

17. **YOU BE THE TEACHER** Your friend finds the area of the triangle. Is your friend correct? Explain your reasoning.



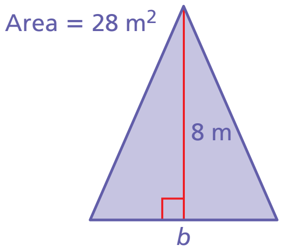


18. **MODELING REAL LIFE** Estimate the area of the cottonwood leaf.

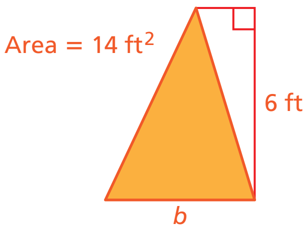
19. **MODELING REAL LIFE** A shelf has the shape of a triangle. The base of the shelf is 36 centimeters, and the height is 18 centimeters. Find the area of the shelf in square inches.

FINDING A MISSING DIMENSION **Find the missing dimension of the triangle.**

20.



21.



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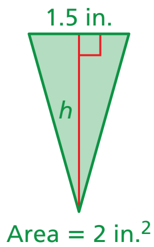
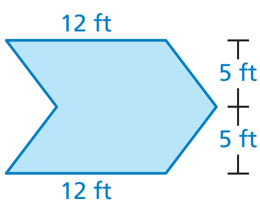
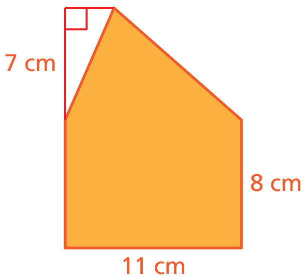


COMPOSITE FIGURES **Find the area of the figure.**

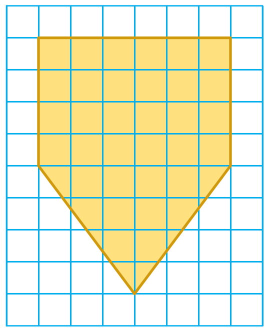
23.

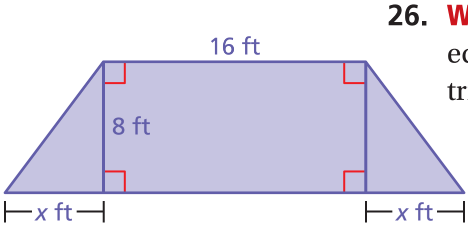


24.



25.



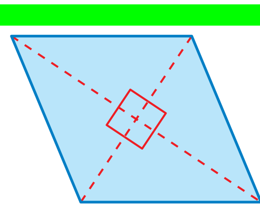


26. **WRITING** You know the height and the perimeter of an equilateral triangle. Explain how to find the area of the triangle. Draw a diagram to support your reasoning.

27. **CRITICAL THINKING** The total area of the polygon is 176 square feet. What is the value of x?

28. **MP REASONING** The base and the height of Triangle A are one-half the base and the height of Triangle B. How many times greater is the

29. **MP STRUCTURE** Use what you know about finding areas of triangles to write a formula for the area of a rhombus in terms of its diagonals. Compare the formula with your answer to Section 7.1 Exercise 34.



The symbols $\times$ and $\bullet$ are used to indicate multiplication. You can also use parentheses to indicate multiplication. For example, $3(2 + 7)$ is the same as $3 \times (2 + 7)$.

EXAMPLE 3 Using Order of Operations

Remember

You can interpret a fraction as division of the numerator by the denominator.

$\dfrac{a}{b} = a \div b$

a. Evaluate $9 + \dfrac{8-2}{3}$.

$9 + \dfrac{8-2}{3} = 9 + (8 - 2) \div 3$	Rewrite fraction as division.
$= 9 + 6 \div 3$	Perform operation in parentheses.
$= 9 + 2$	Divide 6 by 3.
$= 11$	Add 9 and 2.

b. Evaluate $10 - 8(13 + 7) \div 4^2$.

$10 - 8(13 + 7) \div 4^2 = 10 - 8(20) \div 4^2$	Perform operation in parentheses.
$= 10 - 8(20) \div 16$	Evaluate 4^2.
$= 10 - 160 \div 16$	Multiply 8 and 20.
$= 10 - 10$	Divide 160 by 16.
$= 0$	Subtract 10 from 10.

Try It **Evaluate the expression.**

7. $50 + 6(12 \div 4) - 8^2$ **8.** $5^2 - \frac{1}{5}(10 - 5)$ **9.** $\dfrac{8(2 + 5)}{7}$

Self-Assessment *for Concepts & Skills*

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING ORDER OF OPERATIONS **Evaluate the expression.**

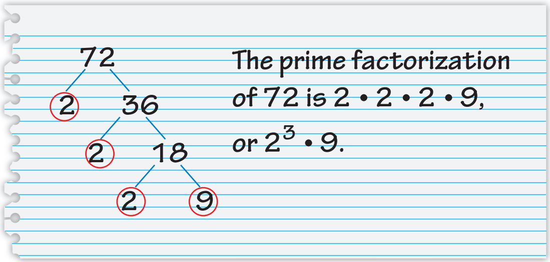
10. $7 + 2 \bullet 4$ **11.** $8 \div 4 \times 2$ **12.** $3(5 + 1) \div 3^2$

13. **WRITING** Why does $12 - 8 \div 2 = 8$, but $(12 - 8) \div 2 = 2$?

14. MP **REASONING** Describe the steps in evaluating the expression $8 \div (6 - 4) + 3^2$.

15. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$5^2 - 8 \times 2$ $5^2 - (8 \times 2)$ $5^2 - 2 \times 8$ $(5^2 - 8) \times 2$



52. **YOU BE THE TEACHER** Your friend finds the prime factorization of 72. Is your friend correct? Explain your reasoning.

USING A PRIME FACTORIZATION **Find the greatest perfect square that is a factor of the number.**

53. 250	**54.** 275	**55.** 392	**56.** 338
57. 244	**58.** 650	**59.** 756	**60.** 1290
61. 2205	**62.** 1890	**63.** 495	**64.** 4725

65. **VOCABULARY** A botanist separates plants into equal groups of 5 for an experiment. Is the total number of plants in the experiment *prime* or *composite*? Explain.

66. MP **REASONING** A teacher divides 36 students into equal groups for a scavenger hunt. Each group should have at least 4 students but no more than 8 students. What are the possible group sizes?

67. **CRITICAL THINKING** Is 2 the only even prime number? Explain.

68. MP **LOGIC** One table at a bake sale has 75 cookies. Another table has 60 cupcakes. Which table allows for more rectangular arrangements? Explain.

69. **PERFECT NUMBERS** A *perfect number* is a number that equals the sum of its factors, not including itself. For example, the factors of 28 are 1, 2, 4, 7, 14, and 28. Because $1 + 2 + 4 + 7 + 14 = 28$, 28 is a perfect number. What are the perfect numbers between 1 and 27?

70. MP **REPEATED REASONING** Choose any two perfect squares and find their product. Then multiply your answer by another perfect square. Continue this process. Are any of the products perfect squares? What can you conclude?

71. MP **PROBLEM SOLVING** The stage manager of a school play creates a rectangular stage that has whole number dimensions and an area of 42 square yards. String lights will outline the stage. What is the least number of yards of string lights needed to enclose the stage?

Rectangular Prism

Volume = 40 cubic inches

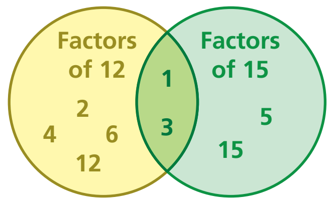
72. **DIG DEEPER!** Consider the rectangular prism shown. Using only whole number dimensions, how many different prisms are possible? Explain.

1.4 Greatest Common Factor

Learning Target: Find the greatest common factor of two numbers.

Success Criteria:
- I can explain the meaning of factors of a number.
- I can use lists of factors to identify the greatest common factor of numbers.
- I can use prime factors to identify the greatest common factor of numbers.

A **Venn diagram** uses circles to describe relationships between two or more sets. The Venn diagram shows the factors of 12 and 15. Numbers that are factors of both 12 and 15 are represented by the overlap of the two circles.



EXPLORATION 1

Identifying Common Factors

Work with a partner. In parts (a)–(d), create a Venn diagram that represents the factors of each number and identify any *common factors*.

a. 36 and 48

b. 16 and 56

c. 30 and 75

d. 54 and 90

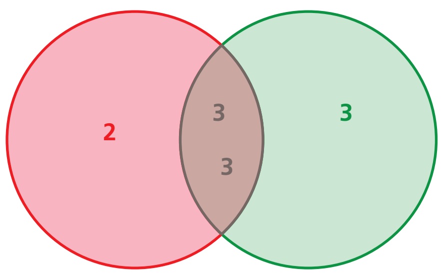
e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *greatest common factor* of each pair of numbers. Then circle it in each diagram.

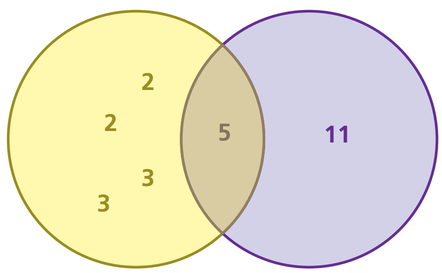
EXPLORATION 2

Using Prime Factors

Work with a partner.

a. Each Venn diagram represents the prime factorizations of two numbers. Identify each pair of numbers. Explain your reasoning.





b. Create a Venn diagram that represents the prime factorizations of 36 and 48.

c. Repeat part (b) for the remaining number pairs in Exploration 1.

d. MP **STRUCTURE** Make a conjecture about the relationship between the greatest common factors you found in Exploration 1 and the numbers in the overlaps of the Venn diagrams you just created.

Math Practice

Interpret a Solution

What does the diagram representing the prime factorizations mean?

1.5 Least Common Multiple

Learning Target: Find the least common multiple of two numbers.

Success Criteria:
- I can explain the meaning of multiples of a number.
- I can use lists of multiples to identify the least common multiple of numbers.
- I can use prime factors to identify the least common multiple of numbers.

EXPLORATION 1 Identifying Common Multiples

Work with a partner. In parts (a)–(d), create a Venn diagram that represents the first several multiples of each number and identify any *common multiples*.

a. 8 and 12 **b.** 4 and 14

c. 10 and 15 **d.** 20 and 35

e. Look at the Venn diagrams in parts (a)–(d). Explain how to identify the *least common multiple* of each pair of numbers. Then circle it in each diagram.

EXPLORATION 2 Using Prime Factors

Work with a partner.

a. Create a Venn diagram that represents the prime factorizations of 8 and 12.

b. Repeat part (a) for the remaining number pairs in Exploration 1.

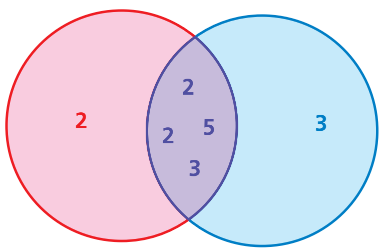
c. MP **STRUCTURE** Make a conjecture about the relationship between the least common multiples you found in Exploration 1 and the numbers in the Venn diagrams you just created.

Math Practice

Analyze Conjectures

How can you test your conjecture in part (c)?

d. The Venn diagram shows the prime factors of two numbers.

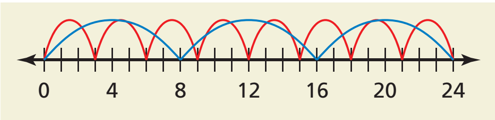


Use the diagram to complete the following tasks.

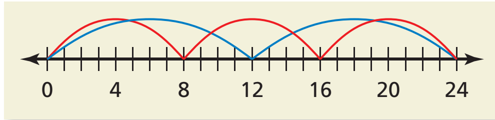
- Identify the two numbers.
- Find the greatest common factor.
- Find the least common multiple.

34. MP **REASONING** Which model represents an LCM that is different from the other three? Explain your reasoning.

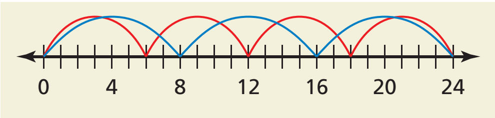
A.



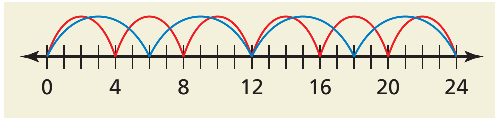
B.



C.



D.



FINDING THE LCM **Find the LCM of the numbers.**

35. 2, 3, 7 **36.** 3, 5, 11 **37.** 4, 9, 12

38. 6, 8, 15 **39.** 7, 18, 21 **40.** 9, 10, 28

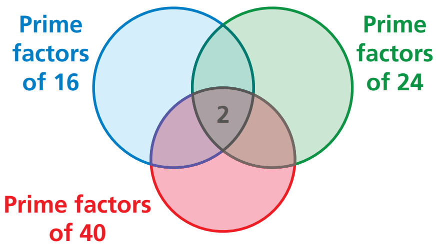
Indicator 2g.i - In #45-47, students have to construct arguments about whether the statements are always, sometimes, or never true.

MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

41. MP **PROBLEM SOLVING** At Unio
you notice that three subway lines
arrived at the same time. How lon
you wait until all three lines arrive
Union Station at the same time ag

42. **DIG DEEPER!** A radio station giv
$25 to every 25th caller, and a free
When will the station first give aw
When this happens, how much m
are given away?

43. MP **LOGIC** You and a friend are r
3 minutes, and your friend runs 2
stop running at the same time and
least possible numbers of miles yo



44.

c. What is the LCM of 16 and 40? 24 and 40? 16 and 24? Explain how you found your answers.

CRITICAL THINKING **Tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

45. The LCM of two different prime numbers is their product.

46. The LCM of a set of numbers is equal to one of the numbers in the set.

47. The GCF of two different numbers is the LCM of the numbers.

31. **YOU BE THE TEACHER** Your friend finds the quotient of $3\frac{1}{2}$ and $1\frac{2}{3}$. Is your friend correct? Explain your reasoning.

$$3\frac{1}{2} \div 1\frac{2}{3} = 3\frac{1}{2} \times 1\frac{3}{2} = \frac{7}{2} \times \frac{5}{2} = \frac{7 \times 5}{2 \times 2} = \frac{35}{4}, \text{ or } 8\frac{3}{4}$$

32. **MP PROBLEM SOLVING** A platinum nugget weighs $3\frac{1}{2}$ ounces. How many $\frac{1}{4}$-ounce pieces can be cut from the nugget?

ORDER OF OPERATIONS **Evaluate the expression. Write the answer in simplest form.**

33. $3 \div 1\frac{1}{5} + \frac{1}{2}$

34. $4\frac{2}{3} - 1\frac{1}{3} \div 2$

35. $\frac{2}{5} + 2\frac{1}{6} \div \frac{5}{6}$

36. $5\frac{5}{6} \div 3\frac{3}{4} - \frac{2}{9}$

37. $6\frac{1}{2} - \frac{7}{8} \div 5\frac{11}{16}$

38. $9\frac{1}{6} \div 5 + 3\frac{1}{3}$

39. $3\frac{3}{5} + 4\frac{4}{15} \div \frac{4}{9}$

40. $\frac{3}{5} \times \frac{7}{12} \div 2\frac{7}{10}$

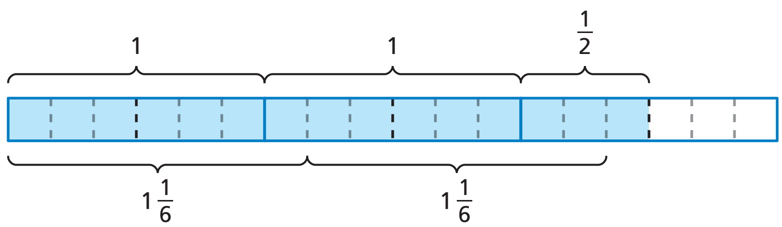
41. $4\frac{3}{8} \div \frac{3}{4} \bullet \frac{4}{7}$

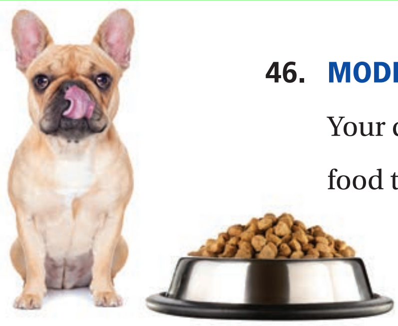
42. $1\frac{9}{11} \times 4\frac{7}{12} \div \frac{2}{3}$

43. $3\frac{4}{15} \div \left(8 \bullet 6\frac{3}{10}\right)$

44. $2\frac{5}{14} \div \left(2\frac{5}{8} \times 1\frac{3}{7}\right)$

45. **MP LOGIC** Your friend uses the model shown to state that $2\frac{1}{2} \div 1\frac{1}{6} = 2\frac{1}{6}$. Is your friend correct? Justify your answer using the model.





46. **MODELING REAL LIFE** A bag contains 42 cups of dog food. Your dog eats $2\frac{1}{3}$ cups of dog food each day. Is there enough food to last 3 weeks? Explain.

47. **DIG DEEPER!** You have 12 cups of granola and $8\frac{1}{2}$ cups of peanuts to make trail mix. What is the greatest number of full batches of trail mix you can make? Explain how you found your answer.

48. **MP REASONING** At a track and field meet, the longest shot-put throw by a boy is 25 feet 8 inches. The longest shot-put throw by a girl is 19 feet 3 inches. How many times greater is the longest shot-put throw by the boy than by the girl?

71. **DIG DEEPER!** The table shows the top three times in a swimming event at the Summer Olympics. The event consists of a team of four women swimming 100 meters each.

Women's 4 × 100 Freestyle Relay		
Medal	**Country**	**Time (seconds)**
Gold	Australia	210.65
Silver	United States	211.89
Bronze	Canada	212.89

a. Suppose the times of all four swimmers on each team were the same. For each team, how much time does it take a swimmer to swim 100 meters?

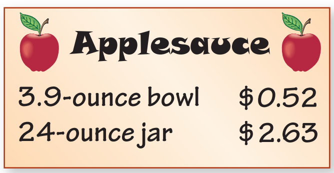
b. Suppose each U.S. swimmer completed 100 meters a quarter second faster. Would the U.S. team have won the gold medal? Explain your reasoning.

72. **MP PROBLEM SOLVING** To approximate the number of bees in a hive, multiply the number of bees that leave the hive in one minute by 3 and divide by 0.014. You count 25 bees leaving a hive in one minute. How many bees are in the hive?

73. **MP PROBLEM SOLVING** You are saving money to buy a new bicycle that costs \$155.75. You have \$30 and plan to save \$5 each week. Your aunt decides to give you an additional \$10 each week.

a. How many weeks will you have to save until you have enough money to buy the bicycle?

b. How many more weeks would you have to save to buy a new bicycle that costs \$203.89? Explain how you found your answer.



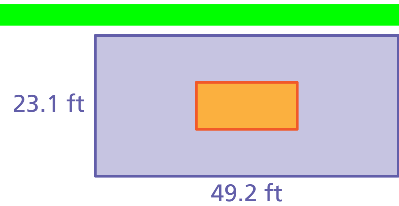
74. **MP PRECISION** A store sells applesauce in two sizes.

a. How many bowls of applesauce fit in a jar? Round your answer to the nearest hundredth.

b. Explain two ways to find the better buy.

c. Which is the better buy?

75. **GEOMETRY** The large rectangle's dimensions are three times the dimensions of the small rectangle.



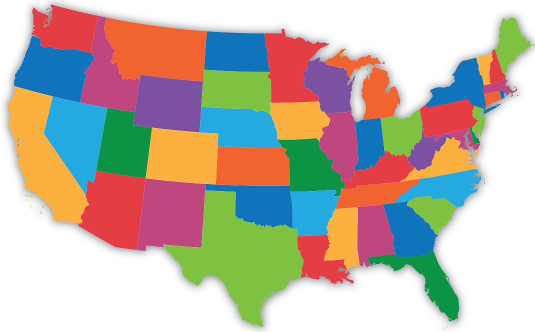
a. How many times greater is the perimeter of the large rectangle than the perimeter of the small rectangle?

b. How many times greater is the area of the large rectangle than the area of the small rectangle?

c. Are the answers to parts (a) and (b) the same? Explain why or why not.

d. What happens in parts (a) and (b) if the dimensions of the large rectangle are two times the dimensions of the small rectangle?

43. MP **PROBLEM SOLVING** In the contiguous United States, the ratio of states that border an ocean to states that do not border an ocean is 7 : 9. How many of the states border an ocean?



44. MP **REASONING** The value of a ratio is $\frac{4}{3}$. The second quantity in the ratio is how many times the first quantity in the ratio? Explain your reasoning.

45. **MODELING REAL LIFE** A train moving at a constant speed travels 3 miles every 5 minutes. A car moving at a constant speed travels 12 miles every 20 minutes. Are the vehicles traveling at the same speed? If not, which is faster?

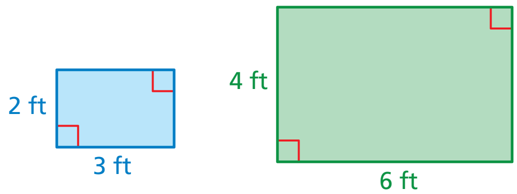
46. **CRITICAL THINKING** To win a relay race, you must swim 200 yards before your opponent swims 190 yards. You swim at a pace of 50 yards every 40 seconds. Your opponent swims at a pace of 10 yards every 8.5 seconds. Who wins the race? Justify your answer.

47. **DIG DEEPER!** There are 3 boys for every 2 girls in a dance competition. Does it make sense for there to be a total of 9 people in the competition? Explain.

48. **GEOMETRY** Use the blue and green rectangles.

a. Find the ratio of the length of the blue rectangle to the length of the green rectangle. Repeat this for width, perimeter, and area.

b. Compare your ratios in part (a).



49. MP **STRUCTURE** The ratio of the side lengths of a triangle is 2 : 3 : 4. The shortest side is 15 inches. What is the perimeter of the triangle? Explain.

TOKENS	
1 Token	$0.50
10 Tokens	$5.00
25 Tokens	$10.00
50 Tokens	$25.00
90 Tokens	$40.00

50. MP **PROBLEM SOLVING** A restaurant sells tokens that customers use to play games while waiting for their orders.

a. Which option is the best deal? Justify your answer.

b. What suggestions, if any, would you give to the restaurant about how it could modify the prices of tokens?

51. **DIG DEEPER!** There are 12 boys and 10 girls in your gym class. If 6 boys joined the class, how many girls would need to join for the ratio of boys to girls to remain the same? Justify your answer.

EXAMPLE 2 Writing Decimals as Percents

a. Write 0.47 as a percent.

$0.47 = 0.47 = 47\%$

b. Write 0.663 as a percent.

$0.663 = 0.663 = 66.3\%$

c. Write 1.8 as a percent.

$1.8 = 1.80 = 180\%$

d. Write 0.009 as a percent.

$0.009 = 0.009 = 0.9\%$

Try It

Write the decimal as a percent.

5. 0.94 **6.** 1.2 **7.** 0.316 **8.** 0.005

EXAMPLE 3 Writing a Fraction as a Percent and a Decimal

On a math test, you earn 92 out of a possible 100 points. Which of the following is *not* another way of expressing 92 out of 100?

A. $\frac{23}{25}$ **B** 92% **C** $\frac{17}{20}$ **D.** 0.92

Write "92 out of 100" as a fraction, a decimal, and a percent.

92 out of 100 $= \frac{92}{100}$

$= 92\%$ Eliminate Choice B.

$= \frac{23}{25}$ Eliminate Choice A.

$= 0.92$ Eliminate Choice D.

So, the correct answer is **C**.

Try It

9. WHAT IF? You earn 90 out of a possible 100 points on the test. Write "90 out of 100" as a fraction, a decimal, and a percent.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

WRITING PERCENTS AS DECIMALS Write the percent as a decimal. Use a model to represent the decimal.

10. 32% **11.** 54.5% **12.** 108%

WRITING DECIMALS AS PERCENTS Write the decimal as a percent.

13. 0.71 **14.** 0.052 **15.** 9.66

16. WRITING Explain why the decimal point moves left when dividing a number by 100.

35. **MODELING REAL LIFE** The table shows the approximate portions of the world population that live in four countries. Order the countries by population from least to greatest.

Country	Brazil	India	Russia	United States
Portion of World Population	2.8%	$\frac{7}{40}$	$\frac{1}{50}$	0.044

MP PRECISION **Order the numbers from least to greatest.**

36. 66.1%, 0.66, $\frac{133}{200}$, 0.667

37. $\frac{111}{500}$, 21%, 0.211, $\frac{11}{50}$

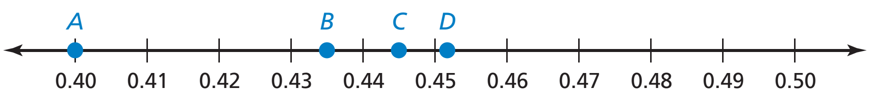
MATCHING **Tell which letter shows the graph of the number.**

38. $\frac{2}{5}$

39. 45.2%

40. 0.435

41. $\frac{89}{200}$



42. **MP PRECISION** The Tour de France is a bicycle road race. The whole race is made up of 21 small races called *stages*. The table shows how several stages compare to the whole Tour de France in a recent year. Order the stages by distance from shortest to longest.

Stage	1	7	8	17	21
Portion of Total Distance	$\frac{11}{200}$	0.044	$\frac{6}{125}$	0.06	4%

43. **MP PRECISION** The table shows the portions of the day that several animals sleep.

a. Order the animals by sleep time from least to greatest.

b. Estimate the portion of the day that you sleep.

c. Where do you fit on the ordered list?

Animal	Portion of Day Sleeping
Dolphin	0.433
Lion	56.3%
Rabbit	$\frac{19}{40}$
Squirrel	$\frac{31}{50}$
Tiger	65.8%

44. **MP NUMBER SENSE** Tell what whole number you can substitute for a in each list so the numbers are ordered from least to greatest. If there is none, explain why.

a. $\frac{1}{a}$, $\frac{a}{20}$, 28%

b. $\frac{3}{a}$, $\frac{a}{5}$, 75%

EXAMPLE 5 Evaluating Expressions with Two Operations

a. Evaluate $3x - 14$ when $x = 5$.

$3x - 14 = 3(5) - 14$ Substitute 5 for x.

$= 15 - 14$ Using order of operations, multiply 3 and 5.

$= 1$ Subtract 14 from 15.

b. Evaluate $n^2 + 8.5$ when $n = 2$.

$n^2 + 8.5 = 2^2 + 8.5$ Substitute 2 for n.

$= 4 + 8.5$ Using order of operations, evaluate 2^2.

$= 12.5$ Add 4 and 8.5.

Try It **Evaluate the expression when $y = 6$.**

13. $5y + 1$

14. $30 - 24 \div y$

15. $y^2 - 7$

16. $1.5 + y^2$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

17. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$2x + 1$ $5w \cdot c$ $3(4) + 5$ $2y \cdot z$

18. **ALGEBRAIC EXPRESSIONS** Identify the terms, coefficients, and constants in the expression $9h + 1$.

EVALUATING EXPRESSIONS **Evaluate the expression when $m = 8$.**

19. $m - 7$

20. $5m + 4$

21. MP **NUMBER SENSE** Does the value of the expression $20 - x$ *increase*, *decrease*, or *stay the same* as x increases? Explain.

22. **OPEN-ENDED** Write an algebraic expression using more than one operation. When you evaluate the expression, how do you know which operation to perform first?

23. MP **STRUCTURE** Is the expression $8.2 \div m \cdot m \cdot m \cdot m$ the same as the expression $8.2 \div m^4$? Explain your reasoning.

5.3 Properties of Addition and Multiplication

Learning Target: Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria:
- I can explain the meaning of equivalent expressions.
- I can use properties of addition to generate equivalent expressions.
- I can use properties of multiplication to generate equivalent expressions.

EXPLORATION 1 Identifying Equivalent Expressions

Work with a partner.

a. Choose four values for a variable x. Then evaluate each expression for each value of x. Are any of the expressions *equivalent*? Explain your reasoning.

x				
$4 + x + 4$				

x				
$16x$				

x				
$4 \cdot (x \cdot 4)$				

x				
$x + 4 + 4$				

x				
$x + 8$				

x				
$(4 \cdot x) \cdot 4$				

b. You have used the following properties in a previous course. Use the examples to explain the meaning of each property.

Commutative Property of Addition: $3 + 5 = 5 + 3$

Commutative Property of Multiplication: $9 \cdot 3 = 3 \cdot 9$

Associative Property of Addition: $8 + (3 + 1) = (8 + 3) + 1$

Associative Property of Multiplication: $12 \cdot (6 \cdot 2) = (12 \cdot 6) \cdot 2$

Are these properties true for algebraic expressions? Explain your reasoning.

Math Practice

Use Counterexamples

Use a counterexample to show that the Commutative Property is not true for division.

EXAMPLE 3 Solving Equations Using Subtraction

a. Solve $x + 2 = 9$.

$x + 2 = 9$	Write the equation.	
Undo the addition. → $\underline{-2} \quad \underline{-2}$	Subtraction Property of Equality	
$x = 7$	Simplify.	

The solution is $x = 7$.

Check

$x + 2 = 9$

$7 + 2 \stackrel{?}{=} 9$

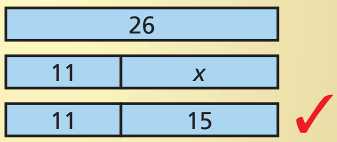
$9 = 9$ ✓

b. Solve $26 = 11 + x$.

$26 = 11 + x$	Write the equation.
$\underline{-11} \quad \underline{-11}$	Subtraction Property of Equality
$15 = x$	Simplify.

The solution is $x = 15$.

Another Method



Try It **Solve the equation. Check your solution.**

8. $s + 8 = 17$ **9.** $9 = y + 6$ **10.** $13 + m = 20$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

CHECKING SOLUTIONS **Tell whether the given value is a solution of the equation.**

11. $n + 8 = 42; n = 36$ **12.** $g - 9 = 24; g = 35$

SOLVING EQUATIONS **Solve the equation. Check your solution.**

13. $x - 8 = 12$ **14.** $b + 14 = 33$

15. **WRITING** When solving $x + 5 = 16$, why do you subtract 5 from the left side of the equation? Why do you subtract 5 from the right side of the equation?

16. MP **REASONING** Do the equations have the same solution? Explain your reasoning.

$$x - 8 = 6$$
$$x - 6 = 8$$

17. MP **STRUCTURE** Just by looking at the equation $x + 6 + 2x = 2x + 6 + 4$, find the value of x. Explain your reasoning.

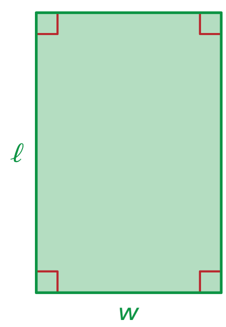
7.1 Areas of Parallelograms

Learning Target: Find areas and missing dimensions of parallelograms.

Success Criteria:
- I can explain how the area of a rectangle is used to find the area of a parallelogram.
- I can use the base and the height of a parallelogram to find its area.
- I can use the area of a parallelogram and one of its dimensions to find the other dimension.

A **polygon** is a closed figure in a plane that is made up of three or more line segments that intersect only at their endpoints. Several examples of polygons are parallelograms, rhombuses, triangles, trapezoids, and kites.

The formula for the area of a parallelogram can be derived from the definition of the area of a rectangle. Recall that the area of a rectangle is the product of its length ℓ and its width w. The process you use to derive this and other area formulas in this chapter is called *deductive reasoning*.



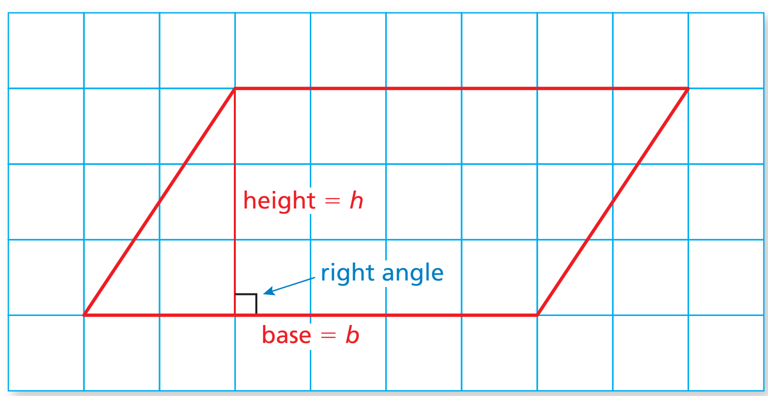
Area $= \ell w$

EXPLORATION 1 Deriving the Area Formula of a Parallelogram

Work with a partner.

a. Draw *any* rectangle on a piece of centimeter grid paper. Cut the rectangle into two pieces that can be arranged to form a parallelogram. What do you notice about the areas of the rectangle and the parallelogram?

b. Copy the parallelogram below on a piece of centimeter grid paper. Cut the parallelogram and rearrange the pieces to find its area.



c. Draw *any* parallelogram on a piece of centimeter grid paper and find its area. Does the area change when you use a different side as the base? Explain your reasoning.

d. Use your results to write a formula for the area A of a parallelogram.

Math Practice

Justify Conclusions

How does decomposing the parallelogram into other figures help you justify your formula?

7.2 Areas of Triangles

Learning Target: Find areas and missing dimensions of triangles, and find areas of composite figures.

Success Criteria:
- I can explain how the area of a parallelogram is used to find the area of a triangle.
- I can use the base and the height of a triangle to find its area.
- I can use the area of a triangle and one of its dimensions to find the other dimension.
- I can use decomposition to find the area of a figure.

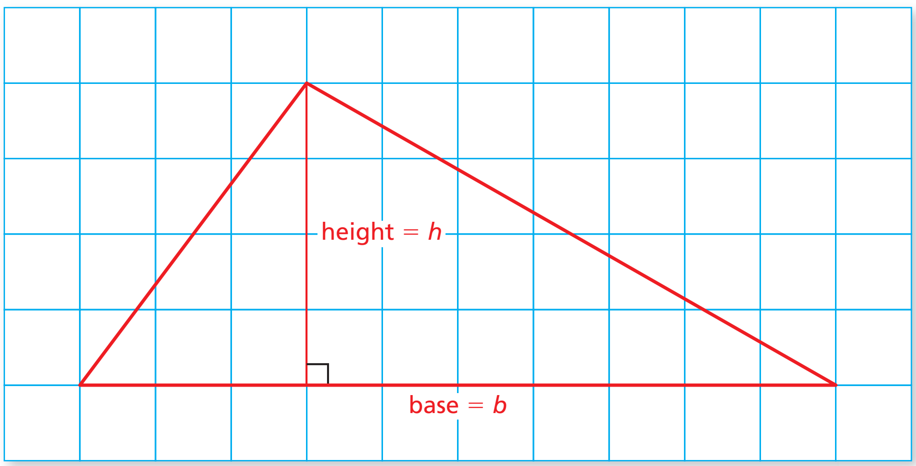
EXPLORATION 1

Deriving the Area Formula of a Triangle

Work with a partner.

a. Draw *any* parallelogram on a piece of centimeter grid paper. Cut the parallelogram into two identical triangles. How can you use the area of the parallelogram to find the area of each triangle?

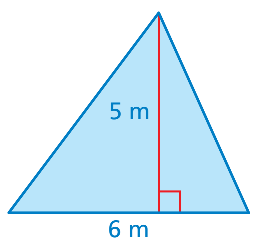
b. Copy the triangle below on a piece of centimeter grid paper. Find the area of the triangle. Explain how you found the area.



c. Draw *any* acute triangle on a piece of centimeter grid paper and find its area. Repeat this process for a right triangle and an obtuse triangle.

d. Do the areas change in part (c) when you use different sides as the base? Explain your reasoning.

e. Use your results to write a formula for the area A of a triangle. Use the formula to find the area of the triangle shown.



Math Practice

Calculate Accurately

If you use the base and the height to calculate the area in part (d), how can you estimate the dimensions so that your calculations are accurate?

7.3 Areas of Trapezoids and Kites

Learning Target: Find areas of trapezoids, kites, and composite figures.

Success Criteria:
- I can explain how the area of a parallelogram is used to find the area of a trapezoid.
- I can decompose trapezoids and kites into smaller shapes.
- I can use decomposition to find the area of a figure.
- I can use the bases and the height of a trapezoid to find its area.

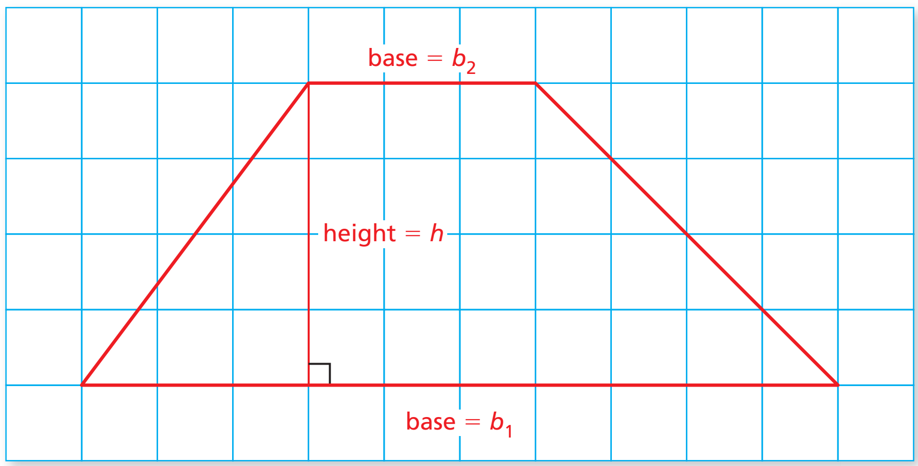
EXPLORATION 1

Deriving the Area Formula of a Trapezoid

Work with a partner.

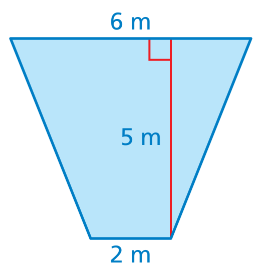
a. Draw *any* parallelogram on a piece of centimeter grid paper. Cut the parallelogram into two identical trapezoids. How can you use the area of the parallelogram to find the area of each trapezoid?

b. Copy the trapezoid below on a piece of centimeter grid paper. Find the area of the trapezoid. Explain how you found the area.



c. Draw *any* trapezoid on a piece of centimeter grid paper and find its area.

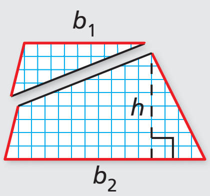
d. Use your results to write a formula for the area A of a trapezoid. Use the formula to find the area of the trapezoid shown.



Math Practice

Make a Plan

How can you use the diagram below to justify the formula you wrote in part (d)?



Laurie's Notes

Scaffolding Instruction

- Students should have a sense of why a standard order of operations is necessary, as stated in the first success criterion. Exponents and grouping symbols are now included in the order of operations.
- **Emerging:** Students understand that there is an order in which operations must be performed, but they may make errors in computations or identifying the operations. Going through Examples 1 and 2, either independently or with guided instruction, will help students become proficient with the first two success criteria.
- **Proficient:** Students
but need to practice
will help students a
- All students should
exercises, before a

Key Idea

- **FYI:** Students shoul
The order is now ex
- **Common Error:** Stu
division, or addition
- If you introduce the
tool for the order of
are important. Whe
symbols, not just pa

EXAMPLE 1

- "How many operati
should be performe
- "Tell a partner how
There are two types
you think this proble
- Give sufficient time
discuss their results
- **Teaching Strategy:**

EXAMPLE 2

- "How many operati

Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The note for Example 2 encourages teachers to have a volunteer explain how to simplify the expression and have other students critique the reasoning.

MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask a volunteer to explain the order in which the expression should be evaluated. Have other students critique the reasoning.
- Refer to the push-pin note to reinforce the left-to-right rule.

Try It

- Each student should work independently before checking his or her work with a neighbor. Encourage students to show their work instead of trying to evaluate the expressions in their heads.
- Have students display work on whiteboards so you can quickly assess where students are in their progress with the success criteria. Offer feedback as needed.

Teaching Strategy

Calculators are now, and will continue to be, a commonly used tool. Students should be familiar with calculator apps they have on their phones, tablets, or computers. It is important to understand how
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Extra Example 2

Evaluate $15 \times (12 - 3^2) \div 9$. 5

Try It

4. 21 **5.** 45
6. 22

1.2 Lesson

Key Vocabulary
numerical expression, *p. 10*
evaluate, *p. 10*
order of operations, *p. 10*

A **numerical expression** is an expression that contains numbers and operations. To **evaluate**, or find the value of, a numerical expression, use a set of rules called the **order of operations**.

Key Idea

Order of Operations

1. Perform operations in grouping symbols.
2. Evaluate numbers with exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

EXAMPLE 1 Using Order of Operations

a. Evaluate $12 - 2 \times 4$.

$12 - 2 \times 4 = 12 - 8$ — Multiply 2 and 4.

$= 4$ — Subtract 8 from 12.

b. Evaluate $60 \div [(4 + 2) \times 5]$.

$60 \div [(4 + 2) \times 5] = 60 \div [6 \times 5]$ — Perform operation in parentheses.

$= 60 \div 30$ — Perform operation in brackets.

$= 2$ — Divide 60 by 30.

Try It **Evaluate the expression.**

1. $7 \cdot 5 + 3$ **2.** $(28 - 20) \div 4$ **3.** $[6 + (15 - 10)] \times 5$

EXAMPLE 2 Using Order of Operations with Exponents

Remember to multiply and divide from left to right. In Example 2, you should divide before multiplying because the division symbol comes first when reading from left to right.

Evaluate $30 \div (7 + 2^3) \times 6$.

$30 \div (7 + 2^3) \times 6 = 30 \div (7 + 8) \times 6$ — Evaluate power in parentheses.

$= 30 \div 15 \times 6$ — Perform operation in parentheses.

$= 2 \times 6$ — Divide 30 by 15.

$= 12$ — Multiply 2 and 6.

Try It **Evaluate the expression.**

4. $6 + 2^4 - 1$ **5.** $4 \cdot 3^2 + 18 - 9$ **6.** $16 + (5^2 - 7) \div 3$

Extra Example 3

Find the LCM of 3, 8, and 16. 48

Try It

7. 40

8. 60

9. *Sample answer:* 4, 10, 25

Self-Assessment *for Concepts & Skills*

10. 18

11. 120

12. 55

13. *Sample answer:* 2, 3;
$18 = 2 \cdot 9$, $18 = 3 \cdot 6$,
$30 = 2 \cdot 15$, $30 = 3 \cdot 10$

14. *Sample answer:* Prime factorization because 13 and 14 do not have many prime factors.

15. GCF: Multiply the prime factors that appear in both rows of the table;
$\text{GCF} = 2 \cdot 2 = 4$

LCM: Multiply the factors that appear in at least one row of the table;
$\text{LCM} = 2 \cdot 2 \cdot 2 \cdot 3 = 24$

16. Find the LCM of the denominators and rewrite the fractions as equivalent fractions having the LCM as a denominator.

Laurie's Notes

EXAMPLE 3

- "You want to find the LCM of three numbers. Which method do you think will be the most efficient and why?" Using prime factorizations, because listing the multiples of 18 is more challenging than listing the multiples of 4 and 15. Writing the prime factorizations of all three numbers is fairly quick.
- Work through the problem as shown.
- **Turn and Talk:** "Is it possible for the LCM of two numbers to be one of the numbers? Explain." Yes, the greater of the two numbers can be the LCM if the greater number is a multiple of the lesser number.
- **Big Idea:** The LCM of two numbers will always be greater than or equal to the greater of the two original numbers.

Try It

- Ask students to explain which method they used to find the LCM, and why they chose that method.
- **MP2 Reason Abstractly and Quantitatively & MP3 Construct Viable Arguments and Critique the Reasoning of Others:** In Exercise 9, students are given the LCM and they need to find the three original numbers. This is not a trivial problem, so ask students to describe their reasoning.

Self-Assessment for Concepts & Skills

- Students may be able to list multiples and write the prime factorizations of numbers. Can they explain what it means to find the least common multiple of two numbers? Can they extend this understanding to more than two numbers?
- Exercise 14 is key in helping students assess their understanding of which method to use, and why. Knowing that 13 is a prime number, students are more likely to choose prime factorization.
- Discuss student responses to Exercise 15. Are the responses procedural, or is there a conceptual understanding that relates to the GCF and the LCM?

ELL Support

Have students check comprehension by working in pairs. Remind them to use the methods they have learned. When they have completed Exercises 10–12, have them check their answers with another pair.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

EXAMPLE 3 Finding the LCM of Three Numbers

Find the LCM of 4, 15, and 18.

Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$4 = 2 \cdot 2$	2 appears most often here, so circle both 2s.
$15 = 3 \cdot 5$	5 appears here only, so circle 5.
$18 = 2 \cdot 3 \cdot 3$	3 appears most often here, so circle both 3s.
$2 \cdot 2 \cdot 5 \cdot 3 \cdot 3 = 180$	Find the product of the circled factors.

So, the LCM of 4, 15, and 18 is 180.

Try It

Find the LCM of the numbers.

7. 2, 5, 8 **8.** 6, 10, 12

9. Write three numbers that have a least common multiple of 100.

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

FINDING THE LCM **Find the LCM of the numbers.**

10. 6, 9 **11.** 30, 40 **12.** 5, 11

13. MP **REASONING** Write two numbers such that 18 and 30 are multiples of the numbers. Justify your answer.

14. MP **REASONING** You need to find the LCM of 13 and 14. Would you rather list their multiples or use their prime factorizations? Explain.

15. MP **CHOOSE TOOLS** A student writes the prime factorizations of 8 and 12 in a table as shown. She claims she can use the table to find the greatest common factor and the least common multiple of 8 and 12. How is this possible?

8 =	2	2	2	
12 =	2	2		3

16. **CRITICAL THINKING** How can you use least common multiples to add or subtract fractions with different denominators?

Learning Target

Compute quotients with mixed numbers and solve problems involving division with mixed numbers.

Success Criteria

- Draw a model to explain division of mixed numbers.
- Write a mixed number as an improper fraction.
- Divide with mixed numbers.
- Evaluate expressions involving mixed numbers using the order of operations.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

A mixed number is a mixture of a whole number and a fraction, such as $1\frac{1}{2}$. In everyday language, the word *proper* means appropriate and *improper* means inappropriate. In math, however, the term *improper fraction* means a fraction in which the numerator is greater than the denominator, such as $\frac{3}{2}$. Like a mixed number, an improper fraction represents more than a whole.

Exploration 1

a. *Sample answer:* You make $\frac{3}{4}$ of a bracelet in 1 hour. How many bracelets do you make in $4\frac{1}{2}$ hours? 6

b.–g. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

COMMON CORE STATE STANDARDS
6.NS.A.1

Preparing to Teach

- This is a different type of exploration, one in which students use their literacy skills as well as their math skills. The goal is for students to write a real-life problem that provides context for a division expression. Many students will find this challenging.
- After dividing fractions in the previous section, students will apply those skills to dividing with mixed numbers. Models are still an important connection.

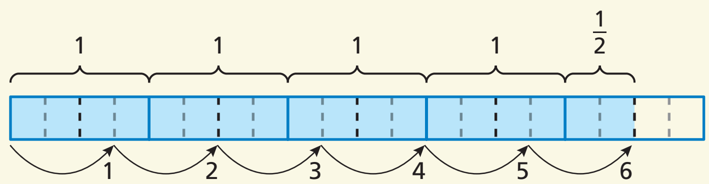
Motivate

- Share a colorful story while holding a travel coffee mug. The essence of the story should result in the following facts: a coffee pot holds 12 cups and the mug holds $1\frac{1}{2}$ cups. How many times can the travel mug be filled?

? "What expression can you use to answer the question?" $12 \div 1\frac{1}{2}$ "What is the answer?" 8 times

Exploration 1

- Partners need to write a real-life problem that represents each division expression. Allow time for students to think about where these numbers may occur in real life. Students should contextualize the expressions faster as they progress through the parts.
- If students are struggling to create contexts, provide an example for part (a). Example: A city bus completes its route in $\frac{3}{4}$ of an hour. How many times does the bus complete its route in $4\frac{1}{2}$ hours?
- Visual models are drawn to represent division of mixed numbers. The labels above each diagram represent the quantity you have (the dividend). Below the diagram students should mark the size of the unit they want to fit into the quantity (the divisor). In part (a), they are counting the number of three-fourths in four and one-half.



- In parts (d)–(g), students may have more difficulty because the quotient is not a whole number. Help students by asking them to think about what portion of the unit they are counting is represented by the amount that is left over. For example, in part (d), the remaining piece is $\frac{6}{7}$ of the $\frac{7}{6}$ unit they are counting.
- Ask volunteers to share their real-life problems.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Take time to have a student talk through his or her approach to each problem in the exploration. Other students should listen carefully to the language used and the explanation given. You want students to be in the habit of communicating their thinking and having others critiquing their arguments.

2.3 Dividing Mixed Numbers

Learning Target: Compute quotients with mixed numbers and solve problems involving division with mixed numbers.

Success Criteria:
- I can draw a model to explain division of mixed numbers.
- I can write a mixed number as an improper fraction.
- I can divide with mixed numbers.
- I can evaluate expressions involving mixed numbers using the order of operations.

EXPLORATION 1

Dividing Mixed Numbers

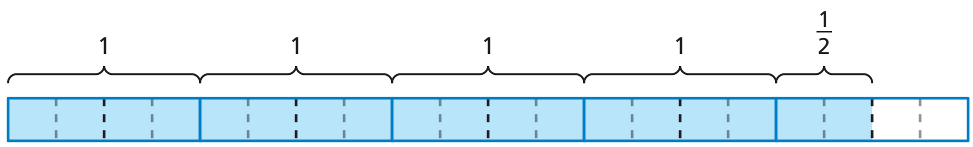
Work with a partner. Write a real-life problem that represents each division expression described. Then solve each problem using a model. Check your answers.

Math Practice

Make Sense of Quantities

What values do the parts of the model represent?

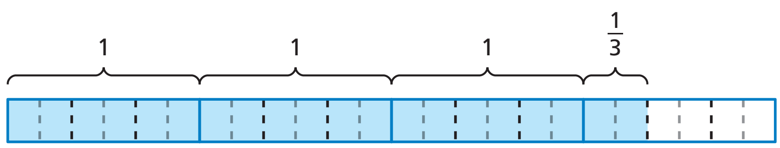
a. How many three-fourths are in four and one-half?



b. How many three-eighths are in two and one-fourth?

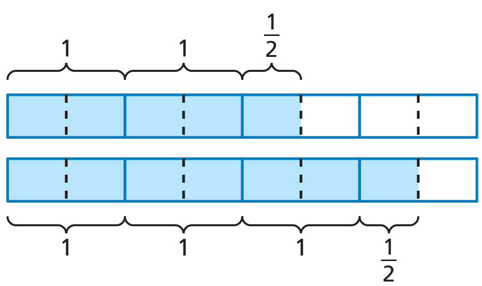
c. How many one and one-halves are in six?

d. How many seven-sixths are in three and one-third?



e. How many one and one-fifths are in five?

f. How many three and one-halves are in two and one-half?



g. How many four and one-halves are in one and one-half?

Learning Target

Use ratio tables to represent equivalent ratios and solve ratio problems.

Success Criteria

- Use various operations to create tables of equivalent ratios.
- Use ratio tables to solve ratio problems.
- Use ratio tables to compare ratios.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Explain that the word *double* means two times or twice. A number line is a line with numbers. A double number line is two number lines put together that show values for two things that are being compared.

Exploration 1

a. See Additional Answers.

b. *Sample answers:* As cups of milk increase by 2, calories increase by 180; The number of calories is 90 times the number of cups of milk.

c. *Sample answers:* Multiply or divide each quantity in one ratio by the same number; Add the numbers in the given ratio to the respective quantities in one ratio.

Exploration 2

a–b. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.RP.A.1, 6.RP.A.3, 6.RP.A.3a

Preparing to Teach

- You can find and organize equivalent ratios in a **ratio table**. In this lesson, various operations are used to create ratio tables.
- Another way to organize equivalent ratios is to make a double number line. Help students see the connection between the double number line and the ratio table.
- **Common Misconception:** Students sometimes think there is only one correct ratio table when solving a problem. Address this misconception when two students have the same answer but different tables.

Motivate

- **Story Time:** Tell students that you took a five-mile float trip down a river. For each mile, the drop in elevation was the same.
- Draw the table as shown and ask students to fill in the missing values. 40, 120, 200

Number of Miles	1	2	3	5
Drop in Elevation (feet)		80		

- Ask students to explain how they found the missing values.

Exploration 1

- Students should not have difficulty with the context of this problem, though they may skip over the direction line.
- When asked to think about any relationships in part (b), students may use ratio language.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** There are many different ways to find equivalent ratios. Having students share their thinking will likely preview several of the strategies used in this lesson. If students think a ratio is a fraction, it won't make sense to them that you can add the values from the two columns. You want students to explain their thinking. Expect others to ask questions if the explanation is unclear.

Exploration 2

- Students may label the increments on the double number line with the same values as the ratio table. Others may think that the increments must be multiples of 10 or 100. This is a good class discussion.
- **Connection:** The double number line can help students realize there are infinitely many ratios equivalent to a given ratio. Before part (b), ask, "How many calories are in 8 cups?" 720 calories "How many cups of milk contain 1800 calories?" 20 cups
- "Can the ratio table be used to find the number of calories in 3 cups of milk? 3.5 cups of milk?" Students may find it easier to think about these questions on a double number line because they know there are other numbers between 2 and 4, such as 3 and 3.5. If students haven't recognized that the columns in a ratio table represent equivalent ratios, it will not make sense to them that the columns can be in any order.

3.3 Using Ratio Tables

Learning Target: Use ratio tables to represent equivalent ratios and solve ratio problems.

Success Criteria:
- I can use various operations to create tables of equivalent ratios.
- I can use ratio tables to solve ratio problems.
- I can use ratio tables to compare ratios.

EXPLORATION 1 Making a Table of Equivalent Ratios

Work with a partner. You buy milk that contains 180 calories per 2 cups.

a. You measure 2 cups of the milk for a recipe and pour it into a pitcher. You repeat this four more times. Make a table to show the numbers of calories and cups in the pitcher as you add the milk.

b. Describe any relationships you see in your table.

c. Describe ways that you can find equivalent ratios using different operations.

Math Practice

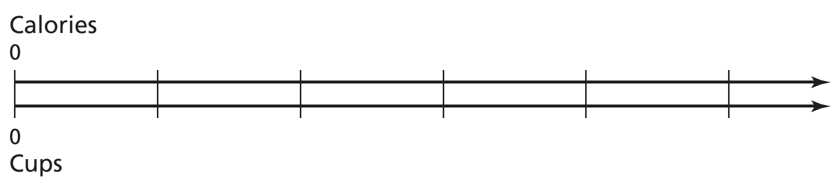
Compare Arguments

Compare your explanations in part (c) with another group. If they are different, are they both correct?

EXPLORATION 2 Creating a Double Number Line

Work with a partner.

a. Represent the ratio in Exploration 1 by labeling the increments on the *double number line* below. Can you label the increments in more than one way?



b. How can you use the double number line to find the number of calories in 3 cups of milk? 3.5 cups of milk?

Learning Target

Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria

- Explain the meaning of equivalent expressions.
- Use properties of addition to generate equivalent expressions.
- Use properties of multiplication to generate equivalent expressions.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com.*

ELL Support

Discuss the meaning of the word *property*. In everyday language, property refers to things that people own. For example, your clothing and toys are your property. A house or land is also commonly referred to as property. In mathematics, a property is a rule.

Exploration 1

a–b. See Additional Answers.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

COMMON CORE STATE STANDARDS
6.EE.A.3, 6.EE.A.4

Preparing to Teach

- Students will identify expressions that are equivalent and then review the properties of addition and multiplication with numbers before applying them to algebraic expressions.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Mathematically proficient students justify their conclusions, communicate them to others, and respond to the arguments of others. In this lesson, students will make conjectures about which operations are commutative and associative. Expect clear communication of their thinking.

Motivate

- **Acting Time:** If there is a bit of an actor in you, start the class by pretending that you're brushing your teeth (about 10 seconds). Then take a tube of toothpaste out and apply some to the toothbrush. This will clearly evoke a few comments about the order in which you performed the two tasks.
- ? As students begin to comment, say, "Oh, does the order matter? Hmmm…"
- If brushing your teeth doesn't work for you, put a sock over a shoe or anything obviously in the wrong order. Do something that students will remember. Catch their attention!
- Have students think of other examples where order matters and ask volunteers to share.
- ? Then have students think of examples where order does *not* matter and ask volunteers to share. If students are struggling to think of ideas, say, "When you make a ham and cheese sandwich, does it matter if you put the ham on the bread and then the cheese?"

Exploration 1

- **Turn and Talk:** Ask students to discuss the meaning of the word *equivalent*. Students may say, "the same" or "equal." Refer to a balance, a visual that students used in prior courses. A balance shows that although each side looks different, they have the same "weight" when the sides are raised to the same height. Tell students to keep this in mind as they explore possible **equivalent expressions**.
- ? In part (a), tell students to read the directions carefully and skim the tables. Ask, "What kind of numbers do you want to choose as your four values for x?" They will probably mention "easy" numbers. Discuss what makes a number easy to compute with. Remind students that the four values they choose for the first table will be used for the other tables as well.
- After completing the tables, partners should discuss whether any expressions are equivalent and explain their reasoning.
- ? "What helps you remember the meaning of the Commutative and Associative Properties?" Listen for the root words *commute* and *associate*. "In part (b), how do you know the expressions in each example are equivalent?" The values of the expressions are the same on both sides of the equal sign.
- ? Discuss where the properties apply in part (a). "How do you know whether the algebraic expressions are equivalent?" The operations are the same in both expressions and the values of the expressions are the same for any value of x.

Extra Example 2

a. Simplify the expression $0 + (s + 4)$.
$s + 4$

b. Simplify the expression $(h \cdot 1) \cdot 8$.
$8h$

Try It

4. $12 \cdot b \cdot 0 = 12 \cdot (b \cdot 0)$
Assoc. Prop. of Mult.
$= 12 \cdot 0$
Mult. Prop. of Zero
$= 0$
Mult. Prop. of Zero

5. $1 \cdot m \cdot 24 = (1 \cdot m) \cdot 24$
Assoc. Prop. of Mult.
$= m \cdot 24$
Mult. Prop. of One
$= 24m$
Comm. Prop. of Mult.

6. $(t + 15) + 0 = t + (15 + 0)$
Assoc. Prop. of Add.
$= t + 15$
Add. Prop. of Zero

ELL Support

Have students work independently on Self-Assessment for Concepts & Skills Exercises 7–9. Then have students compare their answers with a partner to check understanding and practice language. Discuss Exercises 10 and 11 with students to verify understanding.

Self-Assessment
for Concepts & Skills

7–9. See Additional Answers.

10. Expressions are equivalent when they result in the same number for any values of their variables; *Sample answer:* $(4 + 3)x$, $7x$

11. *Sample answer:* $(5 \cdot x) \cdot 1$

Laurie's Notes

Key Ideas

- Write and discuss the three new properties. Students may say (or at least think) that these properties are obvious.
- Adding zero, in any form, does not change the value of the quantity.
- Multiplying by zero, in any form, produces a product of 0.
- Multiplying by 1, in any form, does not change the value of the quantity.

  Remind students that 1 can be represented in different ways: 1, $\left(\frac{1}{2} + \frac{1}{2}\right)$, or $\frac{3}{3}$.
- The Addition Property of Zero is also known as the Identity Property of Addition (or the Additive Identity Property). The Multiplication Property of One is also know as the Identity Property of Multiplication (or the Multiplicative Identity Property). Meaning you end up with the same value you started with, so the values are *identical*.

EXAMPLE 2

- Work through both parts and point out the properties as they are used. Mention to students that the first steps in both parts involve the Associative Property of Multiplication. In part (a), you get the same result if you group $(0 \cdot p)$ first.
- In part (b), students may apply the Commutative Property of Multiplication first: writing $r \cdot 1$ as $1 \cdot r$ to group $(4.5 \cdot 1)$ and then apply the Multiplication Property of One. Encourage students to share different methods with the class.

Try It

Think-Pair-Share: Students should read each exercise independently and then work in pairs to simplify the expressions. After completing the exercises, have each pair compare their answers with another pair and discuss any discrepancies. As students are working on the last two success criteria, listen to their conversations to identify any common errors or misconceptions.

Self-Assessment for Concepts & Skills

- Have students work independently.
- Exercises 7–9 are accessible to all students because they are similar to Examples 1 and 2.
- Exercises 10 and 11 provide an indication of students' understanding of the success criteria. Do students understand what *equivalent* means in terms of algebraic expressions? Can they write an expression that can be simplified?
- Allow time for students to test their expressions in Exercise 11 to ensure they meet the requirements.
- **MP3 Construct Viable Arguments and Critique the Reasoning of Others:** Ask volunteers to write their expressions on the board for Exercise 11. Then ask the class if they agree or disagree with each expression.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Key Ideas

Addition Property of Zero

Words The sum of any number and 0 is that number.

Numbers $7 + 0 = 7$ **Algebra** $a + 0 = a$

Multiplication Properties of Zero and One

Words The product of any number and 0 is 0.

The product of any number and 1 is that number.

Numbers $9 \cdot 0 = 0$ **Algebra** $a \cdot 0 = 0$

$4 \cdot 1 = 4$ $a \cdot 1 = a$

EXAMPLE 2 Using Properties to Write Equivalent Expressions

a. Simplify the expression $9 \cdot 0 \cdot p$.

$9 \cdot 0 \cdot p = (9 \cdot 0) \cdot p$ Associative Property of Multiplication

$= 0 \cdot p$ Multiplication Property of Zero

$= 0$ Multiplication Property of Zero

b. Simplify the expression $4.5 \cdot r \cdot 1$.

$4.5 \cdot r \cdot 1 = 4.5 \cdot (r \cdot 1)$ Associative Property of Multiplication

$= 4.5 \cdot r$ Multiplication Property of One

$= 4.5r$ Rewrite.

Try It **Simplify the expression. Explain each step.**

4. $12 \cdot b \cdot 0$ **5.** $1 \cdot m \cdot 24$ **6.** $(t + 15) + 0$

Self-Assessment for Concepts & Skills

Solve each exercise. Then rate your understanding of the success criteria in your journal.

USING PROPERTIES **Simplify the expression. Explain each step.**

7. $(7 + c) + 4$ **8.** $4(b \cdot 6)$ **9.** $0 \cdot b \cdot 9$

10. **WRITING** Explain what it means for expressions to be equivalent. Then give an example of equivalent expressions.

11. **OPEN-ENDED** Write an algebraic expression that can be simplified using the Associative Property of Multiplication and the Multiplication Property of One.

Extra Example 2

Find the value of each power.

a. 8^3 512 **b.** 13^2 169

Try It

5. 216 **6.** 81

7. 81 **8.** 324

Extra Example 3

Determine whether each number is a perfect square.

a. 50 not a perfect square

b. 9 perfect square

Try It

9. perfect square

10. not a perfect square

11. not a perfect square

12. perfect square

ELL Support

Check comprehension by having ELLs work in pairs to complete the Self-Assessment for Concepts & Skills exercises. Then have two pairs present their answers to each other and revise them if there is any disagreement.

Self-Assessment for Concepts & Skills

13. 64

14. 243

15. 1331

16–17. See Additional Answers.

18. $3 + 3 + 3 + 3 = 3 \times 4$; the others show powers as products of repeated factors.

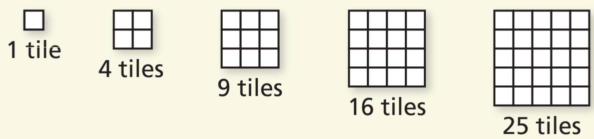
Laurie's Notes

EXAMPLE 2

- Students should be able to evaluate some powers using mental math.
- To practice using precise language, ask a volunteer to read the problem and the answer. You should hear, "7 squared is 49" and "5 cubed is 125."
- A common error many students make is to multiply the exponent by the base. For example, saying $7^2 = 7 \times 2 = 14$ rather than $7 \times 7 = 49$. The simpler the problem, the more often they seem to make this error.
- Evaluating a power is the second success criterion. Connect the work students are doing with the success criterion so that they understand what success looks like. This is how their learning becomes visible.

Discuss

- As an introduction to the definition of a perfect square, write the following sequence on the board: 1, 4, 9, 16, 25, 36, . . .
- ? "What are the next three numbers in the sequence?" 49, 64, 81 "What is the pattern?" Square the whole numbers in order. Students will often answer that you add 3, add 5, add 7, and so on. This is also a correct pattern, so you may need to probe further to get students to recognize that the numbers in the sequence are squares of the whole numbers.
- **MP5 Use Appropriate Tools Strategically:** Define **perfect square.** Give students a pile of square tiles and ask them to make a larger square. If you don't have square tiles, students can color squares on grid paper of various sizes. How many tiles were used? The number will always be a perfect square. The square tiles are a tool to help students visualize square numbers.



EXAMPLE 3

- Work through each part of the example.
- Give students 20 tiles and ask them to use all of the tiles to make a square. They will not be able to make a square because 20 is not a perfect square.

Try It

- Ask students to explain their answers either visually or verbally, so that others can understand the different strategies.
- Some students may reason that 99 is not a perfect square because $9^2 = 81$ and $10^2 = 100$. The base is between 9 and 10, so it is not a whole number.

Self-Assessment for Concepts & Skills

- Review the success criteria with students and have them complete the exercises. Discuss any common errors that may arise.
- The vocabulary in this lesson is important. Students should be able to explain the difference between exponents and powers, either by description or example.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Laurie's Notes

Scaffolding Instruction

- Students have used a powerful visual representation of **common factors** to find the **greatest common factor** (GCF). Now they will move on to more efficient methods: lists of factors and factor trees.
- **Emerging:** Students who want to continue using Venn diagrams, or who struggled with the two different approaches in the explorations, will benefit from guided instruction in Examples 1 and 2. The Try It exercises will allow you to assess their progress.
- **Proficient:** Students who were confident in both explorations, and were able to make the connection between the greatest common factor and common prime factors, can self-assess using the Try It exercises.

EXAMPLE 1

- Using lists of factors to find the GCF is similar to the method used in Exploration 1.
- ? "What are the factors of 24?" 1, 2, 3, 4, 6, 8, 12, 24
- ? "What are the factors of 40?" 1, 2, 4, 5, 8, 10, 20, 40
- ? "Which factors appear in both lists?" 1, 2, 4, 8
- **FYI:** Students should not just say, "The greatest common factor is 8." The complete answer is, "The greatest common factor of 24 and 40 is 8."

Try It

- **Neighbor Check:** Students should complete the exercises on their own, and then share their methods and answers with a partner or group. Listen to their reasoning in Exercises 2 and 3.
- ? "How do you find the GCF of three numbers?" Find all the common factors of all three numbers and choose the greatest.

EXAMPLE 2

- Using prime factorizations to find the GCF is similar to the method used in Exploration 2.
- ? "Which prime factors do 12 and 56 have in common?" two factors of 2
- The greatest common factor of two (or more) numbers is the product of the prime factors that they have in common. The greatest common factor of 12 and 56 is 2 • 2, or 4.
- ? "Does it matter which method you use? Explain." No, each method will give you the correct answer.
- ? "How do you decide which method to use?" If the numbers have relatively few factors, use the lists of factors method. If the numbers have relatively large numbers of factors, use the prime factorizations method.

Try It

- Have students use whiteboards to complete the exercises. As they share their factor trees, encourage students to study other boards and compare their factor trees to other factor trees. This is a good opportunity to show that even though factor trees may differ, they result in the same prime factorization.

Extra Example 1

Find the GCF of 36 and 54 using lists of factors. 18

ELL Support

Have students work in pairs to complete Try It Exercises 1–3. Each partner should factor one of the given numbers by listing factors, as demonstrated in Example 1. Then partners should compare their lists and circle the common factors.

Beginner: State the greatest common factor.

Intermediate: Use a complete sentence to verbally identify the greatest common factor.

Advanced: Identify the greatest common factor, and explain the process they used to find it.

Try It

1. 4
2. 18
3. 7

Extra Example 2

Find the GCF of 40 and 48 using prime factorizations. 8

Try It

4. 5
5. 2
6. 15

1.4 Lesson

Factors that are shared by two or more numbers are called **common factors**. The greatest of the common factors is called the **greatest common factor** (GCF). One way to find the GCF of two or more numbers is by listing factors.

Key Vocabulary
Venn diagram, *p. 21*
common factors, *p. 22*
greatest common factor, *p. 22*

EXAMPLE 1 Finding the GCF Using Lists of Factors

Find the GCF of 24 and 40.

List the factors of each number.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40

Circle the common factors.

The common factors of 24 and 40 are 1, 2, 4, and 8. The greatest of these common factors is 8.

So, the GCF of 24 and 40 is 8.

Try It **Find the GCF of the numbers using lists of factors.**

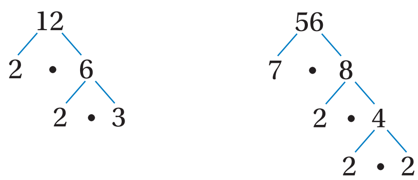
1. 8, 36 **2.** 18, 72 **3.** 14, 28, 49

Another way to find the GCF of two or more numbers is by using prime factors. The GCF is the product of the common prime factors of the numbers.

EXAMPLE 2 Finding the GCF Using Prime Factorizations

Find the GCF of 12 and 56.

Make a factor tree for each number.



Write the prime factorization of each number.

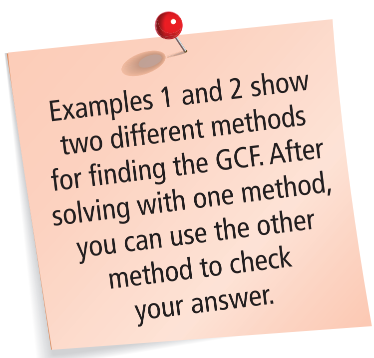
$12 = 2 \cdot 2 \cdot 3$

$56 = 2 \cdot 2 \cdot 2 \cdot 7$

Circle the common prime factors.

$2 \cdot 2 = 4$ Find the product of the common prime factors.

So, the GCF of 12 and 56 is 4.



Try It **Find the GCF of the numbers using prime factorizations.**

4. 20, 45 **5.** 32, 90 **6.** 45, 75, 120

Multi-Language Glossary at *BigIdeasMath.com*

Laurie's Notes

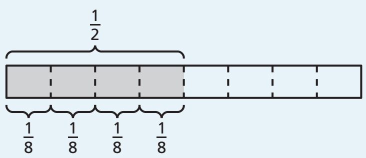
Extra Example 2

a. Find $\frac{5}{6} \div \frac{7}{12}$. $1\frac{3}{7}$

b. Find $\frac{1}{4} \div \frac{11}{20}$. $\frac{5}{11}$

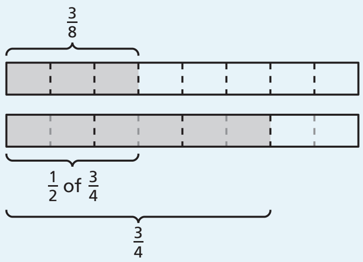
Try It

5. 4

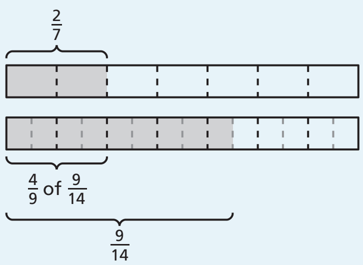


6. See Additional Answers.

7. $\frac{1}{2}$



8. $\frac{4}{9}$



EXAMPLE 2

- In part (a), students need to find how many five-twelfths are in three-fourths.
- Remind students that $\frac{3}{4} \div \frac{3}{4} = 1$. Because five-twelfths is less than three-fourths, there is at least 1 five-twelfths in three-fourths. Some students may realize that $\frac{3}{4} = \frac{9}{12}$, so the answer is less than 2.
- **Turn and Talk:** Have students explain how the model represents the answer. You may need to guide their thinking as you listen to conversations.
- Work through part (a) using the algorithm to verify the answer.
- In part (b), students need to find how many two-thirds are in one-sixth. Because one-sixth is much less than two-thirds, there aren't many. The answer must be very small.
- **Turn and Talk:** Again, have students describe how the model represents the answer. Listen for understanding that only $\frac{1}{4}$ of the $\frac{2}{3}$ piece will match the $\frac{1}{6}$ piece.
- Work through part (b) using the algorithm to verify the answer.
- **Common Error:** Some students may have the misconception that if you divide a fraction by a number greater than 1, the quotient is less than 1. Or, if you divide a fraction by a number less than 1, the quotient is greater than 1. This is not necessarily true. In parts (a) and (b), you divide a fraction by a number less than 1, but one quotient was greater than 1 and the other was less than 1.
  ? To clarify, ask, "Is a whole number divided by a smaller whole number *greater than 1* or *less than 1*?" greater than 1 "Is a whole number divided by a larger whole number *greater than 1* or *less than 1*?" less than 1 Tell students that the same holds true with fractions. The result depends upon the comparison of the divisor to the dividend.

Try It

- Ask students to predict whether the answer is *greater than 1*, *equal to 1*, or *less than 1*. This will give you an indication of their understanding of comparing fractions.

These exercises are instrumental to the first and third success criteria.

ELL Support

Have students work in pairs to complete the exercises. Remind students that they perform division of fractions by inverting the divisor, and then multiplying. Tell them to follow the process used in Example 2.
Beginner: Find the quotient using the algorithm or a model.
Intermediate: State the answer using a complete sentence. For example, "One-half divided by one-eighth is four."
Advanced: Explain each step.

Laurie's Notes

Scaffolding Instruction

- In a previous course, students divided decimals to the hundredths place. In this course, students will become fluent with division and multiply by powers of 10 when dividing by decimals. In the next course, students will convert a fraction or mixed number to a decimal using long division.
- **Emerging:** Students may understand that decimals can be divided by whole numbers and decimals but struggle with the procedure. Guided instruction with the examples will help students understand the success criteria.
- **Proficient:** Students understand the need for making the divisor a whole number, and can insert zeros in the dividend when necessary. They can estimate their answers to check for reasonableness. After reviewing the Key Ideas, have students self-assess using Try It Exercises 1–10.

Key Idea

- Placing the decimal point in the quotient above the decimal point in the dividend makes sense to the students.
- **FYI:** Some students may have difficulty lining up the columns when they perform long division. Have them use grid paper as an aid.

EXAMPLE 1

- Work through both parts of the example. Begin by estimating the quotient and end with judging the reasonableness of your answer.
- Students often ask where the decimal point is written in the work below the dividend. "Why don't you bring down the decimal point?" In theory, you can bring the decimal point down. But, once the decimal point is placed in the quotient, you treat the problem as if you were dividing whole numbers.
- ❓ "If the divisor is greater than the dividend, what do you know about the quotient?" It will be less than 1.
- You may want to share whole number examples to illustrate the above question. For example, use $8 \div 4$ and $4 \div 8$.
- ❓ In part (b), place the decimal point in the quotient. Then ask, "Does 12 go into 4?" no "Does 12 go into 43?" yes
- ❓ In this problem, a zero must be inserted at the end of the dividend to continue dividing. Ask, "Is 4.374 equivalent to 4.3740? 4.37400?" yes; yes
- Students may want to know how many zeros to insert. Tell students to continue to insert zeros until the division ends with a zero remainder.

Extra Example 1

a. Find $3.5 \div 7$. 0.5

b. Find $16.92 \div 8$. 2.115

Try It

1. 18.2 **2.** 3.7

3. 8.52 **4.** 0.195

5. 1.556 **6.** 3.1005

2.7 Lesson

Key Idea

Dividing Decimals by Whole Numbers

Words Place the decimal point in the quotient above the decimal point in the dividend. Then divide as you would with whole numbers. Continue until there is no remainder.

Numbers

$$\begin{array}{r} 1.83 \\ 4\overline{)7.32} \end{array}$$

Place the decimal point in the quotient above the decimal point in the dividend.

EXAMPLE 1 Dividing Decimals by Whole Numbers

a. Find 7.6 ÷ 4. **Estimate** $8 \div 4 = 2$

$$\begin{array}{r} 1.9 \\ 4\overline{)7.6} \\ -4 \\ \hline 3\,6 \\ -3\,6 \\ \hline 0 \end{array}$$

Place the decimal point in the quotient above the decimal point in the dividend.

So, 7.6 ÷ 4 = 1.9. **Reasonable?** $1.9 \approx 2$ ✓

b. Find 4.374 ÷ 12.

$$\begin{array}{r} 0.3645 \\ 12\overline{)4.3740} \\ -3\,6 \\ \hline 77 \\ -72 \\ \hline 54 \\ -48 \\ \hline 60 \\ -60 \\ \hline 0 \end{array}$$

Place the decimal point in the quotient above the decimal point in the dividend.

Insert a zero and continue to divide.

So, 4.374 ÷ 12 = 0.3645. **Check** $0.3645 \times 12 = 4.374$ ✓

Try It Divide. Use estimation to check your answer.

1. 36.4 ÷ 2 **2.** 22.2 ÷ 6 **3.** 59.64 ÷ 7

4. 3.12 ÷ 16 **5.** 6.224 ÷ 4 **6.** 43.407 ÷ 14

Learning Target

Understand the concepts of ratios and equivalent ratios.

Success Criteria

- Write and interpret ratios using appropriate notation and language.
- Recognize multiplicative relationships in ratios.
- Describe how to determine whether ratios are equivalent.
- Name ratios equivalent to a given ratio.

Warm Up

Cumulative, vocabulary, and prerequisite skills practice opportunities are available in the *Resources by Chapter* or at *BigIdeasMath.com*.

ELL Support

Clarify the meaning of the word *relationship*. Explain that the mother of Sam's mother is Sam's grandmother. The word *grandmother* describes Sam's relationship to his mother's mother. If necessary, draw a family tree on the board to clarify and describe other relationships within a family. Explain that ratios describe relationships between quantities, which are the amounts of different items. Tell students that they will be creating tables to show ratio relationships. In a mathematical context, a *table* is a type of chart, not a type of furniture.

Exploration 1

a–c. See Additional Answers.

Exploration 2

a. yes

b. Add 3 parts of iced tea for every 1 part of lemonade added.

Laurie's Notes

Check out the Dynamic Classroom.
BigIdeasMath.com

STATE STANDARDS
6.RP.A.1, 6.RP.A.3

Preparing to Teach

- Students are familiar with comparing measurable attributes using language such as *longer than*, *less than*, *heavier than*, and so on.
- A **ratio** is a comparison of two quantities, and there is language and notation associated with ratios.
- Represent the **value of the ratio** $a : b$ as the number $\frac{a}{b}$. Meaning, the quantity a is $\frac{a}{b}$ times b (also the quantity b is $\frac{b}{a}$ times a). For example, the ratio $1 : 3$ means 1 is $\frac{1}{3}$ times 3 (or 3 is $\frac{3}{1}$ times 1). This sounds like awkward language and unnecessarily complicated, however, when interpreting ratios and using ratios to solve problems, this is the type of reasoning students often have to use.

Motivate

- Ask for two volunteers. Hand 3 blocks to Student A and 1 to Student B.
- Ask the other students to describe the relationship between the numbers of blocks. Student A has 2 more blocks (additive relationship) or 3 times as many blocks (multiplicative relationship).
- Hand each student 1 more block. "Describe the relationship now." The additive relationship is still the same but the multiplicative relationship changed.
- Continue to add 1 block and ask about the relationships. You want students to realize that the relationship is never 3 to 1 again.
- You will revisit this scenario in the Closure.

Exploration 1

- State the learning target and success criteria for this section, and then relate these to the Motivate activity.
- Although the language and notation of ratios will likely be revealed in the Motivate, remind students that they should always read introductory text and directions. The definition of a ratio is provided before Exploration 1.
- **MP4 Model with Mathematics:** Remind students that they can use a table to organize the possible numbers of girls and boys in the science class.
- Part (b) is not asking students to compare actual numbers, just relative

MP3 Construct Viable Arguments and Critique the Reasoning of Others: When discussing part (b), focus on student reasoning. Solicit several comments. Ask other students if they agree with explanations offered. "Student B, do you agree with what Student A said? Why?"

Exploration 2

- Expect students to read the problem and work with a partner or group. You are listening to conversations, not teaching the problem.
- **Common Misconception:** Students may believe that if you add or subtract the same quantity to each number in a ratio, it is still the same relationship.
- Students may not have the language of **equivalent ratios**, yet it seems logical to double or triple the recipe. Record student answers so that you may reference these answers when discussing equivalent ratios.

3.1 Ratios

Learning Target: Understand the concepts of ratios and equivalent ratios.

Success Criteria:
- I can write and interpret ratios using appropriate notation and language.
- I can recognize multiplicative relationships in ratios.
- I can describe how to determine whether ratios are equivalent.
- I can name ratios equivalent to a given ratio.

A **ratio** is a comparison of two quantities. Consider two quantities a and b. The ratio $a : b$ indicates that there are a units of the first quantity for every b units of the second quantity.

EXPLORATION 1 Writing Ratios

Work with a partner. A science class has two times as many girls as it has boys.

a. Discuss possible numbers of boys and girls in the science class.

b. What comparisons can you make between your class and the science class? Can you determine which class has more girls? more boys? Explain your reasoning.

c. Write three ratios that you observe in your classroom. Describe what each ratio represents.

Math Practice

Use a Table

How can you use a table to represent the relationship between the numbers of girls and boys?

EXPLORATION 2 Using Ratios in a Recipe

Work with a partner. The ratio of iced tea to lemonade in a recipe is 3 : 1. You begin by combining 3 cups of iced tea with 1 cup of lemonade.

a. You add 1 cup of iced tea and 1 cup of lemonade to the mixture. Does this change the taste of the mixture?

b. Describe how you can make larger amounts without changing the taste.

Extra Example 4

You and your friend make tea. You add 1.5 teaspoons of sugar for every 8 fluid ounces of tea. Your friend adds 1.75 teaspoons of sugar for every 10 fluid ounces of the same tea. Whose tea is sweeter? your tea

Self-Assessment for Problem Solving

9. $26\frac{1}{4}$ tbsp

10. your friend's

Learning Target

Use ratio tables to represent equivalent ratios and solve ratio problems.

Success Criteria

- Use various operations to create tables of equivalent ratios.
- Use ratio tables to solve ratio problems.
- Use ratio tables to compare ratios.

Laurie's Notes

EXAMPLE 4

- There are different approaches rather than one correct approach for solving ratio problems using a ratio table or a double number line.
- **Teaching Tip:** You can model this problem by using red and blue food coloring. Fill two clear glasses with the same amount of water. Add 3 drops of red and 1 drop of blue into one glass. Stir. It will appear to be a shade of

"Are the shades of purple the same? Which is redder? Which is bluer?" Have students share their thinking as a class. Without a ratio table, many students will rely on their eyesight.

"How will you know that the frosting is redder?" Listen for understanding of the need to compare equal amounts of something—drops of red, drops of blue, or total amount of drops.

Self

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Closure

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Indicator 2g.ii - The Teaching Edition encourages teachers to ask probing questions to engage students in constructing arguments and analyzing the arguments of others. The notes for Example 4 offer teachers several questions to ask about the example to get students to construct arguments.

MP3 Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and — if there is a flaw in an argument — explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades.... Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

EXAMPLE 4 Modeling Real Life

You and your teacher make colored frosting. You add 3 drops of red food coloring for every 1 drop of blue food coloring. Your teacher adds 5 drops of red for every 3 drops of blue. Whose frosting is redder?

Understand the problem.

You are given the numbers of drops of food coloring that you and your teacher use to make frosting. You are asked to determine whose frosting is redder.

Make a plan.

Use ratio tables to compare the frostings. Find ratios in which the number of drops of red, the number of drops of blue, or the total number of drops is the same. Then compare the quantities to determine which is redder.

Solve and check.

Create ratio tables for 3 : 1 and 5 : 3 using repeated addition. Include a column for the total number of drops in each frosting.

Your Frosting		
Drops of Red	Drops of Blue	Total Drops
3	1	4
6	2	8
9	3	12
12	4	16
15	5	20

Your Teacher's Frosting		
Drops of Red	Drops of Blue	Total Drops
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40

When both frostings have 3 drops of blue, your frosting has $9 - 5 = 4$ more drops of red than your teacher's frosting.

So, your frosting is redder than your teacher's frosting.

Look Back
The tables show that when both frostings have a total of 16 drops, your frosting has 2 more drops of red and 2 fewer drops of blue. So, your frosting is redder. ✓

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

9. You mix 7 tablespoons of vinegar for every 4 tablespoons of baking soda to produce a chemical reaction. You use 15 tablespoons of baking soda. How much vinegar do you use?

10. You make a carbonated beverage by adding 7 ounces of soda water for every 3 ounces of regular water. Your friend uses 11 ounces of soda water for every 4 ounces of regular water. Whose beverage is more carbonated?

Laurie's Notes

EXAMPLE 3

- Ask a student to read the problem aloud. Ask how many T-shirts were purchased and refer students to the Common Error note.
- Ask what information is needed to find the total amount and write a verbal model. Then use the verbal model to write an expression.
- "Can the expression be simplified using properties?"
- Ask students to explain why $7x + 168.25$ is an equivalent expression and how the properties of addition helped solve this problem. Listen for students' understanding of the success criteria.
- **Extension:** "If the sponsor decides to purchase different T-shirts that cost $17.30 each, then what will the new total be?" $289.35 "How much more will the sponsor pay?" $19.60

Self-Assessment for Problem Solving

- Allow time in class for students to practice using the problem-solving plan. Students should work independently. Support students with probing questions and feedback. Encourage students who are struggling to begin by using a verbal model. Remember, some students may only be able to complete the first step.
- Remind students to think back to similar problems they have solved and ask, "How did I approach that problem? Will the same approach work for this problem?"
- As students complete the exercises, ask, "Do your answers seem reasonable? Is there a way to check your answers?"
- Writing an expression to represent a context is an essential skill for writing and solving equations.

The Success Criteria Self-Assessment chart can be found in the *Student Journal* or online at *BigIdeasMath.com*.

Closure

- **Four Square:** Have students place $3.5 + (y + 1.1)$ in the ovals of their *Four Squares*. Then have students label each square as one of the four categories: writing an equivalent expression, identifying properties, evaluating the expression, and application. Students may choose a reasonable value for evaluating the expression and create a context for the application. *Sample answer:*

Writing an Equivalent Expression $3.5 + (y + 1.1) = 3.5 + (1.1 + y)$ $= (3.5 + 1.1) + y$ $= 4.6 + y$	Identifying Properties Commutative Property of Addition; Associative Property of Addition; Add 3.5 and 1.1.
Evaluating the Expression Evaluate $3.5 + (y + 1.1)$ when $y = 4.4$. $3.5 + (y + 1.1) = 3.5 + (4.4 + 1.1)$ $= 3.5 + 5.5$ $= 9$	Application You purchase a stapler for $3.50, a binder for y dollars, and a pen for $1.10. Use an algebraic expression to find the total amount you paid when the binder costs $4.40. $9

$3.5 + (y + 1.1)$

Extra Example 3

You hand out 425 programs on the first night of your school's variety show, p programs on the second night, and 520 programs on the third night. Write an expression that represents the total number of programs you handed out. Then find the total number of programs that you handed out if you handed out 515 programs on the second night.
$p + 945$; 1460 programs

Self-Assessment for Problem Solving

12. $368.50 **13.** $21.21

Formative Assessment Tip

Four Square
A *Four Square* is often used as a study reference, however, it can be used to assess students' understanding of a concept. One way to use a *Four Square* is to write a problem in the oval and label each of the four squares surrounding the oval as a related category. Related categories may include: answer, meaning, application, algebra, numbers, model, graph, or equation. You can also ask students to illustrate each of the four depth of knowledge levels in the squares surrounding the problem. Consider hanging students' *Four Squares* around the classroom so they see a variety of ways to display information about a particular problem.

Learning Target

Identify equivalent expressions and apply properties to generate equivalent expressions.

Success Criteria

- Explain the meaning of equivalent expressions.
- Use properties of addition to generate equivalent expressions.
- Use properties of multiplication to generate equivalent expressions.

EXAMPLE 3 Modeling Real Life

You and six friends play on a basketball team. A sponsor paid $100 for the league fee, *x* dollars for each player's T-shirt, and $68.25 for basketballs. Write an expression that represents the total amount (in dollars) the sponsor paid. Then find the total amount paid when each T-shirt costs $14.50.

Use a verbal model to write an expression that represents the sum of the league fee, the cost of the T-shirts, and the cost of the basketballs. Then evaluate the expression when $x = 14.5$.

Common Error

You **and** six friends are on the team, so use 7, not 6, to represent the number of T-shirts.

League fee (dollars)	+	Number of T-shirts	•	Cost per T-shirt (dollars)	+	Cost of basketballs (dollars)
$100		7		x		$68.25

$$100 + 7x + 68.25 = 7x + 100 + 68.25$$ Commutative Property of Addition

$$= 7x + (100 + 68.25)$$ Associative Property of Addition

$$= 7x + 168.25$$ Add 100 and 68.25.

Evaluate $7x + 168.25$ when $x = 14.5$.

$$7x + 168.25 = 7(14.5) + 168.25 = 101.5 + 168.25 = 269.75$$

An expression that represents the total amount (in dollars) is $7x + 168.25$. When each T-shirt costs $14.50, the sponsor pays $269.75.

Self-Assessment for Problem Solving

Solve each exercise. Then rate your understanding of the success criteria in your journal.

12. You and five friends form a team for an outdoor adventure race. Your team needs to raise money to pay for $130 of travel fees, *x* dollars for each team member's entry fee, and $85.50 for food. Use an algebraic expression to find the total amount your team needs to raise when the entry fee is $25.50 per person.

13. You have $50 and a $15 gift card to spend online. You purchase a pair of headphones for $34.99 and 8 songs for *x* dollars each. Use an algebraic expression to find the amount you have left when each song costs $1.10.